PHYS258

Waves and Related Phenomena

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Introduction

PHYS258 Waves and Related Phenomena (WARP)

Semester: 1	Level: 2	Credits: 15
Lectures: ~24	Tutorials: 4	Assignments: 3

Assessment: Examination = 70 % + Assignments = 3 x 10%

Aims

- To build on material presented in Year 1 modules (especially PHYS126)
- To introduce the use of waves in a wide range of physics
- To develop the concepts of interference and diffraction

Learning Outcomes

- Familiarity with waves and their analysis using complex number notation
- Knowledge of interference and diffraction effects and their use in physics
- Acquired an introduction to the ideas of Fourier techniques
- Acquired an introduction to the basic principles of lasers

Timetable

Week	1	2	3	4	5	6	7	8	9	10	11	12
Lectures	L1							L24				R
Tutorials	T1 X		T2	X		T3	X		T4	X		
Assignments				A1			A2			A3		
Week	1	2	3	4	5	6	7	8	9	10	11	12

X Tutorials 1–4 will be in weeks 1, 4, 7 and 10

Assignments 1–3 will be handed in by the end of weeks 5, 8 and 11

Syllabus

Fundamentals
- Wave Equation
- Phase Velocity
- Superposition (same λ)
- Standing Waves
- Sound Waves

Boundary Conditions
- Waves At Boundaries
- Impedance
- Waves in Cables

Electromagnetic Waves
- EM Waves in Free Space
- EM Waves in Dielectrics
- Polarisation of EM Waves
- Reflection of EM Waves

Superposition (different λ)
- Beats
- Wavepackets
- Bandwidth Theorem
- Group Velocity

Interference
- Young's Slits
- Lloyd's Mirror
- Optical Coatings
- Interference Filters

Diffraction
- Fraunhofer Diffraction
- Single-Slit Diffraction
- Rayleigh Criterion
- Multiple-Slit Diffraction

Fourier Methods
- Fourier Analysis
- Fourier Transforms
- Diffraction and FTs

Lasers
- Principles
- Applications

Java Applets

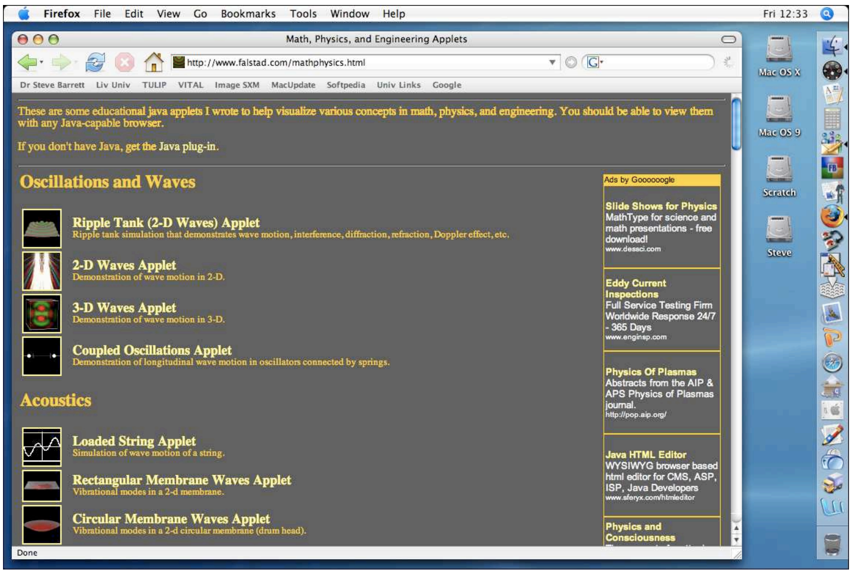
http://www.lon-capa.org/~mmp/applist/applets.htm

http://www.hazelwood.k12.mo.us/~grichert/sciweb/waves.htm

Java Applets

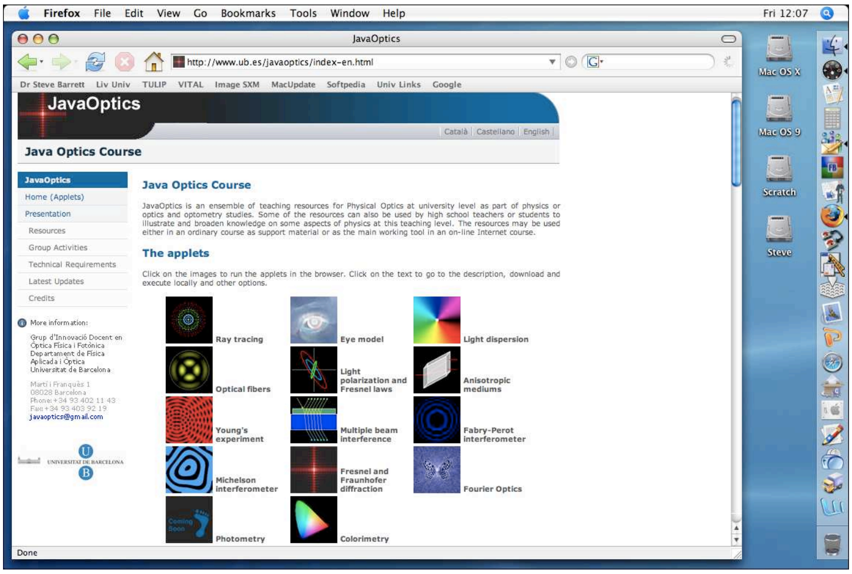
http://www.falstad.com/mathphysics.html

Java Applets



http://www.ub.es/javaoptics/index-en.html

Java Applets



Describing Waves

A moving wave must be described as a function of space and time that gives the amplitude at each point for each moment in time

$$\psi(x,t)$$

where ψ is the   of the wave.

If the wave is travelling at speed v and the time is t then the wave has travelled a distance of vt. So at a later time t the function that describes the wave is

$$x' = x - vt$$

and

$$\psi(x,t) =$$

This is the most general form of a travelling wave function.

Harmonic Waves

Harmonic waves have sine (or cosine) wave functions

$$\psi(x,t)\big|_{t=0} = f(x) = A\sin(kx)$$

kx is in radians (1° = 2π/360 radians)

k [radians/m]

Maximum and minimum values of ψ are

A of the wave

If the wave is moving with speed v in the $+x$ direction then the equation becomes

$$\psi(x,t) = f(x - vt) = \qquad [1.1]$$

Another way to write the wave function is

$$\psi(x,t) = A\sin(kx \pm \omega t) \qquad [1.2]$$

k = propagation number [radians m^{-1}]

λ = wavelength = $2\pi/k$ [m]

κ = $1/\lambda$ = [m^{-1}] (spectroscopy)

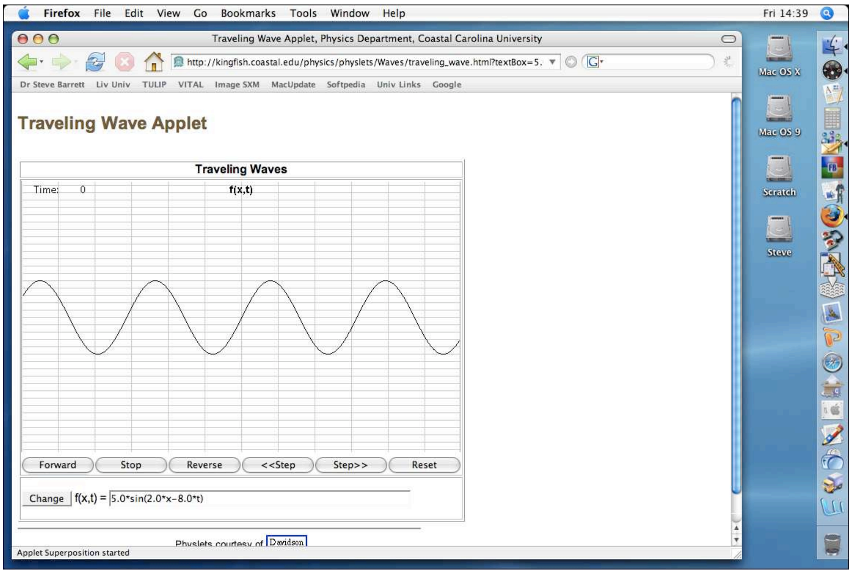
ω = kv = = $2\pi f$ [radians s^{-1}]

v_p = ω/k = [m s^{-1}]

ϕ = $kx - \omega t$ = of the wave [radians]

Note that if ϕ is constant, so ψ is constant, and hence v_p is the velocity of points of http://kingfish.coastal.edu/physics/physlets/Waves/traveling_wave.html?textBox=

Java Applet



The Wave Equation

The wave function that we have been using obeys the wave equation

[1.3]

Show that [1.2] is a solution of the wave equation [1.3].

The wave function $\psi(x,t) = A\sin(kx \pm \omega t)$ has a value of ψ=0 at x=0 and t=0. This is not always the case, and the general expression for a harmonic solution to the wave equation is

where ε is the phase at x=0 and t=0. Note that sin and cos functions differ only in the selection of the value of ε.

Principle of Superposition

The Principle of Superposition states that the sum of any two solutions to the wave equation is another solution to the wave equation.

This is a result of the linear behaviour of the differentials in the wave equation [1.3]. For any two functions f and g

and similarly for the differential with respect to t. Thus, if f and g both solve the wave equation then so does $(f+g)$.

This means that waveforms can be built by adding many wave functions — a result that will be very important later on.

Complex Notation

The Euler formula is

and so another way of writing a travelling wave function is

$$\psi(x,t) = A \cdot \Re\left\{e^{i(kx \pm \omega t + \varepsilon)}\right\} = A\cos(kx \pm \omega t + \varepsilon)$$

Where $\Re\{\ldots\}$ or $\mathrm{Re}\{\ldots\}$ means the of $\{\ldots\}$. Note that this symbol is not usually written explicitly but is implied.

Why Use Complex Numbers?

Waves have an amplitude (A) and a phase (ϕ). So do complex numbers, and so there is a natural tendency (though not a necessity) to use complex numbers to describe waves.

A reminder about complex numbers:

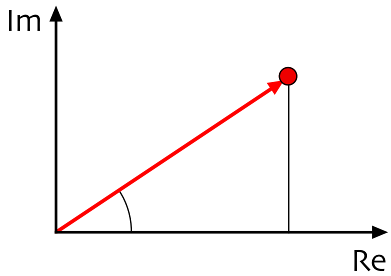
Cartesian form Polar form

where

a = real part $r^2 = a^2+b^2$

b = imaginary part $\tan\phi = b/a$

The algebra of complex numbers mirrors the waves that it describes. This makes calculations much simpler.



Waves in 3d

If points with the same phase form a (2d) plane in (3d) space then the wave is called a .

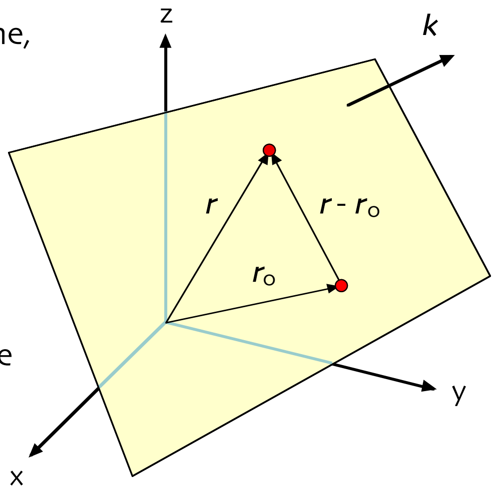
Consider a vector $\boldsymbol{k}$ and a plane, perpendicular to $\boldsymbol{k}$, passing through the point $\boldsymbol{r}_o$.

For all points in the plane

$$\Rightarrow \quad \boldsymbol{k}\cdot\boldsymbol{r} = \boldsymbol{k}\cdot\boldsymbol{r}_o =$$

So we can define a plane wave

$$\psi(\boldsymbol{r}) =$$



The time dependence for a 3d wave is just the same as we have seen for a 1d wave.

$$\psi(r) =$$

The planes of constant phase ϕ are called , which move with the phase velocity v_p.

Superposition of Waves

When two waves occupy the same space the wave functions add to create a new function. From the we know that this function is a solution to the wave equation and so also represents a wave.

At this point we will look at the superposition of harmonic waves of the wavelength and frequency. In a later lecture, we will look at the result of the superposition of waves of wavelength and frequency.

Start with two waves ψ_1 and ψ_2 that add to make ψ_T

$$\psi_T(x,t) =$$

Writing $\omega_1 = \omega_2 = \omega$ and for brevity

$$\psi_T(x,t) = A_1 \sin(\alpha_1 + \omega t) + A_2 \sin(\alpha_2 + \omega t)$$

$$= A_1(\sin\alpha_1 \cos\omega t + \cos\alpha_1 \sin\omega t) +$$
$$A_2$$

$$= \sin\omega t\,(A_1 \cos\alpha_1 + A_2 \cos\alpha_2) +$$
$$\cos\omega t \qquad [1.4]$$

We can simplify this if we rewrite the expressions in brackets in [1.4] as simple cos and sin terms

$$A_T \cos\alpha = A_1 \cos\alpha_1 + A_2 \cos\alpha_2 \qquad [1.5]$$
$$A_T \sin\alpha = \qquad [1.6]$$

Taking [1.5]² + [1.6]²

$$A_T{}^2 = A_1{}^2 + A_2{}^2 + 2A_1A_2\cos(\alpha_2 - \alpha_1) \qquad [1.7]$$

Dividing [1.6] by [1.5]

$$\tan\alpha =$$

Substituting [1.5] and [1.6] into [1.4]

$$\psi_T(x,t) = A_T \cos\alpha \sin\omega t + A_T \sin\alpha \cos\omega t$$
$$=$$

Thus, the resultant wave is wave.

Looking at [1.7] for the amplitude of the wave resulting from the superposition of two waves of amplitude A_1 and A_2, when the difference $\alpha_2 - \alpha_1 = 0$ (or an integer multiple of 2π) the amplitude is a

$$A_T^{\,2} = A_1^{\,2} + A_2^{\,2} +$$

$$A_T^{\,2} = A_1^{\,2} + A_2^{\,2} + 2A_1A_2 =$$

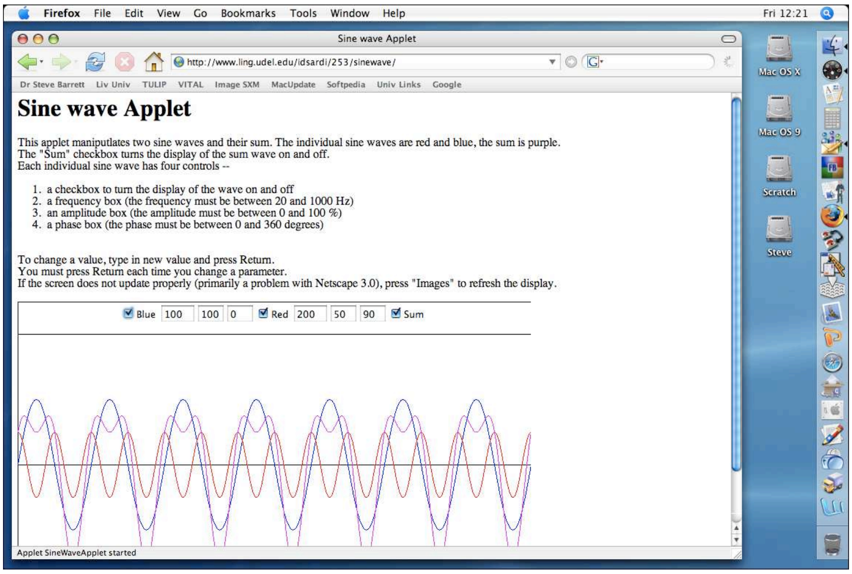
Similarly, the amplitude of the resultant wave is a when the difference $\alpha_2 - \alpha_1 = (2n+1)\pi$ where n is an integer

$$A_T^{\,2} = A_1^{\,2} + A_2^{\,2} + 2A_1A_2\cos(\pi)$$

$$A_T^{\,2} = A_1^{\,2} + A_2^{\,2} - 2A_1A_2 =$$

http://www.ling.udel.edu/idsardi/253/sinewave/

Java Applet



Standing Waves

If two waves of the same wavelength and frequency, but propagating in directions, are added the result is a .

This can happen when, for example, a wave meets its reflection (we will deal with wave reflection later).

Take two harmonic waves travelling in the

$$\psi(x,t) = A_R \sin(kx - \omega t + \varepsilon_R) +$$
$$A_L \qquad [1.8]$$

To simplify the solution, let us take $A_R = A_L = A$ and

$$\psi(x,t) = A\{\sin(kx - \omega t) + \sin(kx + \omega t)\}$$

Use the trig identity

$$\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$$

to write the wave function as

$$\psi(x,t) =$$

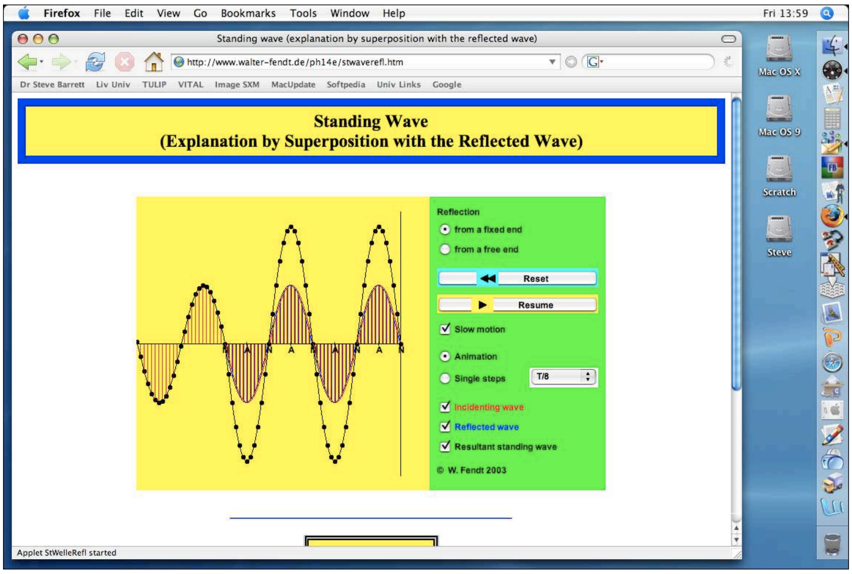
This equation describes a .

Note that the x and t variables are now in separate factors of the expression (there are no " " terms) and so, although the wave is still a function of space and time, it through space.

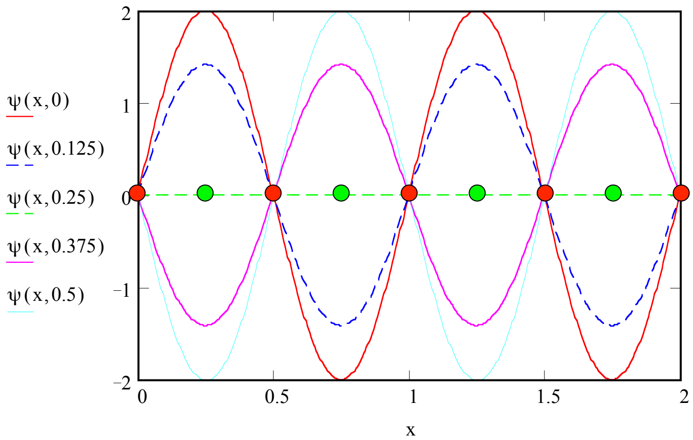
Note also that the amplitude of the wave is now a function of the position in x.

http://www.walter-fendt.de/ph14e/stwaverefl.htm

Java Applet



By watching a standing wave evolve with time, the positions of nodes (red dot) (where $\psi = 0$ at all times) and antinodes (green dot) (where the amplitude is a maximum) can be identified.



Mixed Waves

If the amplitudes of the two waves are the same ($A_R \neq A_L$) then the resultant wave is a combination of a travelling wave and a standing wave.

Rewriting [1.8] with (where $0 \leq R \leq 1$)

$$\psi(x,t) = A_L \{\sin(kx+\omega t) + R\sin(kx-\omega t)\}$$

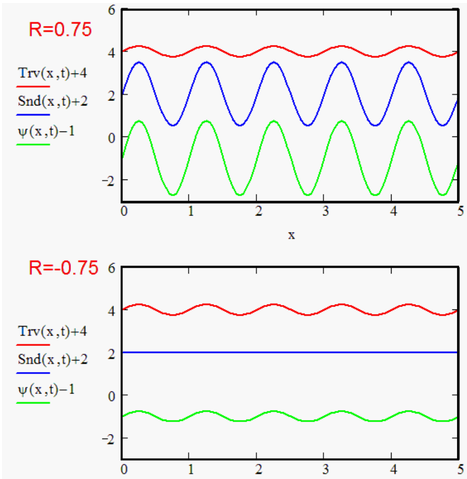
$$\psi(x,t) = A_L \{(1-R)\sin(kx+\omega t) +$$

$$\psi(x,t) = A_L (1-R)\sin(kx+\omega t) +$$
$$2RA_L \sin(kx)\cos(\omega t)$$

Consider three distinct cases for possible values of R:

No reflection	wave
Some reflection	Travelling + Standing
Perfect reflection	wave

Note that if $R < 0$ (negative reflection coefficients will be covered later) the result is still a mix of travelling and standing waves, but the positions of the nodes and antinodes of the standing wave component will be interchanged.



Standing Wave Ratio

For any value of R the maxima and minima in the amplitudes will occur at fixed locations (with a separation between neighbouring maxima, and similarly for neighbouring minima).

Measuring the intensity of standing waves as a function of position is a means of determining wavelength — it is used in PHYS378/478 Advanced Practical Physics with microwaves.

The is defined as

$$\text{SWR} = \frac{\text{Max. amplitude}}{\text{Min. amplitude}} = \frac{A_L + A_R}{A_L - A_R} =$$

If $R = 0 \quad \Rightarrow \quad \text{SWR} =$

If $|R| = 1 \quad \Rightarrow \quad \text{SWR} =$

Waves in Elastic Media

Many materials can be distorted by application of a force. Materials are often under small distortions — an object is restored to its original shape when the force is removed. Forces above the cause permanent deformation.

The general theory of elastic materials (continuum mechanics) is complicated because in many materials the elastic properties depend on the .

Here we will consider only the basic concepts that allow us to understand the propagation of waves in solids, liquids and gases.

Some important definitions:

Stress

$$T = \lim_{Area \to 0} \qquad \left[Nm^{-2} \right]$$

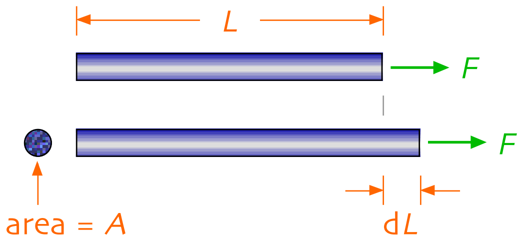
Strain

If parts of a body can be moved relative to each other then the body is said to be in a state of strain. The amount of strain depends on the variation of displacement ψ with distance x.

The propagation of waves is dependent on how stress and strain are related in any given medium.

Young's Modulus

$$Y = \frac{\text{stress}}{\text{strain}} \quad \left[\text{Nm}^{-2}\right]$$



If the material being stressed and strained is a wire, then

For example, steel has $Y \sim$

(*cf* plastics $Y \sim$, diamond $Y =$)

Bulk Modulus

$$B = \quad \left[\text{Nm}^{-2}\right]$$

where $\Delta V/V$ is the fractional change in volume under the applied pressure P.

Compressibility

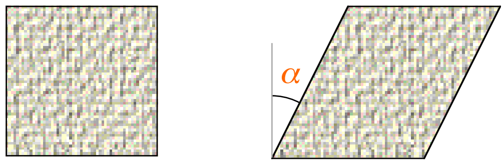
$$K = 1/B = \quad \left[\text{N}^{-1}\text{m}^{2}\right]$$

Water has $K \sim$ $\text{N}^{-1}\,\text{m}^2 \Rightarrow$

Shear Modulus

$$\mu = \qquad [\mathrm{Nm^{-2}}]$$

where T is the applied stress and α is the angle that defines the deformation.



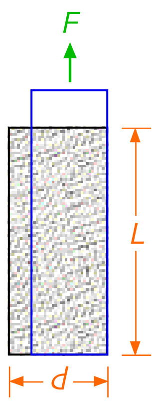
Note that the area used in calculating the stress is the area over which the force is applied (ie, the 'top' of the block).

For liquids and gases, .

Poisson's Ratio

This is a measure of how much a body deforms in the direction
to the direction of stress.

$$\sigma =$$



Note that for isotropic materials, Y, B, μ and σ are all inter-related.

For instance, $B = \dfrac{Y}{3(1-2\sigma)}$ $\quad \mu = \dfrac{Y}{2(1+\sigma)}$

Wave Propagation

The basic principle behind wave propagation in materials is that if there is a in a material then the material will
.

$$F \propto \frac{\partial}{\partial x}\left(\frac{\partial \psi}{\partial x}\right) =$$

where ψ is the displacement of the material.

The force F produces an acceleration which depends on the density of the material ρ

$$F =$$

and so

$$\rho \frac{\partial^2 \psi}{\partial t^2} \propto \qquad \text{(the wave equation)}$$

Longitudinal Waves

By comparison with the wave equation introduced in Lecture 1, we see that the speed of propagation of waves depends on the of the material and on the constant of proportionality between the and the resultant generated.

A full derivation gives (no proof here)

$$v_p{}^2 = \frac{B + 4\mu/3}{\rho} \qquad [1.9]$$

or, using the expressions relating B, μ, Y and σ

$$v_p{}^2 = \frac{Y(1-\sigma)}{\rho(1+\sigma)(1-2\sigma)}$$

Noting that the values of σ for solid materials is typically ~ 0.25 we can make the approximation

$$v_P^{\,2} \approx \frac{6Y}{5\rho} \qquad \text{or } v_P \approx \sqrt{\frac{Y}{\rho}}$$

for the phase velocity of ( ) waves travelling through solid media.

Example

Steel has $Y \sim$

$\rho \sim$

$\Rightarrow \quad v_P =$

Transverse Waves

A derivation for gives (no proof here)

$$v_P^{\,2} = \frac{\mu}{\rho} = \frac{Y}{2\rho(1+\sigma)}$$

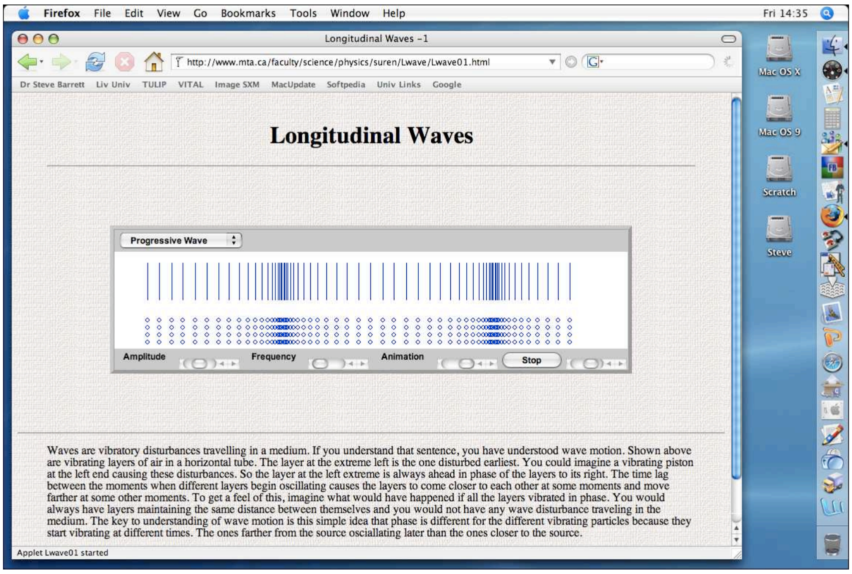
Taking $\sigma \sim 0.25$ we can make the approximation

$$v_P^{\,2} \approx \frac{2Y}{5\rho} \qquad \text{or} \qquad v_P \approx 0.6\sqrt{\frac{Y}{\rho}}$$

Remembering that the shear modulus $\mu = 0$ for liquids and gases, these expressions for the phase velocity for ( ) waves show that these waves cannot propagate in such materials.

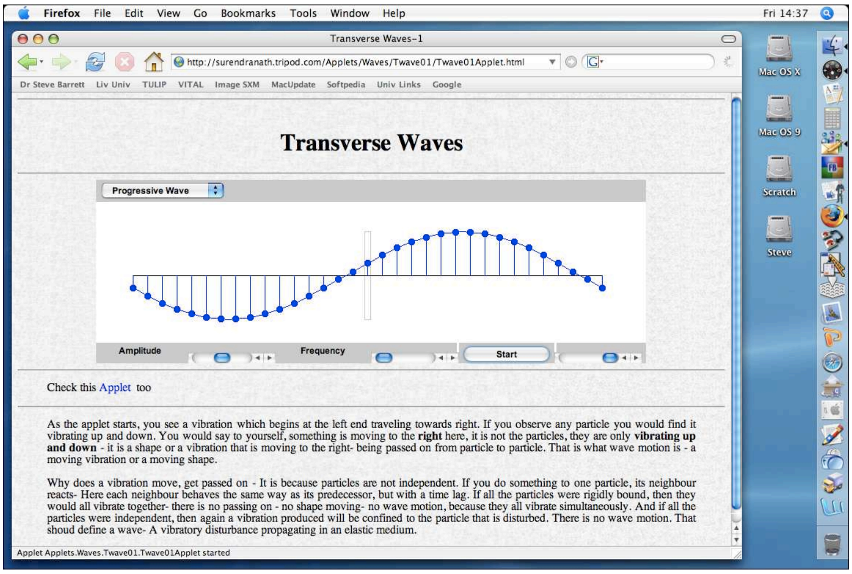
http://www.mta.ca/faculty/science/physics/suren/Lwave/Lwave01.html

Java Applet



http://surendranath.tripod.com/Applets/Waves/Twave01/Twave01Applet.html

Java Applet



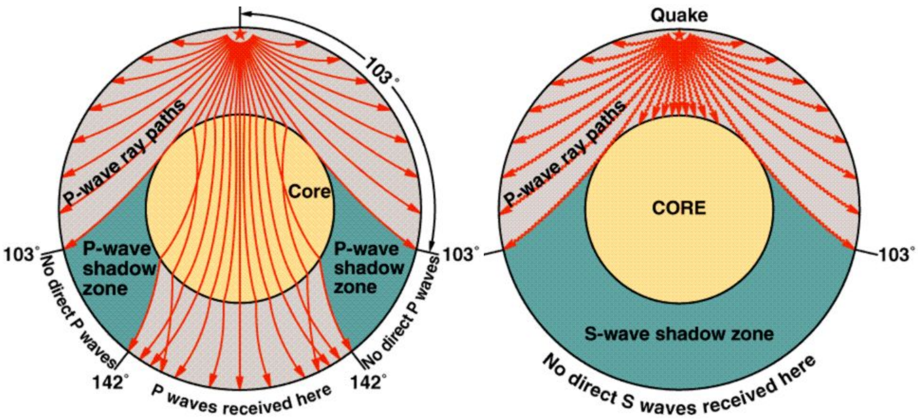
Seismic Waves

The ratio of the phase velocities for (or Pressure or Primary) waves and (or Shear or Secondary) waves in solids is

$$\frac{v_p \text{ for P waves}}{v_p \text{ for S waves}} \approx$$

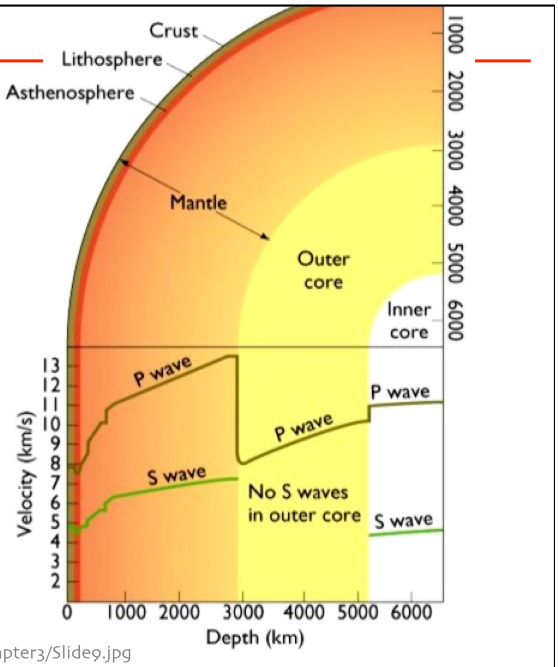
This velocity difference between P and S waves is crucial to our understanding of the interior of the Earth, as it enables seismologists to monitor earthquakes and use differential timings of the arrival of the two types of seismic waves at various locations on the Earth's surface to refine models of the Earth's inner structure.

Two types of surface wave, having displacement components to and to the Earth's surface, also have different velocities.



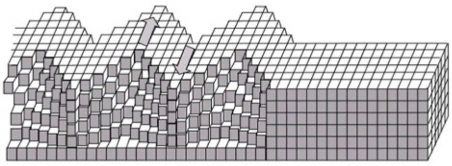
http://ww2.lafayette.edu/~malincol/Geol120/earthquaketopics.html

Measuring the variation of with depth gives information on the in regions that cannot be accessed by any other probe.



http://www.gly.fsu.edu/~salters/GLY1000/Chapter3/Slide9.jpg

waves

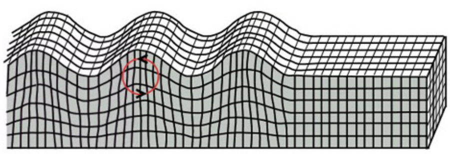


An earthquake can produce two types of surface wave.

An S wave which has only

movement of the ground is called a Love wave.

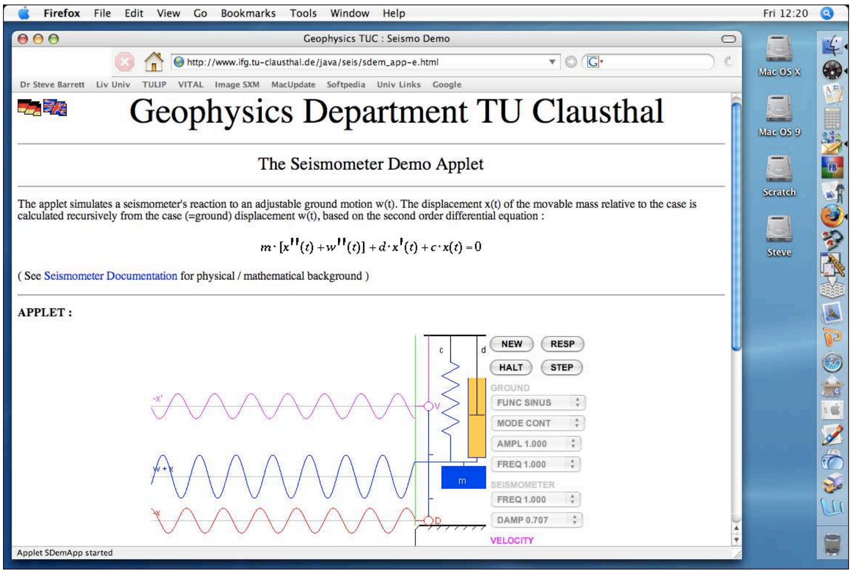
waves



The other kind of surface wave is the Rayleigh wave, which is similar to waves rolling across the ocean. The ground moves and (in the direction that the wave is propagating). Most of the shaking felt from an earthquake is due to the Rayleigh wave.

http://www.ifg.tu-clausthal.de/java/seis/sdem_app-e.html

Java Applet



Waves in Liquids

For waves liquids (but of a liquid) we use [1.9] with

$$v_p{}^2 = \frac{B}{\rho} = \frac{1}{K\rho}$$

For water we find, taking the value for K given earlier,

$$v_p \approx$$

This is than that found for a solid like steel, but still substantially than the speed of sound in air (next slide) because the forces that try to restore the medium after a deformation takes place are much in a liquid than in a gas.

Waves in Gases

For gases, the expression for the phase velocity can be written

$$v_p^2 =$$

where

γ	ratio of specific heats (c_p/c_v)	= 1.4	for air
R	gas constant	=	for all gases
T	temperature in Kelvin	= 300 K	
M	molar mass	=	for air

$$\Rightarrow \quad v_p \approx \qquad \text{for air}$$

The phase velocity of waves propagating through any elastic medium can be written, rather generally, as

$$v_p^2 \sim \frac{\text{“Stiffness” of medium}}{\text{“Mass” of medium}}$$

For a solid $\qquad v_p^2 \sim$

For a liquid $\qquad v_p^2 \sim$