When Are Eddy Tracer Fluxes Directed Downgradient?

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ABSTRACT

The mechanisms controlling the direction of eddy tracer fluxes are examined using eddy-resolving isopycnic experiments for a cyclic zonal channel. Eddy fluxes are directed downgradient on average when either (i) there is a Lagrangian increase in tracer variance or (ii) there is strong dissipation of tracer variance. The effect of the eddies on the mean tracer evolution can be described through an ensemble of eddies that each have a particular life cycle. Local examination of the eddy behavior, such as fluxes, eddy kinetic energy, and tracer variance appears complex, although the cumulative time-mean picture has coherence: eddies are preferentially formed in localized regions with downstream growth and increase in tracer variance concomitant with downgradient eddy tracer fluxes, while eventually the eddies decay with a decrease in tracer variance and upgradient eddy tracer fluxes. During spinup, tracer deformation through flow instability leads to an area-average increase in tracer variance (although locally it is increasing and decreasing with the individual eddy life cycles) and therefore an implied area-average, downgradient tracer flux. At a steady state, part of the pattern in eddy fluxes simply reflects advection of background tracer variance by the time-mean and eddy flows. The eddy flux becomes biased to being directed downgradient if there is a strong sink in the tracer, which is likely to be the case for eddy heat fluxes along isopycnals outcropping in the mixed layer or for eddy nitrate fluxes along isopycnals intersecting the euphotic zone.

1. Introduction

The time-varying circulations associated with geostrophic eddies in the atmosphere and ocean have a strong effect on tracer distributions and the background climatic state. In the atmosphere, the eddy circulation provides a poleward transport of heat in midlatitudes in the troposphere and a transport of ozone from the summer to winter hemisphere in the stratosphere (Andrews and McIntyre 1976; Plumb and Mahlman 1987; Andrews et al. 1987). In the ocean, the eddy circulations lead to extensive regions of nearly uniform potential vorticity along poorly ventilated density surfaces within the main thermocline (McDowell et al. 1982), as well as enabling the meridional transfer of water masses across the Antarctic Circumpolar Current (Danabasoglu et al. 1994; Marshall 1997; Speer et al. 2000; Karsten and Marshall 2002).

Given the importance of the eddy circulations, it is desirable to understand their effects on the time-mean tracer distribution, both to gain insight and improve how eddies are parameterized within coarse-resolution climate models. For the ocean, there is an ongoing debate as to how the eddy transfer should be represented. The eddy transfer is usually interpreted to act in a downgradient manner, which for a thickness closure leads to a flattening of isopycnals (Gent and McWilliams 1990) and for a potential vorticity closure leads to homogenization of conserved tracers within closed geostrophic contours (Rhines and Young 1982a,b). However, model tests of downgradient eddy closures in terms of thickness or potential vorticity only show limited skill when the parameterized fluxes are correlated locally with the eddy fluxes from eddy-resolving models (Roberts and Marshall 2000; Drijfhout and Hazeleger 2001). Indeed, support for downgradient closures of eddy transfer is only obtained if there are special symmetries in the flow, such as the spreading of convective waters from a cylindrical chimney (Visbeck et al. 1997) or the zonal-mean spreading of tracers in a zonal channel (Lee et al. 1997; Marshall et al. 1999). Our aim is to further explore the mechanism that controls why eddy tracer fluxes are sometimes directed downgradient, sometimes upgradient, and sometimes normal to the mean gradient, as demonstrated in our previous basin
study (Wilson and Williams 2004). We make an analogy with how atmospheric storms have a characteristic life cycle involving initial growth and eventual decay. There is a downgradient, poleward heat transfer during the growth phase and an upgradient, horizontal momentum transfer during the decay phase. The development of storms is preferentially located in nearly zonal storm tracks over the western side of the ocean basins (Hoskins and Valdes 1990), which leads to the downgradient heat transfer occurring within the core of the storm track and the upgradient momentum transfer occurring downstream of the growth region. In analogy with the atmospheric storms, we propose that localized regions of down- or upgradient eddy transfer of tracers are associated with a characteristic life cycle of ocean eddies.

In particular, we examine how the direction of the eddy tracer flux relative to the mean tracer gradient is controlled by the Lagrangian evolution of tracer variance. We extend our previous study of tracer variance in an ocean basin (Wilson and Williams 2004) by considering the eddy fluxes and life cycles in an isopycnic, reentrant eddy-resolving channel model, which extends previous tracer variance studies by Rhines and Holland (1979), Holland and Rhines (1980), Marshall and Shutts (1981), Illari and Marshall (1983), and Marshall (1984). This domain includes an almost-zonal jet and topographical interactions and should be viewed as a highly idealized representation of a limited sector of the Southern Ocean. We examine different states of equilibration and develop an intuitive picture of a typical life cycle of an ocean eddy in terms of Lagrangian growth and decay of tracer variance. In section 2, a theoretical context is provided to understand the evolution of tracer variance and the necessary conditions for the eddy tracer flux to be directed downgradient. In section 3, the setup of the numerical experiments is described. In section 4, the mean tracer fields are examined, together with their associated variance and eddy flux components. In section 5, the tracer variance dynamics and their control of eddy tracer fluxes is assessed for limits of negligible tracer dissipation and for several diffusive sinks; Lagrangian diagnostics are exploited to provide a life cycle framework for understanding when eddy fluxes are directed downgradient in the former case. In section 6, the implications of the study are summarized.

2. Theoretical background

Assuming a tracer equation of the form,

$$\frac{DC}{Dt} = \mathcal{J} - \mathcal{D},$$

(1)

where $D/Dt = \partial \bar{C}/\partial t + \mathbf{u} \cdot \nabla$ and $\mathcal{J}$ and $\mathcal{D}$ represent forcing and dissipation of the tracer $C$, then Wilson and Williams (2004) showed how direction of the eddy tracer flux, $\mathbf{u} \cdot \nabla C$, is controlled through the Lagrangian evolution of tracer variance together with forcing and dissipation:

$$\mathbf{u} \cdot \nabla C = -\frac{D}{Dt} \left( \frac{C^2}{2} \right) + \mathcal{F} C - \mathcal{D} C,$$

(2)

where an overbar represents a general mean, a prime a deviation from that mean, and $\mathcal{F} C$ and $\mathcal{D} C$ are written in positive-definite forms representing forcing and dissipation of tracer variance. The mean of the Lagrangian evolution of tracer variance,

$$\frac{D}{Dt} \left( \frac{C^2}{2} \right),$$

includes contributions from the Eulerian time evolution of variance, the advection by the time-mean flow, and the eddy advection of tracer variance,

$$\frac{D}{Dt} \left( \frac{C^2}{2} \right) = \frac{\partial}{\partial t} \left( \frac{C^2}{2} \right) + \mathbf{u} \cdot \nabla \left( \frac{C^2}{2} \right) + \mathbf{u}' \cdot \nabla \left( \frac{C^2}{2} \right).$$

Wilson and Williams (2004) showed the importance of the triple correlation term from eddy advection of tracer variance, while previous studies of this budget (Rhines and Holland 1979; Marshall and Shutts 1981) had argued that this term should be negligible in the tracer variance budget.

In the limit of negligible forcing of tracer variance, $\mathcal{F} C$, the eddy tracer fluxes are only directed downgradient, $\mathbf{u} \cdot \nabla C < 0$, when the right-hand side of (2) is negative, which occurs when either (i) there is an increase in tracer variance following the flow,

$$\frac{D}{Dt} \left( \frac{C^2}{2} \right) > 0,$$

or (ii) the mean dissipation, $\mathcal{D} C$, is large. Consider the following scenarios when these conditions apply:

(i) **Dominance of Lagrangian increase in tracer variance.** For a spinup, tracer variance is initially zero and increases with time, so for a closed volume and time mean,

$$\frac{D}{Dt} \left( \frac{C^2}{2} \right) > 0.$$

Therefore, the eddy tracer fluxes are downgradient on average over this closed volume and time mean.
An example of this case occurs for potential vorticity (PV) in shielded layers within a wind-driven gyre: eddy enstrophy (PV variance) increases and, on average, PV contours are expelled from the region of initial eddy enstrophy growth. This homogenization of PV and expelling of mean PV contours is in accord with that expected from a downgradient eddy PV closure, as advocated by Rhines and Young (1982a,b), despite the local eddy fluxes being directed both up- and downgradient (Rhines and Holland 1979; Holland and Rhines 1980; Roberts and Marshall 2000; Wilson and Williams 2004).

In a Lagrangian frame, fluid may also experience localized changes in tracer deformation and variance (Figs. 1a,b). These Lagrangian changes might be associated with the characteristic life cycles of eddies deforming the tracer contours, as seen for atmospheric storms passing along a storm track. The tracer variance is largest where these tracer contours deviate most from their time-mean position (Fig. 1b). Assuming that tracer forcing and dissipation are small, there are associated Lagrangian increases and decreases in tracer variance. The increase in tracer variance following the flow is, on average, concomitant with a downgradient eddy tracer flux, implied by (2). Conversely, a decrease in tracer variance following the flow is associated with an upgradient eddy tracer flux.

(ii) Dissipation dominance. The tracer fluxes may also be downgradient when $\overline{\nabla C'}$ is large enough to dominate any mean Lagrangian decreases in tracer variance (in a similar sense to that proposed by Rhines and Holland 1979). In this limit, the divergent eddy flux component could be considered to be consistently directed downgradient as long as the eddies are small amplitude, the mean flow is along mean tracer contours, and the tracer forcing is relatively weak (Marshall and Shutts 1981; Illari and Marshall 1983; Marshall 1984). For $\overline{\nabla C'}$ to be large, the temporal or spatial scales (depending on the averaging operator) of $\nabla$ must be comparable with those of $C'$ so that a significant correlation may be formed. This correlation is likely to depend on the particular tracer and its dissipation. Thus, one might expect that the direction of the eddy fluxes of a tracer with a strong sink, such as heat in the mixed layer or nitrate in the euphotic zone, may be different to that of a more conserved tracer, such as salinity. We will investigate the validity of these scenarios and how the dynamics might control them in the following numerical experiments.

![Fig. 1](image-url). A schematic depicting (a) two tracer contours, $C$, with localized variations in concentration and (b) corresponding regions of high variance, $C^2$, in the tracer (shaded). Following a streamline (thick contour) there is a positive and negative advection of tracer variance, respectively, upstream and downstream of the maxima in tracer variance. Since a Lagrangian increase in tracer variance implies a downgradient eddy flux, there should be a down- and upgradient eddy flux, respectively, up- and downstream of the maxima in tracer variance.

### 3. Eddy-resolving isopycnic channel model

#### a. Model formulation

The eddy tracer fluxes and tracer variance dynamics are examined using a three-layer, eddy-resolving 1/6° isopycnic primitive equation model, a parallel version of the Miami Isopycnic Coordinate Ocean Model (Bleck and Smith 1990), similar to that used in a basin setup in Wilson and Williams (2004). The momentum and thickness diffusion have been modified to include biharmonic mixing in order to dissipate enstrophy near the grid scale, allowing large enstrophy gradients to emerge. The coefficients of thickness and momentum diffusivity are $A_h = (\Delta x)^3 (5 \times 10^{-3})$ m s$^{-1}$ $\sim 1.1 \times 10^{10}$ m$^4$ s$^{-1}$ and

$$A_u = \max \left\{ 1 \times 10^{-3} \text{ m s}^{-1}(\Delta x)^3, 0.2 \left[ \left( \frac{\partial u}{\partial x} - \frac{\partial v}{\partial y} \right)^2 + \left( \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} \right)^2 \right]^{0.5} (\Delta x)^4 \right\},$$

respectively, where $\Delta x$ is the grid spacing; $A_u$ is typically $\sim 2 \times 10^{10}$ m$^4$ s$^{-1}$ but can rise to $\sim 10^{12}$ m$^4$ s$^{-1}$ in regions of strong shear. The vorticity equation is formulated to conserve PV and enstrophy even in the limit of layers.
having vanishing thickness (Boudra and Chassignet 1988). Bottom drag is of a quadratic form, with a coefficient $C_D = 3 \times 10^{-3}$, as in Wilson and Williams (2004).

The model domain is a zonally cyclic channel in the Southern Hemisphere with a maximum depth of 5000 m with a partial topographical barrier and submerged ridge, which should be viewed as a highly idealized representation of the Pacific Ocean sector of the Southern Ocean (Fig. 2b, contours). The domain extends from 48.1° to 41.3°S in latitude and spans 45° in longitude. There are vertical sidewalls with a no-slip boundary condition. Initial intermediate interface depths are 1700 and 3500 m. The internal Rossby deformation radius, $\sqrt{g' H f_0}$, is on average 66 km for the lower interface of the top layer and 16 km for the lower interface of the middle layer, where $f_0$ is taken as the midchannel value of the Coriolis parameter $f$, $g'$ is the reduced gravity, and $H$ is the height of the interface above the bottom. These values of the internal deformation radius are unrealistically large for the latitude range of the model, but have been chosen in order to allow experiments that resolve both jets and eddies well and are not prohibitively expensive. However, this choice may prevent the formation of multiple jets (Panetta 1993).

The experiments are forced with a zonal wind stress of the form $\tau_y = \tau_0 \sin(\pi y/L)$, where $\tau_0 = 0.2$ N m$^{-2}$ and $L$ is the meridional basin extent. There is no surface buoyancy forcing and no outcropping. The model is integrated for 50 years and unless otherwise stated, the time means shown correspond to years 40–50 of the uppermost layer after the model has equilibrated.

**b. Time-mean background state**

The wind forces a mean almost-zonal jet with maximum depth-integrated transport of around 120 Sv (Sv $= 10^6$ m$^3$ s$^{-1}$) in the jet (82% by the top layer, 18% by the bottom layer), as well as recirculations to north and south of the jet core (Fig. 2a, contours). The mean eddy kinetic energy is a maximum in the jet core (Fig. 2a, shaded), but there is also an alongstream variation with local maxima, reaching approximately 0.075 m$^2$ s$^{-2}$ in two patches, extending over of order 500 km in length along the jet. For the time-mean state, the area-averaged eddy kinetic energy is highest in the top layer, 0.033 m$^2$ s$^{-2}$, and decreases to 0.004 m$^2$ s$^{-2}$ in the middle and bottom layers. The Eady (1949) theory of baroclinic instability provides a useful linear estimate of growing waves, where the growth rate of the most unstable Eady mode is given by

$$\sigma_{Bl} = 0.31 \left| \frac{f}{N} \frac{\partial U}{\partial z} \right|$$

where $N$ is the buoyancy frequency and $U$ is the basic velocity (Lindzen and Farrell 1980; Hoskins and Valdes 1990). This Eady growth rate predicts where mesoscale eddies are most likely to grow, although this proxy only provides a rough measure of whether there is available potential energy, rather than whether the instability criteria are satisfied (Gill et al. 1974). The Eady growth period, $\sigma_{Bl}^{-1}$, reveals that the jet is the most baroclinically unstable part of the flow, with typical e-folding periods of between 6 and 16 days within that region (Fig. 2b, shaded).

**c. Relationship between Eady growth rate and eddy kinetic energy**

Within the jet, there are noticeable alongstream variations in the Eady growth period that do not always
locally coincide with the alongstream variation in eddy kinetic energy (Figs. 2a,b). The precise phase relationship between these variables is difficult to understand at a steady state. During the spinup, the Eady growth rate first develops (day 180) along the gap in the partial barrier (35°E) and along the downstream meander of the jet (37°–43°E) (Fig. 2c). At the same time, maxima in eddy kinetic energy (Fig. 2d) develop downstream of the Eady growth rate maximum (eastward of 38° to 5°E).

Subsequently, by day 720, the Eady growth rate increases all along the jet although there is still a local maximum rate located close to the gap in the topography (Fig. 2e). The eddy kinetic energy develops by an order of magnitude all along the jet with maxima centered along the jet at 40°–10°E and 20°–30°E (Fig. 2f). Thus, in our view, the maxima in eddy kinetic energy initially develop downstream of the maximum Eady growth rate, but this simple connection is obscured in the time-mean state by the persistence of background eddy kinetic energy. This background or fossil eddy kinetic energy remains since the dissipation of eddy kinetic energy is small in the model.

4. Mean, variance, and eddy fluxes of tracer

The mechanisms controlling the direction of eddy tracer fluxes are examined for two tracers: a passive tracer C, and a dynamic tracer, the PV. The passive tracer is initialized with a linear northward gradient from zero to unity, advected with no explicit diffusion, and has a source value of unity on the northern boundary and a sink value of zero on the southern boundary (with a relaxation time scale of 2.5 days, increasing linearly to 15 days in five grid cells toward the interior of the domain). The PV is conserved on the large scale, with only small explicit dissipation near the grid scale, because of the biharmonic momentum and thickness diffusion. We will focus on the idealized tracer, as it complements the previous gyre study of PV in Wilson and Williams (2004) and provides a simple parallel for fluxes of nutrients and carbon, which do not depend explicitly on velocity derivatives.

a. Mean tracer variations and total eddy fluxes

In the time mean for the upper layer, there is a front in both tracers following the time-mean jet, which acts as a partial dynamical barrier separating both higher C and PV (less negative) to the north and lower C and PV to the south (Figs. 3a,b, contours). The tracers are homogenized to different values to the north and south of the front. Maxima in the variance for both tracers are concentrated along the time-mean jet and associated

![Image](https://example.com/image.png)
\textit{b. Spinup of tracer and its variance}

Despite the simplicity of the model domain, there is a rich structure in the tracer variance and associated pattern of eddy fluxes. To reveal how these patterns emerge, maps of instantaneous tracer and tracer variance, $C^2$, are examined during the spinup at days 180 and 720 using a running 300-day average (Figs. 4a,b, shaded). High variance is first generated downstream of the partial barrier (Fig. 4a) where there is a northward deflection of the jet and a maximum in Eady growth rate (Fig. 2c). Subsequently, variance develops all along the jet (Fig. 4b) with a pattern closely resembling that of eddy kinetic energy (Fig. 2f). Initially, there is a constant gradient in the passive tracer from the south to the north. During the spinup in the top layer, as the tracer variance increases, the tracer gradient becomes weaker on the flanks of the jet and becomes concentrated along the core of the jet (Figs. 4a,b, contours). As for eddy kinetic energy, there is significant background tracer variance that is not locally produced, but instead advected into a region before variance dissipation can act. During the spinup for the middle layer, the gradients in passive tracer instead become weaker within the subbasins between the topographical barriers (Figs. 4c,d). This different response for each layer is due to the PV being strongly forced by the wind over the top model layer, but not being frictionally forced in the middle layer (apart from weak biharmonic terms). Thus, a PV front is maintained in the top layer but not in the middle layer, which is reflected in the different evolution of the passive tracer in each layer.

c. Rotational and divergent eddy flux components and implied diffusivities

Although (2) provides a link between tracer variance and eddy flux direction, it is the total eddy flux that is considered rather than a divergent component. Since tracer evolution is affected by the divergence of the eddy tracer flux, it is useful to separate the eddy tracer flux into rotational and divergent components (e.g., Marshall and Shutts 1981; Roberts and Marshall 2000; Peterson and Greatbatch 2001).

Following Peterson and Greatbatch (2001), we decompose the eddy flux through solution of Poisson equations for each component with boundary conditions chosen for zero normal flux at the boundary. The rotational component is typically 5 times larger than the divergent component for the passive tracer (Figs. 5a,b) and is similar for PV. The rotational component dominates the total flux and circulates with up- and downgradient fluxes, whereas the divergent component is downgradient over most of the jet. This pattern is reflected in the eddy diffusivities, $\kappa$, for the total and the divergent components from the closure of the form \( \mathbf{u} \cdot \nabla C = -\kappa \nabla C \) (Figs. 5c,e). The values of $\kappa$ for the total eddy tracer flux component are larger than 5000 m$^2$ s$^{-1}$ for most of the domain and are negative in large regions. For the divergent component the $\kappa$ is smaller in magnitude, typically 2000 m$^2$ s$^{-1}$, and positive for most of the domain, suggesting that the divergent component is downgradient. Repeating this diagnostic for PV (Figs. 5d,e), the eddy diffusivities are about half as large as those for the passive tracer. This smaller value is probably due to the adjustment of mean PV gradients being more physically constrained than that of the mean passive tracer gradients. The $\kappa$ for the total eddy PV flux component again has large regions corresponding to upgradient eddy transfer, while the $\kappa$ for the divergent eddy PV flux component is positive over most of the jet and has a smaller magnitude.

The divergent eddy tracer flux is the most relevant component for developing eddy parameterizations. As discussed in Wilson and Williams (2004), many com-
To explore the limits discussed in section 2, we perform two integrations: one in which tracer variance forcing and dissipation is negligible and another in which there is large dissipation. These diagnostics follow those of Wilson and Williams (2004) for a double gyre; however, they differ in that the zonally cyclic domain geometry allows many eddies to grow and decay following the flow.

5. Tracer variance budget

The tracer variance budget is now examined at the equilibrated state between years 40 and 50, where (2) is rewritten as

\[
\frac{\partial}{\partial t} \left( \frac{C^2}{2} \right) + \bar{u} \cdot \nabla \left( \frac{C^2}{2} \right) + \mathbf{u}' \cdot \nabla \left( \frac{C^2}{2} \right) = -\bar{\mathbf{u}} \cdot \nabla C' \cdot \nabla C' + \overline{\mathbf{f}'C'} - \overline{\mathbf{D}C'}. \tag{3}
\]

\[
\mathbf{u} \cdot \nabla \left( \frac{C^2}{2} \right) + \mathbf{u}' \cdot \nabla \left( \frac{C^2}{2} \right) = -\bar{\mathbf{u}} \cdot \nabla C' \cdot \nabla C'. \tag{4}
\]

Mean advection of tracer variance by the time-mean flow (Fig. 6a) and eddies (Fig. 6b) shows complex reversing signals where tracer variance increases and decreases following the individual mean and eddy flows. The signal is largest in the jet, where flow is strongest and gradients of tracer variance are largest. The mean and eddy advection components have roughly equal magnitudes, and their smallest spatial scales cancel when added together to give a smoother pattern of mean advection of tracer variance by the total flow (Fig. 6c). The eddy contribution is more important downstream of the gap in the topography (38°E), while the mean flow contribution dominates elsewhere.

This large-scale reversing signal in total advection of tracer variance,

\[
\mathbf{u} \cdot \nabla \left( \frac{C^2}{2} \right),
\]

controls the direction of the eddy tracer flux (Figs. 6c,d). This budget is well closed and there is only a small residual (Fig. 6e). This balance between \(-\bar{\mathbf{u}} \cdot \nabla C' \cdot \nabla C'\) and \(\mathbf{u} \cdot \nabla \left( \frac{C^2}{2} \right)\) is also supported in there being a high correlation of 0.85 between each term (Table 1); this correlation increases to 0.94 if performed between 0°...
and 30°E and omitting the partial barrier. The correlation for \(-\overrightarrow{u} \cdot \overrightarrow{\nabla} C \cdot \overrightarrow{\nabla} \) with either the mean advection of variance, \(\overrightarrow{u} \cdot \overrightarrow{\nabla}(C^{2}/2)\), or the eddy advection of variance, \(\overrightarrow{u}^{e} \cdot \overrightarrow{\nabla}(C^{2}/2)\), becomes much smaller (0.33 and 0.02 respectively), suggesting both contributions have an important role. When the correlations are repeated for the PV variance budget, there is a reasonably good correlation between \(-\overrightarrow{u} \cdot \overrightarrow{\nabla} Q \cdot \overrightarrow{\nabla} \) and \(\overrightarrow{u} \cdot \overrightarrow{\nabla}(Q^{2}/2)\) for the whole domain (0.60) that, away from the partial barrier, has weak contributions from mean (0.08) and eddy (0.18) components. These sum to give a similarly good correlation (0.58) for the total advection of PV variance with the eddy PV flux direction term.

In summary, this analysis suggests that time-mean increases in tracer variance following the flow are accompanied by the eddy tracer flux being directed downstream.

b. A Lagrangian eddy life cycle framework

The previous Eulerian time-mean eddy flux patterns consist of ensembles of many eddy events, including growth and decay of finite amplitude disturbances. For example, atmospheric eddies leave an Eulerian time-mean imprint in the varied distribution of eddy fluxes along storm tracks, while at the same time they are understood in terms of eddy life cycles. We now examine an equivalent idea for the ocean, looking at the characteristic scales and regions of eddy fluxes and their direction throughout the life cycle.

To illustrate typical eddy life cycle behavior in the model, we consider local Lagrangian variation of Eady growth rate, eddy kinetic energy, tracer variance, and the direction of the eddy fluxes. The Lagrangian diagnostics are conducted following floats advected with the instantaneous velocity field for the case of negligible tracer variance dissipation in the top layer. The floats are advected with the 6-hourly velocity field, using a fourth-order Runge–Kutta scheme with a 6-h time step and bilinear interpolation of the velocity and Eulerian mean fields.

First, an ensemble of float trajectories is considered and, second, two different trajectories are discussed that represent two characteristic types of behavior.

(i) Ensemble of trajectories. An ensemble of nine floats, arranged in a 3 by 3 uniform grid of spacing 1/6° centered at 43.6°S, 37.3°E is advected by the instantaneous flow starting at year 40 where the starting position (white square), trajectories, and end points (numbered) for each of the floats over a 2-month period are indicated in Fig. 7a. The ensemble of floats all experience an initial high Eady growth rate over the first 0.2 month, then there is a subsequent increase in eddy kinetic energy to a maximum at 0.75 month, then subsequent decay (Figs. 7b,c; the mean plus and minus a standard deviation, is shown). Likewise, there are two peaks of rate of increase of tracer variance (at 0.2 and 0.5 month) separated by a rate of decrease, which are associated with down- and upgradient eddy tracer fluxes, respectively (Figs. 7d,e). After 2 months, the trajectories become much more complex and there is a divergence in the individual responses of the floats (Fig. 7f), so two of the individual float trajectories from the ensemble are discussed in more detail.

(ii) Single trajectories. The life cycles for two single float trajectories are followed now for 7 months (Figs. 8 and 9), starting at almost identical initial positions (trajectories 9 and 4 of Fig. 7f) downstream of the gap in the topographic barrier at year.
One of these floats loops round in a circuit, almost returning to its initial position just after 3 months and just after 5 months (Fig. 8a), while the other float follows the jet and zonally traverses the whole domain (Fig. 9a) in less than 5 months. For the former trajectory, confined to a region where the background flow is most unstable, the Eady growth rate maxima are followed by eddy kinetic energy maxima with a lag of around 10 days (Figs. 8b,c). Since the float loops through the active region three times, at 0.5, 4, and 6 months, this life cycle pattern of growth following instability is repeated. At each time the eddy kinetic energy is preceded by a reversing pattern of

\[
\frac{D}{Dt} \left( \frac{C^2}{2} \right) \sim - \vec{u} \cdot \nabla \vec{C}.
\]

These reversals are due to flow passing into and out of local maxima in tracer variance, yet they do not completely cancel out. The time-integrated effect (Fig. 8f) is for an overall Lagrangian increase in tracer variance and an overall downgradient eddy tracer flux.

For the latter trajectory, which is no longer confined solely to a region where the background flow is most unstable, the life cycles are more complex because of background eddy kinetic energy and tracer variance being advected by the mean flow. At times when the float...
passes through the region 35°–5°E (0 to 1 month and 4.5 to 5.5 months; Figs. 9a–e), the local instability dominates the eddy kinetic energy signal and the life cycles are similar to the previous case (Fig. 8). Elsewhere, in terms of eddy kinetic energy and Eady growth rate, the life cycles are less coherent although the balance

$$\frac{D}{Dt} \left( \frac{C^2}{2} \right) - \mathbf{u} \cdot \nabla C$$

still holds and shows reversals of large magnitude and similar 5–30-day periods. Again, for this particular trajectory, the time-integrated effect (Fig. 9f) shows that these reversals throughout the trajectory have a net effect of tracer variance increase and downgradient eddy tracer flux.

In summary, this Lagrangian view illustrates that there are characteristic eddy life cycles whose local behavior depends both on the stability of the local flow and on integrated effects such as downstream development of eddy kinetic energy and tracer variance. Over a period of several months, locally reversing changes in tracer variance and eddy tracer flux direction can accumulate to have a coherent signal of a particular sign.

c. Diffusive sink for tracer variance

The role of the tracer sink is now considered, given previous discussions of the tracer variance dissipation (Rhines and Holland 1979; Marshall and Shutts 1981; Illari and Marshall 1983; Marshall 1984). The tracer sink may directly affect the eddy flux direction through a sink of tracer variance on the right-hand side of (2):

$$\overline{\mathbf{u} \cdot \nabla C} = - \frac{D}{Dt} \left( \frac{C^2}{2} \right) + \mathbf{f} \cdot \nabla\overline{C} - D \overline{\nabla C}.$$

We consider three cases with tracer diffusion $\mathcal{D}$ in the form of Newtonian damping and Laplacian and biharmonic diffusion. For the Newtonian damping case, the tracer is governed by

$$\frac{DC}{Dt} = -\frac{C}{\lambda},$$

where $\lambda$ is a relaxation time scale of 30 days (typical of a tracer such as nitrate). For the Laplacian case, the tracer is governed by

$$\frac{DC}{Dt} = -\left(\overline{\Delta p}\right)^{-1} \nabla \cdot (u_d \Delta x \overline{\Delta p} \nabla C),$$

where $u_d$ is a diffusion velocity of $5 \times 10^{-2}$ m s$^{-1}$, $\Delta x$ is the grid spacing, and

$$\overline{\Delta p} = \frac{2}{\Delta p_{i-1} + \Delta p_i}$$

is the harmonic mean of the layer thickness $\Delta p$ (the equation is solved in $i$ and $j$ directions on the Arakawa C grid) following the standard form of temperature diffusion in the model. For the biharmonic case, the governing equation is

$$\frac{DC}{Dt} = \left(\overline{\Delta p}\right)^{-1} \nabla^3 \cdot (u_d \Delta x^3 \overline{\Delta p} \nabla C),$$

where $u_d$ is again a diffusion velocity of $5 \times 10^{-2}$ m s$^{-1}$. The magnitude of the diffusion velocity is chosen to give a similar diffusive effect at the grid cell for all three cases.
steady state. The implied eddy diffusivity, $\kappa$, is diagnosed assuming a closure,

$$\overline{u'C'} = -\kappa \frac{\partial}{\partial y} C^{*t},$$

where the superscripts $x, t$ denote averaging in $x$ and $t$, although the zonal mean excludes the partial barrier region ($32.5^\circ$–$37.5^\circ$E). For each of the model layers, there is a large and positive $\kappa$, typically of order $1000$–$4000$ m$^2$ s$^{-1}$, reflecting how the eddies effectively transfer tracer downgradient between the boundary sources (Fig. 11, full line); also see Lee et al. (1997) for a similar result. However, this zonal- and time-mean $\kappa$ integrates out the negative values seen in the time-mean maps of $\kappa$ in Fig. 5e, associated with local upgradient eddy transfer.

There are also latitude ranges, such as between $46.5^\circ$ and $47.5^\circ$S in Fig. 11a, where $\kappa$ is much larger, of order $15000$ m$^2$ s$^{-1}$. This signal is an artifact of zonal averaging when the flow deviates from latitude circles and is due to the combination of significant zonal-mean fluxes (Fig. 5b) with weak zonal-mean tracer gradient (Fig. 3a).

The positive sign for the zonal-mean $\kappa$, implying downgradient transfer, is broadly unchanged for the different choices in tracer dissipation (Fig. 11), although the Newtonian case has a lower $\kappa$ in the top layer and a higher $\kappa$ in the middle and bottom layers.

While the eddy tracer flux has a complicated pattern in the time mean being directed both up- and downgradient, the eddy tracer flux is consistently directed downgradient in the zonal mean. This simpler result is due to the zonal mean together with the cyclic boundary condition automatically removing the rotational component of the eddy tracer flux, which usually dominates the remaining divergent component. However, the zonal-mean diagnostics of $\kappa$ only have a limited usefulness for the ocean because of the presence of topography.

6. Summary

The direction of eddy tracer fluxes has been considered, exploiting the tracer variance budget. In the limit
of small variance forcing, eddy tracer fluxes are directed downgradient when there is a Lagrangian increase in tracer variance or a dissipation of tracer variance. This response can be understood in terms of a Lagrangian view of ocean eddy life cycles, which is analogous to the life cycle view of atmospheric storms explained in terms of an ensemble of wave packets (Chang et al. 2002).

During spinup, there is an overall increase in the area-averaged tracer variance leading to an area-averaged, downgradient eddy tracer flux. While, locally, the eddy tracer fluxes can still be upgradient, this overall downgradient transfer is also reflected in the systematic smoothing of tracer gradients in unforced layers.

At a steady state, the systematic effect of the eddies depends on (i) whether the eddies are locally growing or decaying and (ii) the degree of tracer variance dissipation. If the dissipation dominates the changes in tracer variance then the eddy fluxes are directed downgradient. For small dissipation and for a nonzero mean tracer gradient, the sign of the steady pattern of eddy fluxes is controlled by whether the ensemble of eddies either respectively grows, that is, when

$$\frac{D}{Dt} \left( \frac{C^2}{2} \right) > 0,$$

or decays. Over parts of the domain the pattern of eddy fluxes can be explained in terms of this eddy life cycle view where there are downgradient fluxes in eddy growth regions. However, there are other regions where the eddy fluxes locally reverse and are controlled by the advection of background variance by the total flow, making it difficult to parameterize the detailed direction of the eddy fluxes.

The steady-state Eulerian mean patterns may be considered to be a composite of many individual eddy life cycles, which are each affected by local flow stability and nonlocal boundary conditions. Again, an obvious analogy is that of the atmospheric storm tracks, whose Eulerian-mean eddy fields are made by the passage of many different individual eddies organized into latitudinal bands of high activity. For the ocean, we see that the rate of change of tracer variance and change in eddy flux direction locally reverse and often compensate in a Lagrangian sense, where the rotational eddy flux component plays a dominant role. However, there are also regions where these local reversals do not cancel when integrated over several eddy lifetimes; for example, there is a systematic downgradient eddy transfer over most of the jet associated with the divergent eddy flux component. Also, there are times during a typical eddy life cycle for which the tracer variance remains constant and there is no eddy tracer flux across mean contours.

The direction of the eddy tracer fluxes is sensitive to the tracer sink, where increasing the dissipation of tracer variance leads to a stronger downgradient flux. Consequently, if the tracer dissipation is concentrated in a surface mixed layer, such as for heat, then the eddy heat flux will be directed downgradient close to the outcrop of density surfaces and instead will have a reversing local pattern of up- and downgradient fluxes in shielded layers. Likewise, for a biogeochemical tracer, such as nitrate, which has a strong sink in the euphotic zone, the fluxes will be directed downgradient on density surfaces intersecting the euphotic zone.

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FIG. 11. Diagnostics of the eddy diffusivity (m$^2$ s$^{-1}$), $\kappa$, for the tracer, assuming both a zonal- and time-mean closure, $\nabla \cdot \nabla \psi (\theta) / \nabla \theta$, for (a) top layer, (b) middle layer, and (c) bottom layer, where the average is applied over all longitudes except the range 32.5$^\circ$–37.3$^\circ$E, for years 40–50. The no-diffusive case is denoted by a solid line, the Newtonian dissipation by a long dashed line, Laplacian by a long–short dashed line, and biharmonic by a dotted line.


