
PHYS258

Waves and Related Phenomena

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Introduction

PHYS258 Waves and Related Phenomena (WARP)

Semester: 1 Level: 2 Credits: 15

Lectures: ~24 Tutorials: 4 Assignments: 3

Assessment: Examination = 70 % + Assignments = 3 x 10%

Aims

- To build on material presented in Year 1 modules (especially PHYS126)
- To introduce the use of waves in a wide range of physics
- To develop the concepts of interference and diffraction

Learning Outcomes

- Familiarity with waves and their analysis using complex number notation
- Knowledge of interference and diffraction effects and their use in physics
- Acquired an introduction to the ideas of Fourier techniques
- Acquired an introduction to the basic principles of lasers

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Timetable

Week	1	2	3	4	5	6	7	8	9	10	11	12	
Lectures	L1 L24											R	
Tutorials	T1 X		T2	X		T3	X		T4	X			
Assignments				A1			A2			A3			
Week	1	2	3	4	5	6	7	8	9	10	11	12	

X Tutorials 1–4 will be in weeks 1, 4, 7 and 10

Assignments 1–3 will be handed in by the end of weeks 5, 8 and 11

Syllabus

Fundamentals

- Wave Equation
- Phase Velocity
- Superposition (same λ)
- Standing Waves
- Sound Waves

Electromagnetic Waves

- EM Waves in Free Space
- EM Waves in Dielectrics
- Polarisation of EM Waves
- Reflection of EM Waves

Boundary Conditions

- Waves At Boundaries
- Impedance
- Waves in Cables

Superposition (different λ)

- Beats
- Wavepackets
- Bandwidth Theorem
- Group Velocity

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Interference

Young's Slits
Lloyd's Mirror
Optical Coatings
Interference Filters

Fourier Methods

Fourier Analysis
Fourier Transforms
Diffraction and FTs

Diffraction

Fraunhofer Diffraction
Single-Slit Diffraction
Rayleigh Criterion
Multiple-Slit Diffraction

Lasers

Principles
Applications

The screenshot shows a Firefox browser window with the address bar displaying <http://www.lon-capa.org/~mmp/applist/applets.htm>. The page content is titled "The Applet Collection" and includes the following text:

These applets enable you to study simple physical systems in a playful way. They are idealizations of realistic scenarios, which follow the proper equations that also govern the real experimental systems.

Here is also a handy calculator (will open in a separate window):

Calculator

Kinematics

- Significant Figures**
Try some exercises with significant figures on your own.
- GraphLab**
Playing around with the parameters (x_0 , y_0 , and a) of the simple kinematic equations for motion in 1D with constant acceleration.
- Vector Addition**
Practice with the concepts of graphical vector addition and subtraction.
- Projectile Simulation**
Shoot a cannon after selecting the initial speed and angle of the projectile.

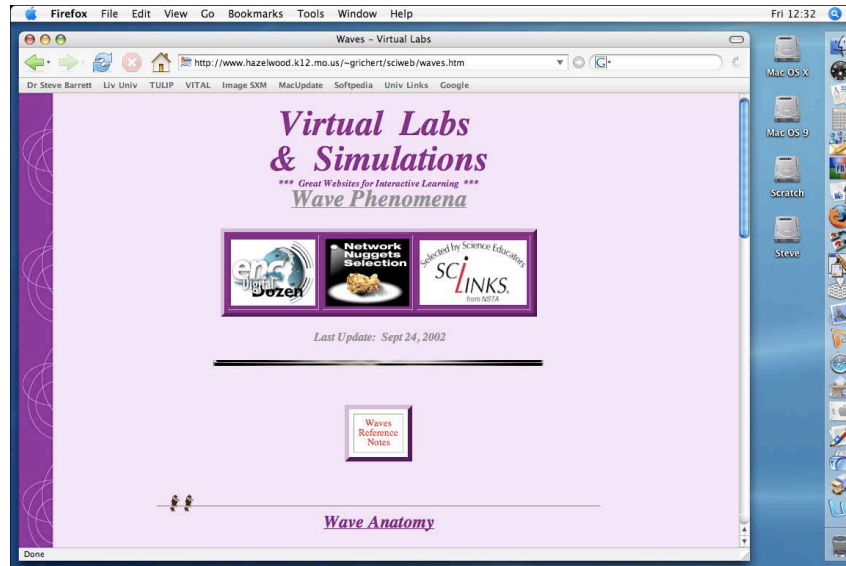
Dynamics

- Net Force**
See how the components of three forces add up to the net force.
- Simple Incline Plane**
Box on an incline - one of the basic physics problems solved.
- Inclined Plane**
Experimentally solve a very complicated inclined plane problem with friction.
- Atwood Machine**
Two masses, a rope, and a pulley.
- Force and Work**
Work you have to do against the forces of gravity and/or friction.
- Virtual Billiards**
Two dimensional collision between two objects of equal mass.

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<http://www.hazelwood.k12.mo.us/~grichert/sciweb/waves.htm>

Java Applets

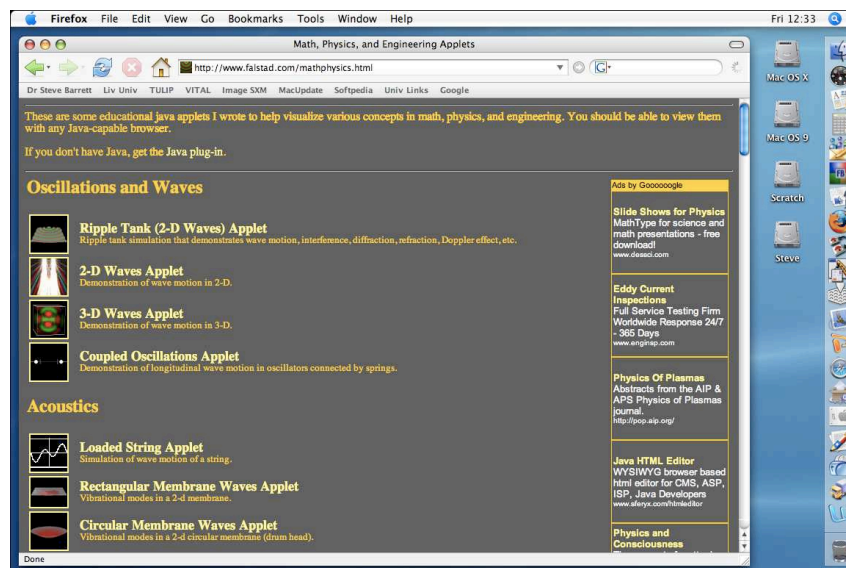


PHYS258 Java Applets

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<http://www.falstad.com/mathphysics.html>

Java Applets



PHYS258 Java Applets

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http://www.ub.es/javaoptics/index-en.html

Java Applets

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Describing Waves

A moving wave must be described as a function of space and time that gives the amplitude at each point for each moment in time

$$\psi(x, t)$$

where ψ is the of the wave.

If the wave is travelling at speed v and the time is t then the wave has travelled a distance of vt . So at a later time t the function that describes the wave is

$$x' = x - vt$$

and

$$\psi(x, t) =$$

This is the most general form of a travelling wave function.

Harmonic Waves

Harmonic waves have sine (or cosine) wave functions

$$\psi(x,t)|_{t=0} = f(x) = A\sin(kx)$$

kx is in radians ($1^\circ = 2\pi/360$ radians)
 k [radians/m]

Maximum and minimum values of ψ are
 A of the wave

If the wave is moving with speed v in the $+x$ direction then the equation becomes

$$\psi(x,t) = f(x - vt) = \tag{1.1}$$

Another way to write the wave function is

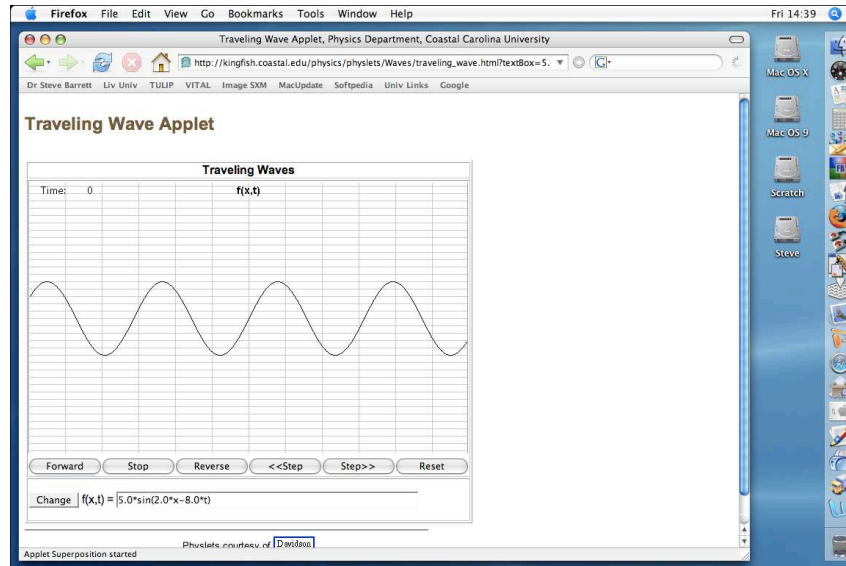
$$\psi(x,t) = A\sin(kx \pm \omega t) \tag{1.2}$$

k = propagation number [radians m^{-1}]
 λ = wavelength = $2\pi/k$ [m]
 κ = $1/\lambda$ = [m $^{-1}$] (spectroscopy)
 ω = $k v$ = = $2\pi f$ [radians s^{-1}]
 v_p = ω/k = [m s^{-1}]
 ϕ = $kx - \omega t$ = of the wave [radians]

Note that if ϕ is constant, so ψ is constant, and hence v_p is the velocity of points of

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http://kingfish.coastal.edu/physics/physlets/Waves/traveling_wave.html?textBox= **Java Applet**



PHYS258 Fundamentals / Travelling Waves / Harmonic Waves

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The Wave Equation

The wave function that we have been using obeys the **wave equation**



[1.3]

Show that [1.2] is a solution of the wave equation [1.3].

The wave function $\psi(x,t) = A\sin(kx \pm \omega t)$ has a value of $\psi=0$ at $x=0$ and $t=0$. This is not always the case, and the general expression for a harmonic solution to the wave equation is

where ε is the phase at $x=0$ and $t=0$. Note that \sin and \cos functions differ only in the selection of the value of ε .

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Principle of Superposition

The **Principle of Superposition** states that the sum of any two solutions to the wave equation is another solution to the wave equation.

This is a result of the linear behaviour of the differentials in the wave equation [1.3]. For any two functions f and g

and similarly for the differential with respect to t . Thus, if f and g both solve the wave equation then so does $(f + g)$.

This means that waveforms can be built by adding many wave functions — a result that will be very important later on.

Complex Notation

The Euler formula is



and so another way of writing a travelling wave function is

$$\psi(x, t) = A \cdot \Re \left\{ e^{i(kx \pm \omega t + \varepsilon)} \right\} = A \cos(kx \pm \omega t + \varepsilon)$$

Where $\Re \{ \dots \}$ or $\text{Re} \{ \dots \}$ means the real part of $\{ \dots \}$. Note that this symbol is not usually written explicitly but is implied.

Why Use Complex Numbers?

Waves have an amplitude (A) and a phase (ϕ). So do complex numbers, and so there is a natural tendency (though not a necessity) to use complex numbers to describe waves.

A reminder about complex numbers:

Cartesian form

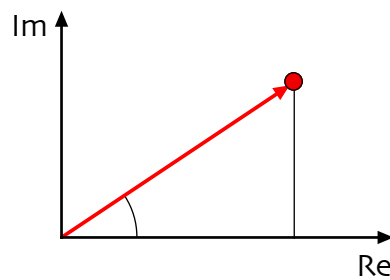
Polar form

where

$a = \text{real part}$
 $b = \text{imaginary part}$

$r^2 = a^2 + b^2$
 $\tan \phi = b/a$

The algebra of complex numbers mirrors the waves that it describes. This makes calculations much simpler.



Waves in 3d

If points with the same phase form a (2d) plane in (3d) space then the wave is called a plane wave.

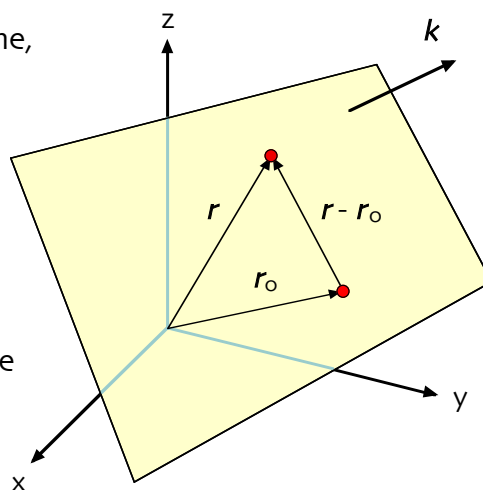
Consider a vector k and a plane, perpendicular to k , passing through the point r_0 .

For all points in the plane

$$\Rightarrow k \cdot r = k \cdot r_0 =$$

So we can define a plane wave

$$\psi(r) =$$



The time dependence for a 3d wave is just the same as we have seen for a 1d wave.

$$\psi(\mathbf{r}) =$$

The planes of constant phase ϕ are called wavefronts , which move with the phase velocity v_p .

Superposition of Waves

When two waves occupy the same space the wave functions add to create a new function. From the $\text{principle of superposition}$ we know that this function is a solution to the wave equation and so also represents a wave.

At this point we will look at the superposition of harmonic waves of the same wavelength and frequency. In a later lecture, we will look at the result of the superposition of waves of different wavelength and frequency.

Start with two waves ψ_1 and ψ_2 that add to make ψ_T

$$\psi_T(x, t) =$$

Writing $\omega_1 = \omega_2 = \omega$ and for brevity

$$\begin{aligned}\psi_T(x,t) &= A_1 \sin(\alpha_1 + \omega t) + A_2 \sin(\alpha_2 + \omega t) \\ &= A_1 (\sin \alpha_1 \cos \omega t + \cos \alpha_1 \sin \omega t) + \\ &\quad A_2 \\ &= \sin \omega t (A_1 \cos \alpha_1 + A_2 \cos \alpha_2) + \\ &\quad \cos \omega t\end{aligned}\tag{1.4}$$

We can simplify this if we rewrite the expressions in brackets in [1.4] as simple cos and sin terms

$$A_T \cos \alpha = A_1 \cos \alpha_1 + A_2 \cos \alpha_2\tag{1.5}$$

$$A_T \sin \alpha =\tag{1.6}$$

Taking [1.5]² + [1.6]²

$$A_T^2 = A_1^2 + A_2^2 + 2A_1A_2 \cos(\alpha_2 - \alpha_1)\tag{1.7}$$

Dividing [1.6] by [1.5]

$$\tan \alpha =$$

Substituting [1.5] and [1.6] into [1.4]

$$\begin{aligned}\psi_T(x,t) &= A_T \cos \alpha \sin \omega t + A_T \sin \alpha \cos \omega t \\ &= \end{aligned}$$

Thus, the resultant wave is wave.

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Looking at [1.7] for the amplitude of the wave resulting from the superposition of two waves of amplitude A_1 and A_2 , when the difference $\alpha_2 - \alpha_1 = 0$ (or an integer multiple of 2π) the amplitude is a

$$A_T^2 = A_1^2 + A_2^2 +$$

$$A_T^2 = A_1^2 + A_2^2 + 2A_1A_2 =$$

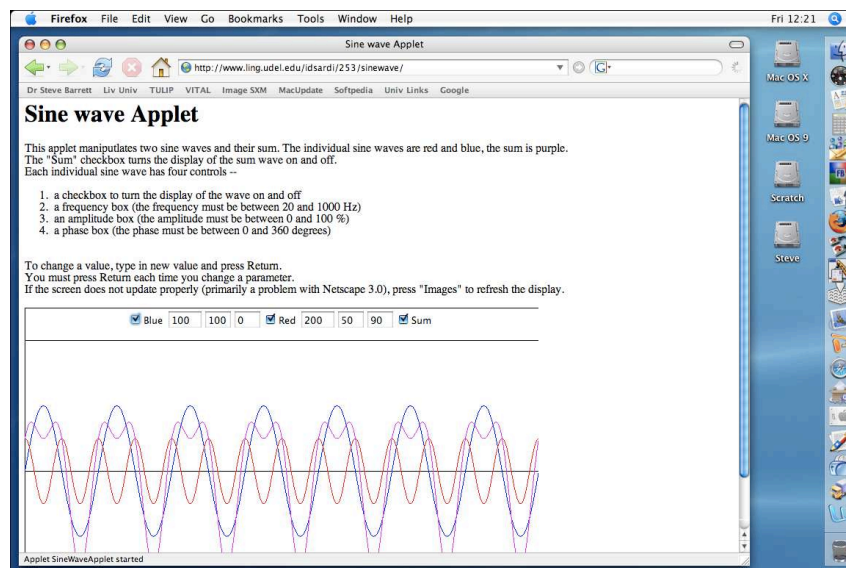
Similarly, the amplitude of the resultant wave is a when the difference $\alpha_2 - \alpha_1 = (2n+1)\pi$ where n is an integer

$$A_T^2 = A_1^2 + A_2^2 + 2A_1A_2 \cos(\pi)$$

$$A_T^2 = A_1^2 + A_2^2 - 2A_1A_2 =$$

<http://www.ling.udel.edu/idsardi/253/sinewave/>

Java Applet



Standing Waves

If two waves of the same wavelength and frequency, but propagating in opposite directions, are added the result is a standing wave.

This can happen when, for example, a wave meets its reflection (we will deal with wave reflection later).

Take two harmonic waves travelling in the opposite directions

$$\psi(x,t) = A_R \sin(kx - \omega t + \epsilon_R) + A_L \sin(kx + \omega t + \epsilon_L) \quad [1.8]$$

To simplify the solution, let us take $A_R = A_L = A$ and $\epsilon_R = \epsilon_L = 0$

$$\psi(x,t) = A\{\sin(kx - \omega t) + \sin(kx + \omega t)\}$$

Use the trig identity

$$\sin A + \sin B = 2 \sin\left(\frac{A+B}{2}\right) \cos\left(\frac{A-B}{2}\right)$$

to write the wave function as

$$\psi(x,t) = 2A \cos(\omega t) \sin(kx)$$

This equation describes a standing wave.

Note that the x and t variables are now in separate factors of the expression (there are no x or t terms) and so, although the wave is still a function of space and time, it is stationary through space.

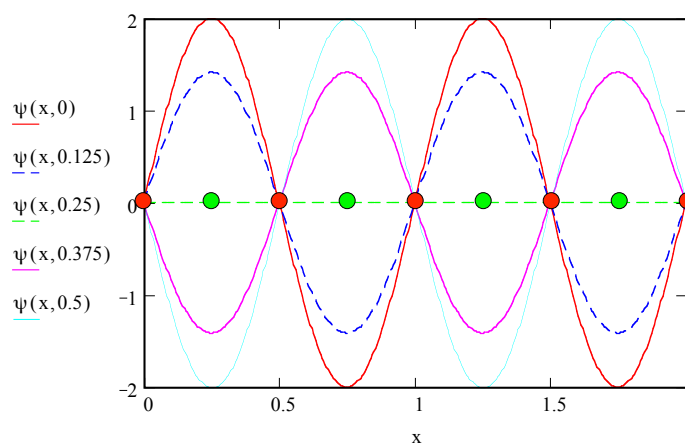
Note also that the amplitude of the wave is now a function of the position in x .

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http://www.walter-fendt.de/ph14e/stwaverefl.htm Java Applet

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By watching a standing wave evolve with time, the positions of nodes ● (where $\psi = 0$ at all times) and antinodes ● (where the amplitude is a maximum) can be identified.



Mixed Waves

If the amplitudes of the two waves are the same ($A_R = A_L$) then the resultant wave is a combination of a travelling wave and a standing wave.

Rewriting [1.8] with $R = A_R/A_L$ (where $0 \leq R \leq 1$)

$$\psi(x, t) = A_L \{ \sin(kx + \omega t) + R \sin(kx - \omega t) \}$$

$$\psi(x, t) = A_L \{ (1 - R) \sin(kx + \omega t) +$$

$$\psi(x, t) = A_L (1 - R) \sin(kx + \omega t) + 2RA_L \sin(kx) \cos(\omega t)$$

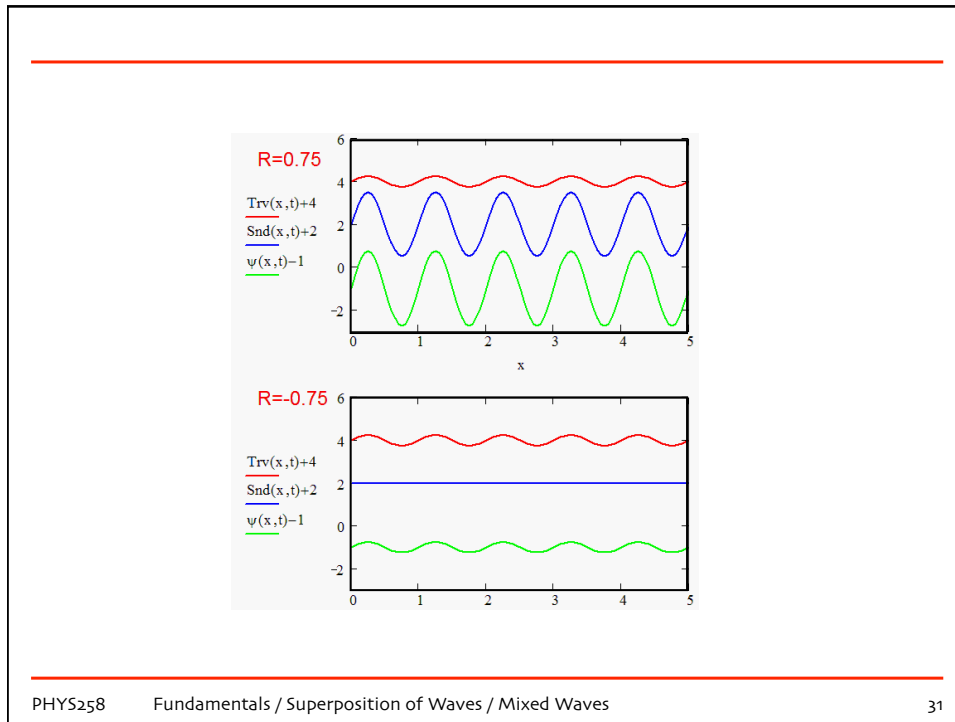
Consider three distinct cases for possible values of R :

No reflection wave

Some reflection Travelling + Standing

Perfect reflection wave

Note that if $R < 0$ (negative reflection coefficients will be covered later) the result is still a mix of travelling and standing waves, but the positions of the nodes and antinodes of the standing wave component will be interchanged.



Standing Wave Ratio

For any value of R the maxima and minima in the amplitudes will occur at fixed locations (with a separation between neighbouring maxima, and similarly for neighbouring minima).

Measuring the intensity of standing waves as a function of position is a means of determining wavelength — it is used in PHYS378/478 Advanced Practical Physics with microwaves.

The SWR is defined as

$$SWR = \frac{\text{Max. amplitude}}{\text{Min. amplitude}} = \frac{A_L + A_R}{A_L - A_R}$$

If $R = 0 \Rightarrow SWR =$

If $|R| = 1 \Rightarrow SWR =$

Waves in Elastic Media

Many materials can be distorted by application of a force. Materials are often elastic under small distortions — an object is restored to its original shape when the force is removed. Forces above the elastic limit cause permanent deformation.

The general theory of elastic materials (continuum mechanics) is complicated because in many materials the elastic properties depend on the material's history.

Here we will consider only the basic concepts that allow us to understand the propagation of waves in solids, liquids and gases.

Some important definitions:

Stress

$$T = \lim_{Area \rightarrow 0} \frac{F}{Area} \quad [Nm^{-2}]$$

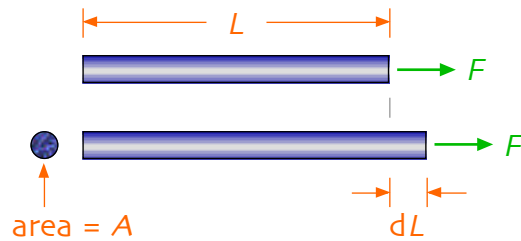
Strain

If parts of a body can be moved relative to each other then the body is said to be in a state of strain. The amount of strain depends on the variation of displacement ψ with distance x .

The propagation of waves is dependent on how stress and strain are related in any given medium.

Young's Modulus

$$Y = \frac{\text{stress}}{\text{strain}} \quad [\text{Nm}^{-2}]$$



If the material being stressed and strained is a wire, then

For **example**, steel has $Y \sim$ (cf plastics $Y \sim$, diamond $Y =$)

Bulk Modulus

$$B = \quad [\text{Nm}^{-2}]$$

where $\Delta V/V$ is the fractional change in volume under the applied pressure P .

Compressibility

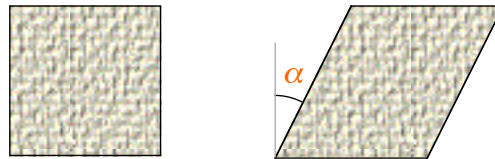
$$K = 1/B = \quad [\text{N}^{-1}\text{m}^2]$$

Water has $K \sim$ $\text{N}^{-1}\text{m}^2 \Rightarrow$

Shear Modulus

$$\mu = \quad [Nm^{-2}]$$

where T is the applied stress and α is the angle that defines the deformation.



Note that the area used in calculating the stress is the area over which the force is applied (ie, the 'top' of the block).

For liquids and gases, .

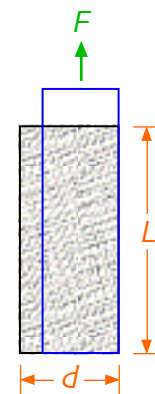
Poisson's Ratio

This is a measure of how much a body deforms in the direction to the direction of stress.

$$\sigma =$$

Note that for isotropic materials, Y , B , μ and σ are all inter-related.

For instance, $B = \frac{Y}{3(1-2\sigma)}$ $\mu = \frac{Y}{2(1+\sigma)}$



Wave Propagation

The basic principle behind wave propagation in materials is that if there is a $\frac{\partial \psi}{\partial x}$ in a material then the material will

$$F \propto \frac{\partial}{\partial x} \left(\frac{\partial \psi}{\partial x} \right) =$$

where ψ is the displacement of the material.

The force F produces an acceleration which depends on the density of the material ρ

$$F =$$

and so

$$\rho \frac{\partial^2 \psi}{\partial t^2} \propto \quad (\text{the wave equation})$$

Longitudinal Waves

By comparison with the wave equation introduced in Lecture 1, we see that the speed of propagation of waves depends on the $\frac{B+4\mu/3}{\rho}$ of the material and on the constant of proportionality between the $\frac{Y(1-\sigma)}{\rho(1+\sigma)(1-2\sigma)}$ and the resultant v_p generated.

A full derivation $v_p = \sqrt{\frac{B+4\mu/3}{\rho}}$ gives (no proof here)

$$v_p^2 = \frac{B+4\mu/3}{\rho} \quad [1.9]$$

or, using the expressions relating B, μ, Y and σ

$$v_p^2 = \frac{Y(1-\sigma)}{\rho(1+\sigma)(1-2\sigma)}$$

Noting that the values of σ for solid materials is typically ~ 0.25 we can make the approximation

$$v_p^2 \approx \frac{6Y}{5\rho} \quad \text{or} \quad v_p \approx \sqrt{\frac{Y}{\rho}}$$

for the phase velocity of () waves travelling through solid media.

Example

Steel has $Y \sim$

$\rho \sim$

$\Rightarrow v_p =$

Transverse Waves

A derivation for () gives (no proof here)

$$v_p^2 = \frac{\mu}{\rho} = \frac{Y}{2\rho(1+\sigma)}$$

Taking $\sigma \sim 0.25$ we can make the approximation

$$v_p^2 \approx \frac{2Y}{5\rho} \quad \text{or} \quad v_p \approx 0.6 \sqrt{\frac{Y}{\rho}}$$

Remembering that the shear modulus $\mu = 0$ for liquids and gases, these expressions for the phase velocity for () waves show that these waves cannot propagate in such materials.

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<http://www.mta.ca/faculty/science/physics/suren/Lwave/Lwave01.html>

Java Applet

Longitudinal Waves

Progressive Wave

Amplitude Frequency Animation Stop

Waves are vibratory disturbances travelling in a medium. If you understand that sentence, you have understood wave motion. Shown above are vibrating layers of air in a horizontal tube. The layer at the extreme left is the one disturbed earliest. You could imagine a vibrating piston at the left end causing these disturbances. So the layer at the left extreme is always ahead in phase of the layers to its right. The time lag between the moments when different layers begin oscillating causes the layers to come closer to each other at some moments and move farther at some other moments. To get a feel of this, imagine what would have happened if all the layers vibrated in phase. You would always have layers maintaining the same distance between themselves and you would not have any wave disturbance traveling in the medium. The key to understanding of wave motion is this simple idea that phase is different for the different vibrating particles because they start vibrating at different times. The ones farther from the source oscillating later than the ones closer to the source.

Applet Lwave01 started

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<http://surendranath.tripod.com/Applets/Waves/Twave01/Twave01Applet.html>

Java Applet

Transverse Waves

Progressive Wave

Amplitude Frequency Start Stop

Check this Applet too

As the applet starts, you see a vibration which begins at the left end traveling towards right. If you observe any particle you would find it vibrating up and down. You would say to yourself, something is moving to the right here, it is not the particles, they are only vibrating up and down - it is a shape or a vibration that is moving to the right - being passed on from particle to particle. That is what wave motion is - a moving vibration or a moving shape.

Why does a vibration move, get passed on - It is because particles are not independent. If you do something to one particle, its neighbour reacts. Here each neighbour behaves the same way as its predecessor, but with a time lag. If all the particles were rigidly bound, then they would all vibrate together - there is no passing on - no shape moving - no wave motion, because they all vibrate simultaneously. And if all the particles were independent, then again a vibration produced will be confined to the particle that is disturbed. There is no wave motion. That should define a wave - A vibratory disturbance propagating in an elastic medium.

Applet Applets.Waves.Twave01.Twave01Applet started

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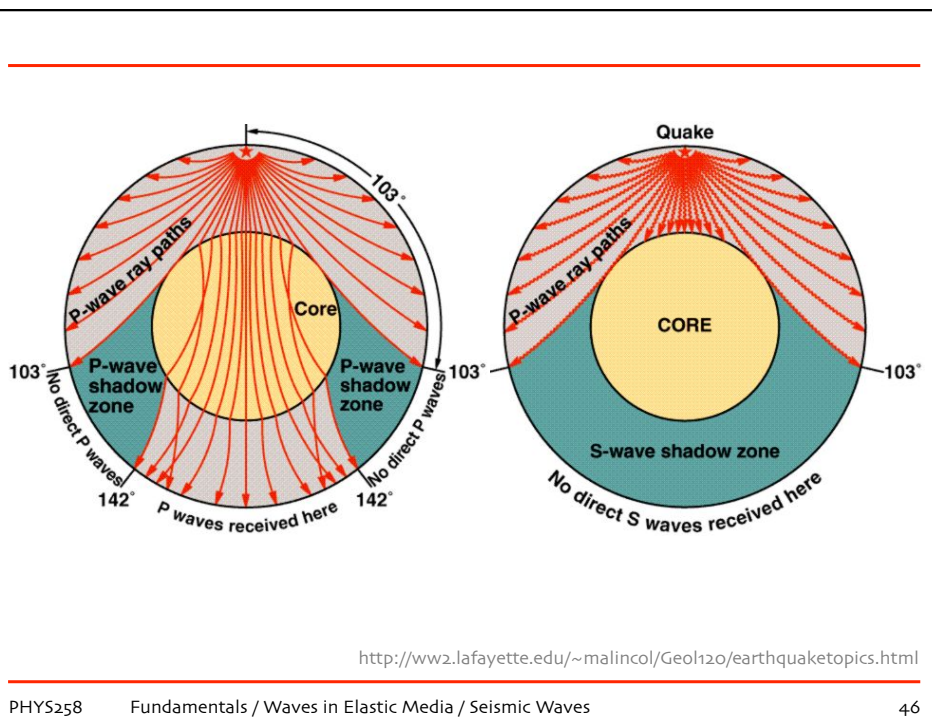
Seismic Waves

The ratio of the phase velocities for (or Pressure or Primary) waves and (or Shear or Secondary) waves in solids is

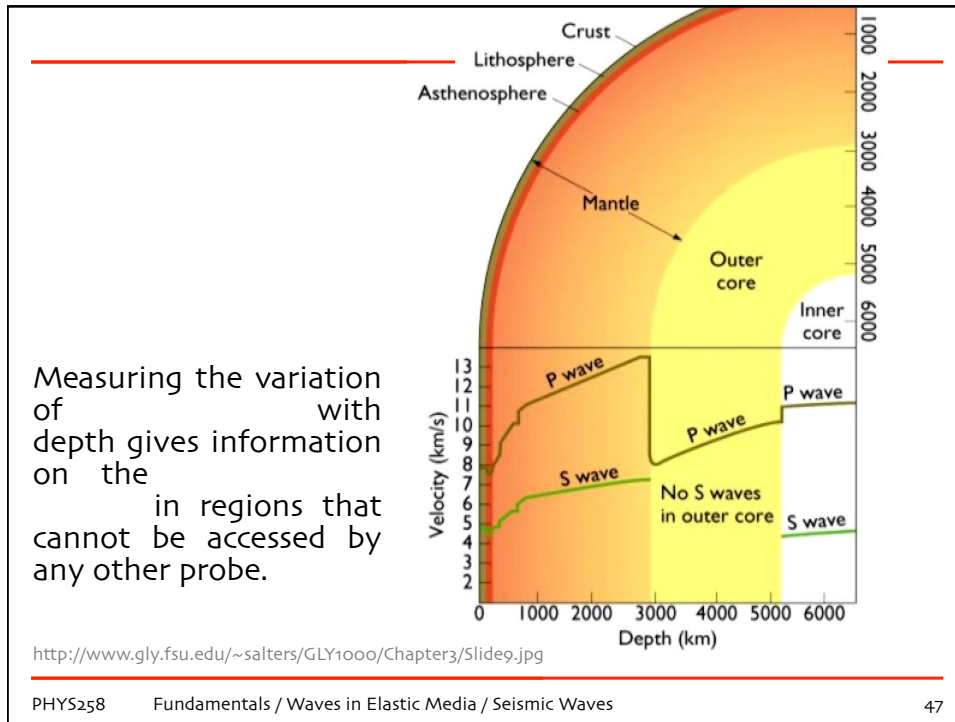
$$\frac{v_p \text{ for P waves}}{v_p \text{ for S waves}} \approx$$

This velocity difference between P and S waves is crucial to our understanding of the interior of the Earth, as it enables seismologists to monitor earthquakes and use differential timings of the arrival of the two types of seismic waves at various locations on the Earth's surface to refine models of the Earth's inner structure.

Two types of surface wave, having displacement components to and to the Earth's surface, also have different velocities.



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waves

An earthquake can produce two types of surface wave.

An S wave which has only movement of the ground is called a Love wave.

The other kind of surface wave is the Rayleigh wave, which is similar to waves rolling across the ocean. The ground moves vertically and horizontally (in the direction that the wave is propagating). Most of the shaking felt from an earthquake is due to the Rayleigh wave.

waves

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Java Applet

http://www.ifg.tu-clausthal.de/java/seis/sdem_app-e.html

Geophysics Department TU Clausthal

The Seismometer Demo Applet

The applet simulates a seismometer's reaction to an adjustable ground motion $w(t)$. The displacement $x(t)$ of the movable mass relative to the case is calculated recursively from the case (=ground) displacement $w(t)$, based on the second order differential equation :

$$m \cdot x''(t) + d \cdot x'(t) + c \cdot x(t) = 0$$

(See [Seismometer Documentation](#) for physical / mathematical background)

APPLET :

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Waves in Liquids

For waves in liquids (but not a liquid) we use [1.9] with

$$v_p^2 = \frac{B}{\rho} = \frac{1}{K\rho}$$

For water we find, taking the value for K given earlier,

$$v_p \approx$$

This is ≈ 1500 m/s than that found for a solid like steel, but still substantially less than the speed of sound in air (next slide) because the forces that try to restore the medium after a deformation takes place are much smaller in a liquid than in a gas.

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Waves in Gases

For gases, the expression for the phase velocity can be written

$$v_p^2 =$$

where

- γ ratio of specific heats (c_p/c_v) = 1.4 for air
- R gas constant = for all gases
- T temperature in Kelvin = 300 K
- M molar mass = for air

$$\Rightarrow v_p \approx \text{for air}$$

The phase velocity of waves propagating through any elastic medium can be written, rather generally, as

$$v_p^2 \sim \frac{\text{"Stiffness" of medium}}{\text{"Mass" of medium}}$$

For a **solid** $v_p^2 \sim$

For a **liquid** $v_p^2 \sim$