Celebrating James William (Bill) Bruce 60th birthday

Farid Tari, ICMC-USP, São Carlos

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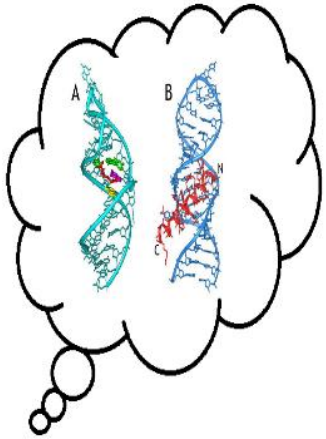
All or nothing: Climbing the Suicide Wall in North Wales

He wanted to become a biochemist



...became a miner

...a builder

...an international footballer

...a babysitter

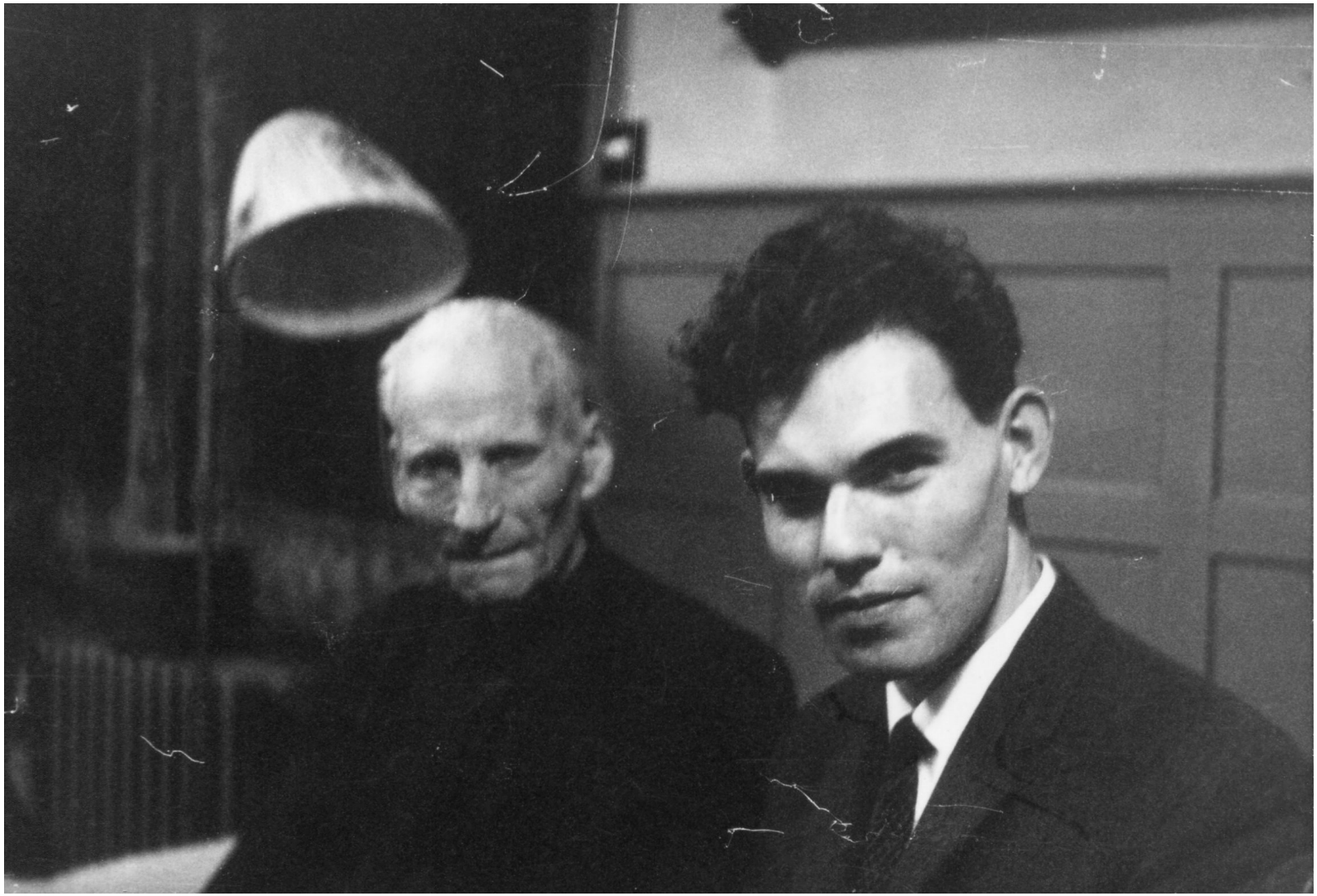
Bill's family

- Born in Liverpool on 11th January, 1952
- Married to Linda, two children Emily and Isaac and a granddaughter Imogen

Bill's family

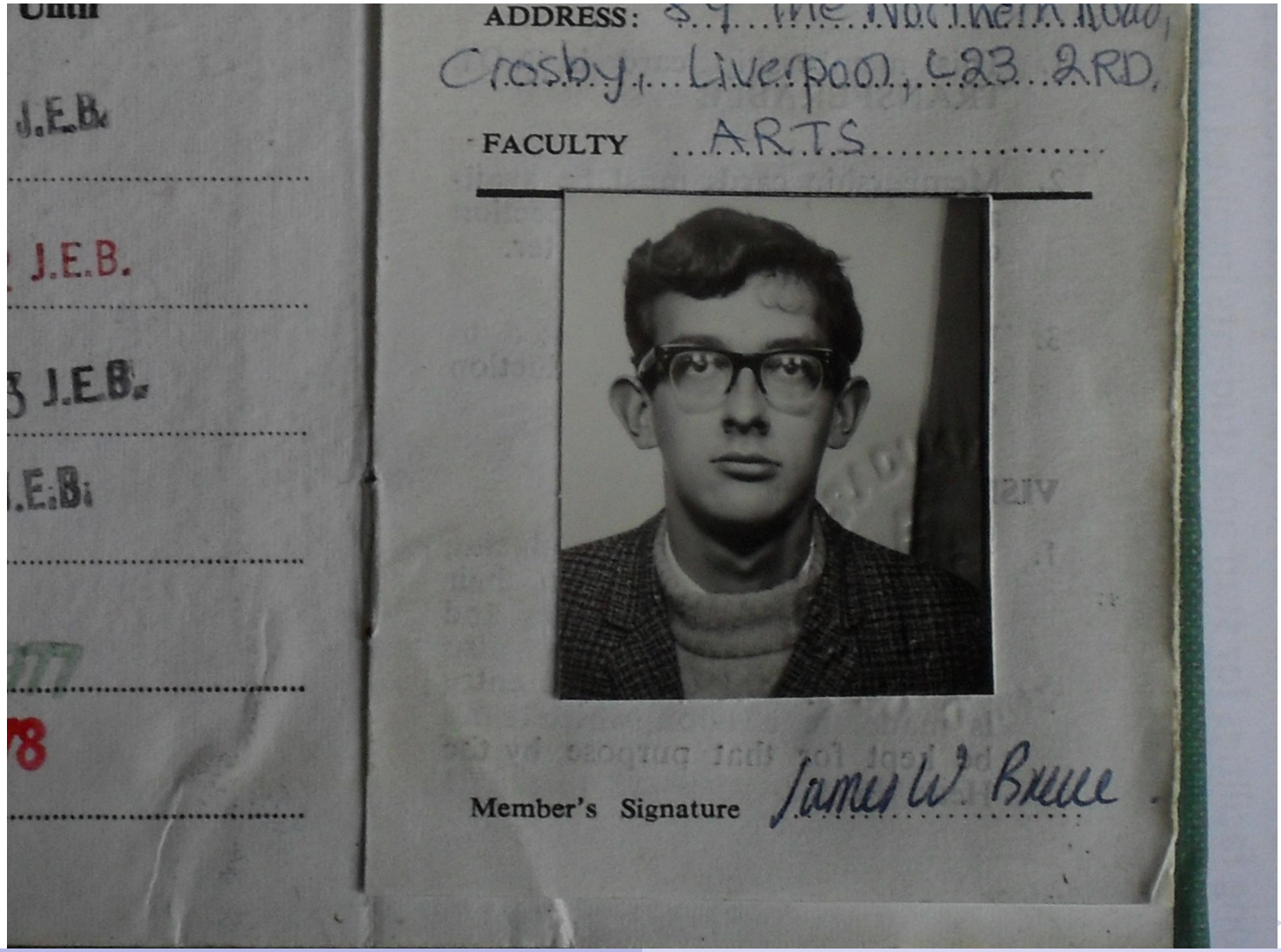
Bill's family

Rescued by Chris Gibson! Chris with L. E. J. Brouwer



Undergraduate studies

Had intended to become a biochemist and ended up in Liverpool because he changed subjects so late!



Postgraduate studies

- Did an MSc with Peter Giblin, got married to Linda and went into teaching in a school for 2 years.
- But he could not resist being away from mathematics!
  Back to Liverpool to do a PhD (in 2 years!) with Terry Wall.

First recollection of Terry:
"*Bit of a daunting figure for a naive first year student, remember him dressed in a stylish green corduroy jacket at one of the reading weeks students used to have in those days. He must have been in the midst of his great work on surgery but he took the time to explain some pretty basic ideas on group theory to me. Even then recognised I was in the presence of a brain the size of the proverbial planet.*"

The PhD thesis

Linda: What do you do for your thesis?

Bill: I have a 19 dimensional space and it is cut up into pieces, I don't not know where the pieces are but I need to show that they fit together nicely.

Linda: um...

She never asked him again about his research.

IHES: if it doesn't break you it makes you.

Terry on Bill

One or two memories:
Had an excellent undergraduate record, but I didn't have much contact at this stage.
Wrote a magnificent M.Sc. thesis about the Milnor fibration, vanishing cycles etc. All in his beautiful handwriting.
Had a break teaching in school before returning to research.
An excellent researcher, full of ideas, leading to lots of papers, many of which could have been (could still be ?) developed much further.
I do remember him saying that when he went on a coach trip with Linda, they would prepare a packed lunch before leaving home, and then eat it before the coach set out!

What Bill says about his teachers

"As an 18 year old I did not realise just how lucky I was to be with my teachers.
Chris Gibson made mathematics sound intriguing and mysterious to me as a 17 year old, and so exciting too. He took enormous trouble to guide me when I was starting out.
Peter Giblin was the best teacher I ever came across: meticulous, clear, witty, engaging, someone who also invested huge amounts of time and effort in his students.
I had the enormous good fortune to have one of the world's great mathematicians as a mentor. Terry Wall taught me that mathematics is an experimental subject, he taught me the difference between learning mathematics and creating it. He also arranged for me to leave Liverpool for the first time and to go off to IHES for a year.
I had the great fortune to write some papers with each of them."

Terry Wall (with Carla Ranicki)

Bill with Chris Gibson and Peter Giblin

J.W. Bruce the mathematician

MathScienet
Total Publications: 103
Total Author/Related Publications: 105
Total Citations: 610

Publications
Algebraic geometry, Differential geometry, Global analysis, analysis on manifolds, Manifolds and cell complexes, Number theory, Numerical analysis, Optics, electromagnetic theory, Ordinary differential equations, Several complex variables and analytic spaces

Research

- Singularity theory
- Generic geometry
- Differential equations

Singularity theory

- Stratifications
- Singular points on complex projective hypersurfaces
- Functions and mappings on singular varieties
- Liftable and lowerable vector fields
- Determinacy
- Classifications

Singularity theory: a selection of papers

- An upper bound for the number of singularities on a projective hypersurface. **Bull Lond Math Soc**
- Critical points of functions on analytic varieties. **Topology.** (With R. M. Roberts)
- Vector fields on discriminants and bifurcation varieties. **Bull. London Math. Soc.**
- Discriminants and liftable vector fields. **J. Algebraic Geom.** (With A. A. du Plessis and L. C. Wilson)
- Functions on discriminants. **J. London Math. Soc.**
- Generic functions on semialgebraic sets. **Quart. J. Math. Oxford Ser.**
- Determinacy and unipotency. **Invent. Math.** (With A. A. du Plessis and C. T. C. Wall)
- Complete transversals and the classification of singularities. **Nonlinearity** (With N. P. Kirk and A.A. du Plessis)
- On families of symmetric matrices. **Mosc. Math. J.**

Finite determinacy

Two map-germs f, g are $\mathcal{A}$-equivalent if the following diagram commutes

$$\begin{array}{ccc} \mathbb{K}^n, 0 & \xrightarrow{f} & \mathbb{K}^p, 0 \\ \downarrow h & & \downarrow k \\ \mathbb{K}^n, 0 & \xrightarrow{g} & \mathbb{K}^p, 0 \end{array}$$

There are other groups: $\mathcal{R}$, $\mathcal{L}$, $\mathcal{C}$, $\mathcal{K}$ (Mather groups)
A map-germ f is k-$\mathcal{G}$-determined if any map-germ g with $j^k g = j^k f$ is $\mathcal{G}$-equivalent to f.
Questions: When is a map-germ (finitely) $\mathcal{G}$-determined? If it is, what is its degree of determinacy?

The answer

Theorem

Let $\mathcal{H}$ be a strongly Z-closed subgroup of $\mathcal{G}$. Then for any $r < \infty$, f is r-$\mathcal{H}$-determined **if and only if** *there is a strongly closed subgroup $U \subset \mathcal{H}$ of $\mathcal{G}$, with J^1U unipotent, for which*

$$m_n^{r+1}.C_n^{\times p} \subset LU \cdot f.$$

Theorem

If f satisfies

$$m_n^{l}.C_n^{\times p} \subset L\mathcal{K} \cdot f$$
$$m_n^{r+1}.C_n^{\times p} \subset L\mathcal{A}_1 \cdot f + m_n^{l+r+1}.C_n^{\times p}$$

then f is r-$\mathcal{A}_1$-determined.

The complete transversal and classification of map-germs

Theorem

Let $f : \mathbb{K}^n, 0 \to \mathbb{K}^p, 0$ be a smooth germ and $h_1, \ldots, h_l$ be homogeneous polynomials of degree $k+1$ with the property that

$$m_n^{k+1}.C_n^{\times p} \subset L\mathcal{G}_1 \cdot f + sp\{h_1, \ldots, h_r\} + m_n^{k+2}.C_n^{\times p}.$$

Then any germ g with $j^k g = j^k f$ is $\mathcal{G}_1$-equivalent to a germ of the form $f(x) + \sum_{i=1}^{l} u_i h_i(x) + \phi(x)$, where $\phi(x) \in \mathcal{M}_n^{k+2}$.
The vector subspace $sp\{h_1, \ldots, h_r\}$ is called the complete $(k+1)$-$\mathcal{G}$-transversal of f.

The Milnor number

$f : \mathbb{C}^n, 0 \to \mathbb{C}$, a holomorphic function.

$$\mu(f) = \dim_{\mathbb{C}} \frac{\mathcal{O}_n}{J(f)}$$

Let $X, 0$ a germ of an analytic subvariety of $\mathbb{C}^n, 0$ $\mathcal{O}_{X,0}$ denote the $\mathcal{O}_n$-module of germs of vector field which are tangent to X. For $f \in \mathcal{O}_n$, define $J_X(f)$ as the ideal in $\mathcal{O}_n$ given by the image of $\mathcal{O}_{X,0} \to \mathcal{O}_n$, given by $\xi \mapsto \xi f$. The Milnor number:

$$\mu_X(f) = \dim_{\mathbb{C}} \frac{\mathcal{O}_n}{J_X(f)}$$

The Milnor number

For many naturally occurring varieties (e.g. free divisors, ICIS, quotients of finite unitary representations) one can define this new "Milnor number", and new "Morse singularity" so that

- "Morse singularities" are characterised by minimal "Milnor number",
- most germs have "Morsifications",
- finite Milnor number implies finite determinacy (preserving X),
- Milnor numbers are preserved under deformation.

Generic geometry

- Gauss maps and duals
- Caustics from mirrors
- Symmetry sets
- Geometry of singular sets
- Duality results
- Geometry of one parameter families of surfaces

Generic geometry: a selection of papers

- The duals of generic hypersurfaces. **Math. Scand.**
- Projections and reflections of surfaces in $\mathbb{R}^3$. **M. Scand.**
- Geometry of singular sets. **Math. Proc. Camb. Philos. Soc.**
- Duality and orthogonal projections of curves and surfaces in Euclidean 3-space. **Quart. J. Math. Oxford Ser.** (With M. C. Romero Fuster)
- Folding maps and focal sets. Singularity theory and its applications. **Lecture Notes in Math.** (With T. C. Wilkinson)
- Generic geometry, transversality and projections. **J. LMS.**
- Images and varieties. **J. Math. Sci.** (With W.L. Marar)
- Functions on a crosscap. Math. **Proc. Camb. Philos. Soc.** (With J.W. West)
- Families of surfaces: height functions, Gauss maps and duals. **Pitman Res. Notes Math. Ser.** (P.J. Giblin and F. T.)

A flavor of the duality results

M a smooth surface in $\mathbb{R}^3$.
$d^2 : M \times \mathbb{R}^3 \to \mathbb{R}$ the family of distance squared functions.
$F : M \times \mathbb{R}^3 \to \mathbb{R}^3$ the family of folding maps.

Theorem

For a generic surface, the bifurcation sets of the families of distance squared functions and of the folding maps are generic and are dual to each other.

Differential equations

- Generic implicit differential equations
- Principle and asymptotic curves on surfaces
- Geometric one-parameter families of IDEs
- Duality
- Bifurcations

Differential equations: a selection of papers

- A note on first order differential equations of degree greater than one and wavefront evolution. **Bull. LMS.**
- On binary differential equations and umbilics. **Proc. Roy. Soc. Edinburgh Sect. A** (With D.L. Fidal)
- On binary differential equations. **Nonlinearity** (With F. T.)
- Generic 1-parameter families of binary differential equations of Morse type. **Dis. Contin. Dynam. Sys.** (With F. T.)
- On the multiplicity of implicit differential equations. **J. Diff. Eqs.** (With F. T.)
- Dupin indicatrices and families of curve congruences. **Trans. Amer. Math. Soc.** (With F. T.)
- Duality and implicit differential equations. **Nonlinearity** (With F. T.)

Bill on Bill's ideas

- Realisation that an envelope is just a discriminant
- Vanishing cycles of singularities on the same fibre don't intersect. (Upper bound for number of singular points of a complex hypersurface of degree d influenced some very interesting work of Arnold and his Moscow school)
- Implicit differential equations need disentangling or blowing up
- Lots of things that you can do for manifolds you can do for nice singular objects
- Determinacy is really about solving equations, not integrating vector fields
- Hartog's theorem is the closest thing to magic that you can get in mathematics
- Duality is really fascinating (and preserves contact)
- Seeing how things change with time is interesting

Books

- J.W.Bruce and P.J.Giblin, Curves and Singularities, Cambridge University Press, 1984. Second Edition, 1992. *Translated into Russian, published by Mir, 1988*.
- J.W.Bruce, P.J.Giblin and P.J.Rippon, Microcomputers and Mathematics, Cambridge University Press, 1990.
- J.W.Bruce, G.Cobb, G.Davidoff, A.H.Durfee, J.A.Gifford, D.O'Shea, M.Peterson, H.Pollatsek, M.Robinson, L.Senechal, R.Weaver, Laboratories in Mathematical Experimentation: A Laboratory Bridge Course, Springer Verlag, 1997. *Translated into Chinese.*
- J.W.Bruce, G.Cobb, G.Davidoff, A.H.Durfee, J.A.Gifford, D.O'Shea, M.Peterson, H.Pollatsek, M.Robinson, L.Senechal, R.Weaver, Instructor's Manual for Laboratories in Mathematical Experimentation: A Laboratory Bridge Course, Springer Verlag, 1997. *Translated into Chinese.*

Collaborators

du Plessis, Andrew A.
Fidal, D. L.
Fletcher, G. J.
Gaffney, Terence
Giblin, Peter J.
Gibson, Christopher George
Goryunov, Victor V.
Hurley, Donal J.
Kirk, Neil
Marar, Washington Luiz
Nogueira, Antônio Carlos
Rippon, Philip J.
Roberts, Richard Mark
Romero Fuster, Mara del Carmen
Ruas, Maria Aparecida Soares
Saia, Marcelo Jos
Tari, Farid
Wall, Charles Terence Clegg
West, Janet Mary
Wilkinson, T. C.
Wilson, Leslie Charles
Zakalyukin, Vladimir M

Bill's main partners in crime

PhD students and Pos-docs

PhD students

- Gareth Haslinger, University of Liverpool
- Neil Kirk, University of Liverpool
- Antonio Nogueira, Universidade de So Paulo
- Christopher Rimmer, University of Liverpool
- Janet West, University of Liverpool
- Timothy Wilkinson, University of Newcastle

Pos-Docs:

- David Fidal
- Neil Kirk
- Kevin Houston
- Me!

One more job: Bill the photographer

A selection of professional activities

- London Mathematical Society Education Committee
- Vice-President London Mathematical Society
- Editor London Mathematical Society Graduate Texts
- Various EPSRC PanelsChair EPSRC Maths Fellowships Committee
- EPSRC Maths Fellowships Committee
- Various EPSRC Panels

In management

Appointments

- PVC (Academic) Edge Hill University, 2009-present
- Deputy VC and PVC Academic, University of Hull, 2004–2009
- Acting Dean Social and Environmental Studies, University of Liverpool, 2003–2004
- Pro-Vice-Chancellor, University of Liverpool, 1999–2003

The man who makes a difference

Hull University before Bill
£2m deficit

Hull University by the time Bill left

- £15m surplus
- research outputs up by 40%
- had increased its student intake by around 20%
- University held a top 10 position in the NSS (mainstream universities in England).

Thank you:

for the beautiful mathematics you have done

for inspiring many of us

for being a great collaborator

for your friendship

Happy Birthday!!!