Victor Goryunov's Mathematical Work

Victor Goryunov was born in Moscow and grew up in a closed military settlement, "behind an iron fence inside the iron curtain". He studied mathematics in Moscow State university with Vladimir Arnol'd, and, extra-officially, with Vladimir Zakalyukin, defending his PhD in 1982. He worked in Moscow Aviation Institute until 1994, with long visits to Warwick, Hawaii, Berkeley, Aarhus, and Athens, Georgia. In 1994 he moved to Liverpool. In 2001 he was joined here by Vladimir Zakalyukin.

Victor's work is characterised by an exceptionally rich interaction between topology and algebra. In the tradition of Arnol'd he is skeptical of abstract topological definitions, instead carving out the objects he wants from "sufficiently high dimensional Euclidean spaces". For him, two dimensional singular chains are "membranes" or "films". His drawings are correspondingly excellent, and I'd like to show a few here. From [17], *Local invariants of mappings from surfaces to 3-space*:

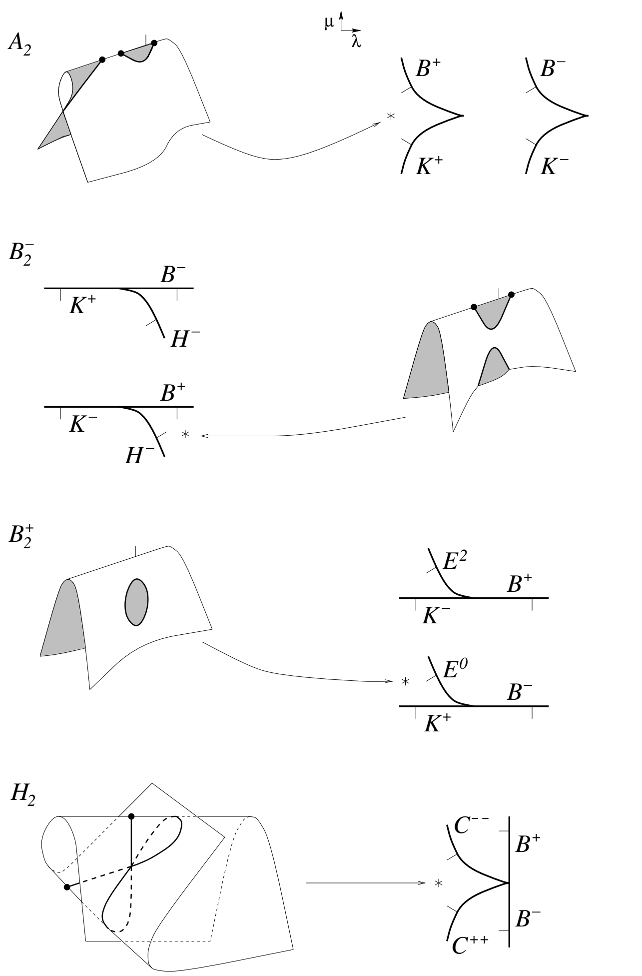

Figure 14: *Bifurcations in generic 2-parameter families of uni-germs*

A comment from the review by Jim Bryan of [17]

> I really enjoyed reading this paper and I would recommend it to any mathematician in any field. I think Gauss would have liked this paper.[. . .] The pictures in this paper are wonderful and the mathematics is clear and concise.

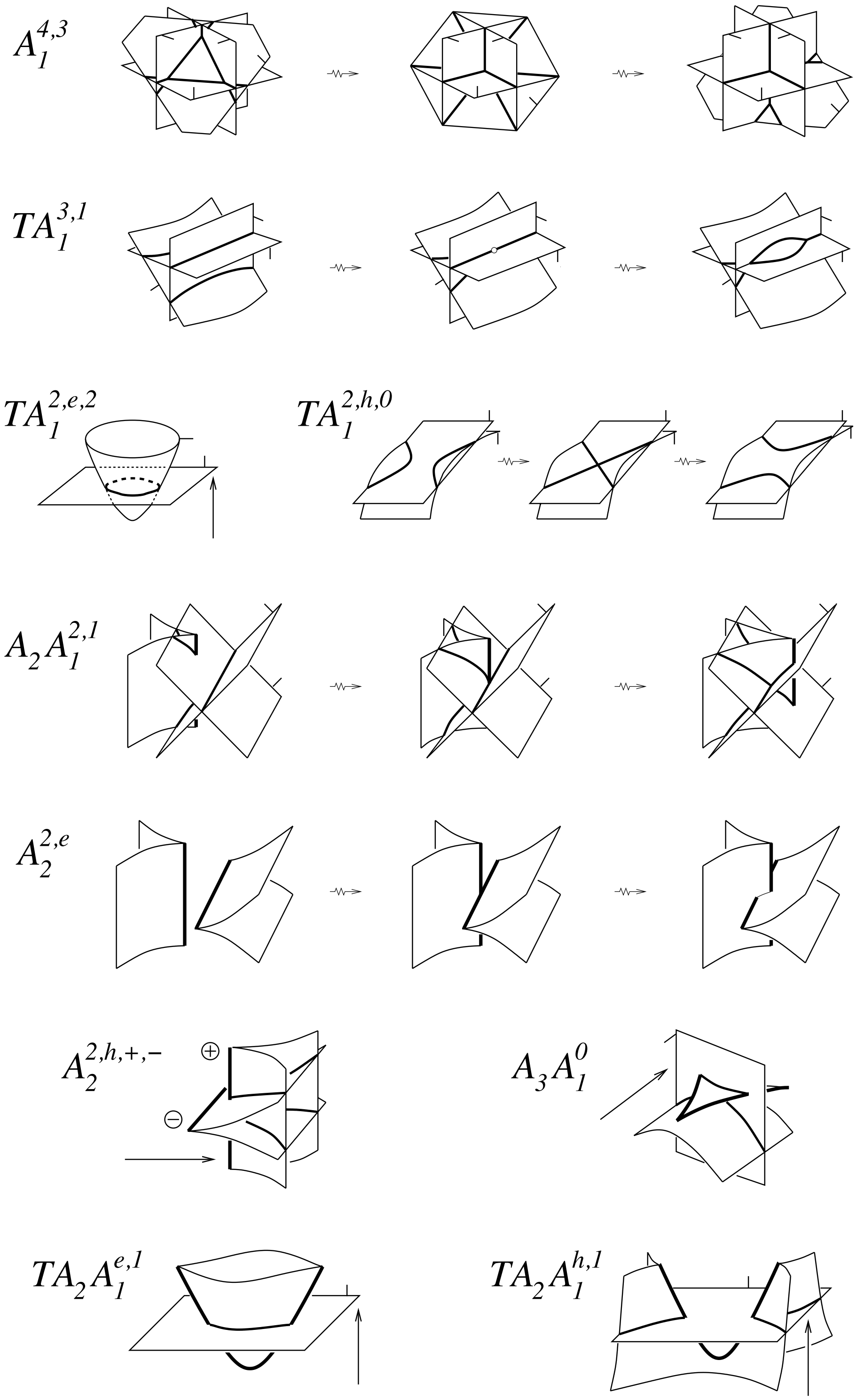

Figure 2: Codimension 1 multi-germs.

1 Discriminants and Bifurcation Sets

Victor's Master's thesis was the computation of the cohomology of the braid groups of type C and D – the cohomology of the complement of the discriminant in the quotient space $\mathbb{C}^n/W$, where W is a Coxeter group of type C_n or D_n. The fundamental groups of these spaces are the "braid groups of type C and D" , and the spaces themselves are $K(\pi, 1)$, so that the group cohomology of the braid groups of type C and D is the topological cohomology of the spaces. The usual braid group is the fundamental group in the case of A_n.

Throughout his career, Victor's work has benefited from this early familiarity with the links between singularities and complex reflection groups, and it is a theme he has returned to several times with his students: Claire Baines, in the late 1990's, Show Man Han in 2006, and, more recently, Joel Haddley. Given a boundary singularity of the form $x + y^k$ (where $\{x = 0\}$ is the boundary), by raising x to a higher power one one obtains a series of more and more complicated singularities. For example with $k = 3$ one obtains successively $A_2, D_4, E_6, E_8, J_{10}$ E_{12}. Reversing the process, Victor proved that the Shephard-Todd groups $G(m, 1, k)$, together with seven exceptional and some other Shephard-Todd groups are realized as the monodromy groups of simple hypersurface singularities equivariant with respect to the action of reflections or finite order elements of SU(2). In [50] he showed that for any simple hypersurface singularity the monodromy in the space of the symmetric smoothings is a Shephard-Todd group. With D. Kerner he worked on automorphisms of P_8 singularities and complex crystallographic groups, [25]. His paper with Show Han Man, *The complex crystallographic groups and symmetries of* J_{10} ([27])) was the first to introduce *affine* reflection groups in singularity theory.

He returned to this theme also in 2003, in a beautiful paper with Zakalyukin, *Simple symmetric matrix singularities and the subgroups of Weyl groups* A_μ, D_μ, E_μ, [56]. Bill Bruce and Farid Tari had classified simple symmetric matrix singularities – symmetric matrices of size 2 or 3, classified by the natural group acting on symmetric matrices, of transpose conjugation by families of invertible matrices together with changes of coordinates in the source, which we call $\mathscr{K}_f$ (where $f = \det M$). Victor and Volodya showed a natural and revealing bijection between the reflection subgroups of the Weyl groups and symmetric matrix singularities. They showed that simple symmetric matrix singularities M are classified by pairs (W, H) where W is a Weyl group of type ADE and H is a reflection subgroup, and $\mathbb{C}^n/W$ is the $\mathscr{R}_e$-versal base space of the composed function $f := \det M$ (which itself must be of type ADE). The choice of H gives a factorisation

$$\mathbb{C}^\mu \to \mathbb{C}^\mu/H \to \mathbb{C}^\mu/W \qquad (1.1)$$

of the quotient map. The size of the symmetric matrix is determined by the vertices of the Dynkin diagram delted in passing form W to H. The quotient space $\mathbb{C}^\mu/H$ can be identified with the $\mathscr{K}_f$ versal base-space for the symmetric matrix M. Then the second term in the factorisation is exactly the inducig map between the $\mathscr{K}_f$-versal base-space of the symmetric matrix M and the $\mathscr{R}_e$-versal base space of the composed function $\det M$. We recover the classical situation from the trivial case where $H = W$.

Following on from Victor's thesis work on the cohomology of discriminant complements was an interest in the geometry of bifurcation sets and discriminants. The papers [11], [33], [36],[37],[43], [47], [19] all deal in one way or another with these topics. One is interested in three main questions here:

1. whether or not bifurcations sets and discriminants are free divisors;
2. whether or not their complements are $K(\pi, 1)$ spaces;

3. in case they are free divisors, finding a basis for the module of logarithmic vector fields.

1. The paper on functions on space curves [47] shows that provided $\mu = \tau$ (in the appropriate sense), the discriminant is a free divisor. The paper on logarithmic vector fields for the discriminants of composite functions, [19], also shows that these are again free divisors, given some fairly standard hypotheses.

2. In his PhD dissertation Victor studied functions on plane curves, and showed that for simple functions, the complement of the bifurcation set is a $K(\pi, 1)$. He returned to this in [47], showing that the same is true for simple functions on space curves, and in [56] for simple symmetric matrix singularities. As in the case of the Lyashko-Looijenga theorem, the key is to see the complement of the bifurcation set as a covering space of the space of regular orbits of the corresponding Weyl group, which is itself a $K(\pi, 1)$.

3. Victor's paper [36] gave us 'Goryunov's algorithm' for a basis for $\mathrm{Der}(-\log D)$ when D is the discriminant of a versal deformation of an ICIS. When X_0 is weighted homogeneous this is especially elegant: by means of a natural isomorphism between two incarnations of $\mathrm{Tor}_1^{\mathcal{O}_S}(T^1_{X/S}, \mathbb{C})$ he shows that the Eu-

ler field generates this module over $\mathbb{C}$, which then leads to a straightforward listing of $\mathcal{O}_S$-generators for $\mathrm{Der}(-\log D)$. This algorithm has been widely used, e.g. by du Plessis and Wall.

1.1 Classification

One of the most-cited papers, [34], based on his PhD thesis, gives an extensive classification, up to left-right equivalence, of singularities $(X, x_0) \to (\mathbb{C}^p, 0)$ where X is an ICIS whose dimension is no less than p. In particular it lists all simple singularities in this context. It contains an example of a map-germ which is simple over $\mathbb{R}$ but not over $\mathbb{C}$.

2 Singularities of mappings $M^n \to N^{n+k}$

In 1989 Victor spent some months in Warwick, at the special year on Singularity Theory. This was under Gorbachev, when Soviet mathematicians were first able to travel to the West. We decided to apply for a grant for him to return to Warwick, to work on the monodromy associated with $\mathscr{A}_e$-versal deformations of map-germs

$$(\mathbb{C}^2, 0) \to (\mathbb{C}^3, 0)$$

of finite codimension, in which the fundamental group of the complement of the bifurcation set in the base of the unfolding acts on the homology of the image. When he arrived for the second visit, he had already written a paper on this topic, finishing what we had proposed doing in the grant application. But it freed up the time to collaborate on the homology of images of maps $f : M^n \to N^{n+k}$ in general. We found a spectral sequence that has played a large role in my own later work, and work of my students. It has

$$E_1^{pq} = \text{Alt } H^p(D^q(f); \mathbb{Q}),$$

and converges to

$$H^{p+q-1}(\text{image}(f)).$$

Here $D^k(f)$ is the closure, in M^k, of the set of k-tuples of pairwise distinct points sharing the same image, and "Alt" means the subspace of the cohomology on which the symmetric group S_k, permuting the copies of M, acts by its sign representation. When f is a stable perturbation of a map-germ f_0 of corank 1, with domain a suitable analogue of a Milnor ball, then for each k, $D^k(f)$ is a Milnor fibre of the ICIS $D^k(f_0)$. In consequence, the spectral sequence collapses at E_1. For the case $k = 1$, it was already known, by Morse theory, that the image had the homotopy type of a wedge of n-spheres, and from the spectral sequence one obtains the very nice formula

$$H^n(\text{image }(f);\mathbb{Q}) \simeq \bigoplus_{k=2}^{n+1} \text{Alt } H^{n-k+1}(D^k(f);\mathbb{Q}). \quad (2.1)$$

When $k > 1$, we do not have good equations for the image, which is not even Cohen-Macaulay, so Morse theory is not available. The spectral sequence gives the first calculation of the cohomology of the image that I am aware of. All of this was written up in [28].

The reason for the *alternating* homology was still mysterious. On a third visit, Victor clarified it by introducing the alternating subcomplex of the cellular chain complex of the multiple-point spaces $D^k(f)$, and showing that its homology (which replaces the alternating part of the rational homology) is isomorphic to the homology groups occurring in the spectral sequence arising from a natural filtration of a Vassiliev-type geometric resolution of the image. The spectral sequence of alternating homology becomes the spectral sequence of homology of the filtered space, converging to the homology of the image. This gives a result in integer homology and cohomology, and is published in [14].

3 Knot theory and finite type invariants

Beginning in the early 1990s, Victor became interested in finite-type invariants of mappings in many different situations. His first paper here was *Local invariants of mappings of oriented surfaces into 3-space*, [15]. In a second paper, [17], the one admired by Jim Bryan, he shows that there are just three local basic first order integer invariants of generic mappings $f : S \to \mathbb{R}^3$. "Local" means that the invariant can be computed by counting crossings of the bifurcation set (with signs) in a generic homotopy between the given mapping f and some fixed (and arbitrary) generic mapping, and that the total should be dependent only on the type of bifurcations encountered (not on the connected component of the regular part of the bifurcation set). The three invariants are

1. $I_1 =$ the number of triple points on the surface;

2. $I_2 =$ the number of pinch points (Whitney umbrellas)

3. I_3, which is given by a remarkable formula, in which one takes sums over the connected components D of the complement of the image, and considers the degree of the map from S to a 2-sphere centred in D, given by composing f with radial projection, which

we denote by deg(D):

$$I_3 = \sum_D \deg(D)\chi(D) - \sum_t \deg(t) - \frac{1}{2}\sum_p \deg(p) \tag{3.1}$$

Here deg(t), for t a triple point, means the average of deg(D) for the eight components of $\mathbb{R}^3 \setminus f(S)$ near t, and deg(p), for p a pinch point, means the average of deg(D) for the three components of $\mathbb{R}^3 \setminus f(S)$ near p. In fact this mysterious formula arises by taking a Legendrian lift of the mapping $S \to \mathbb{R}^3$ to a mapping $S \to ST^*\mathbb{R}^3$ and then computing its self-linking number.

The paper with Chmutov [4], *The Kauffman bracket of plane curves*, applies Kauffman's 1-variable polynomial invariant of knots and links in the solid torus, to give invariants of the fronts of Lagrangian links, in the plane. They construct this bracket on Legendrian links axiomatically in terms of their fronts. The coefficients of the bracket are invariants of finite type. Two more papers with Chmutov, [5] and [6], explore this further.

He returned to this area with two papers on local finite-type invariants of maps $f : M^3 \to N^3$, [21], [22]. Here the space of invariants has rank 7:
I_1, The number of triple points, I_2 and I_3 the numbers of

positive and negative swallowtails, I_4 and I_5 the numbers of $A_2^{\pm}A_1$ points, I_6 half of the Euler characteristic of the critical locus, I_7 the linking number of the 1-jet extension of f with $\Sigma^2 \subset J^1(M, N)$.

In [18], invariants of immersed plane curves without direct self-tangencies are studied by extending them to plane curves with finitely many direct self-tangencies, just as Vassiliev invariants (i.e., finite-type invariants) of knots are studied by extending them to knots with finitely many singular points. The space of order n complex-valued invariants of oriented regular plane curves without direct self-tangencies modulo lower-order invariants is shown to be isomorphic to the dual of the space of all finite formal complex linear combinations of marked n-chord diagrams modulo the marked 4-term relation. Using his analogous result for invariants of oriented framed knots in a solid torus ([46]), Victor shows that the graded spaces of finite-order complex-valued invariants of oriented framed knots in a solid torus and of oriented regular plane curves without direct self-tangencies are isomorphic. The isomorphism is given by the Legendrian lift of the plane curves to the solid torus $ST^*\mathbb{R}^2$, which lifts the bifurcation of a direct self-tangency of a plane curve to a crossing change of a knot in the solid torus.

Further developments in this direction:

1. The Kontsevich integral for framed knots in $\mathbb{R}^3$ and in a solid torus: Legendrian knots are canonically framed, and so in particular one obtains a version of the Kontsevich integral for Legendrian knots – [16].

2. Finite type invariants for Legendrian knots in $\mathbb{R}^3$ and in a solid torus. He showed that the space of finite type invariants is isomorphic to the space of finite type invariants of *arbitrary* framed knots – [24], [49]

3. Legendrian versions of the classical polynomial knot invariants (implying estimates on the Bennequin number of Legendrian knots) – [53].

4 Collaborations

Victor is a sociable mathematician, and collaboration with him is extremely enjoyable, as well as mathematically satisfying. He is a kind and generous PhD supervisor. He is also a tireless and enthusiastic organiser of conferences, something which UK singularists are extremely grateful to him for. It is therefore fitting that as he reaches the young age of 60, we should return the favour. So it's a great pleasure to see everyone here, at this conference in his honour, and to thank him for many contributions, mathematical and personal.

5 Most Cited

Citations	*Ref.*	*Title*
32	[1]	Singularity Theory I (VVG + Arnold, Lyashko, Vassiliev) 1993
21	[28]	Vanishing cohomology of singularities of mappings (VVG+Mond), 1993
20	[34]	Singularities of projections of complete intersections,1983
20	[17]	Local Invariants of mappings of surfaces to 3-space, 1997
20	[14]	Semi-simplicial resolutions and homology of images and discriminants of mappings,1995
19	[39]	Projections of generic surfaces with boundaries, 1990.
12	[47]	Functions on space curves, 2000
11	[7]	Regular Legendrian knots and the HOMFLY polynomial of immersed plane curves, (VVG+Chmutov+Murakami), 2000
10	[5]	Polynomial invariants of Legendrian links and plane fronts, (VVG+Chmutov), 1997.
9	[16]	Finite order invariants of framed knots in a solid torus and in Arnold's J+theory of plane curves, 1997.
9	[18]	Vassiliev type invariants in Arnold's J+ theory of plane curves without direct self-tangencies, 1998
8	[56]	Simple symmetric matrix singularities and the subgroups of Weyl groups A_n, D_n, E_n, (VVG+Zakalyukin), 2003
7	[10]	Cohomology of braid groups of series C and D, 1981
7	[3]	Sectional singularities and geometry of families of planar quadratic forms, (VVG+Bruce+Zakalyukin), 2002
7	[45]	Unitary reflection groups associated with singularities of functions with cyclic symmetry, 1999
6	[11]	Geometry of the bifurcation diagrams of simple projections onto a line, 1981
6	[5]	Polynomial invariants of Legendrian links and their fronts, (VVG+Chmutov), 1997
6	[46]	Vassiliev invariants of knots in R3 and in a solid torus, 1999.
5	[41]	Monodromy of the image of the mapping $\mathbb{C}^2 \to \mathbb{C}^3$, 1991
5	[55]	Tjurina and Milnor numbers of matrix singularities (VVG+Mond), 2005
5	[57]	On the stability of projections of Lagrangian manifolds with singularities, (VVG+Zakalyukin), 2004
4	[50]	Unitary reflection groups and automorphisms of simple hypersurface singularities, 2001
4	[27]	The complex crystallographic groups and symmetries of J_{10}, (VVG+Show Han Man), 2006.
4	[13]	Symmetric quartics with many nodes, 1994.

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