Trigonometric Identities

Pythagoras's theorem

$$\sin^2\theta + \cos^2\theta = 1 \tag{1}$$

$$1 + \cot^2\theta = \mathrm{cosec}^2\theta \tag{2}$$

$$\tan^2\theta + 1 = \sec^2\theta \tag{3}$$

Note that (2) = (1)/ $\sin^2\theta$ and (3) = (1)/ $\cos^2\theta$.

Compound-angle formulae

$$\cos(A+B) = \cos A\cos B - \sin A\sin B \tag{4}$$

$$\cos(A-B) = \cos A\cos B + \sin A\sin B \tag{5}$$

$$\sin(A+B) = \sin A\cos B + \cos A\sin B \tag{6}$$

$$\sin(A-B) = \sin A\cos B - \cos A\sin B \tag{7}$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B} \tag{8}$$

$$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B} \tag{9}$$

$$\cos 2\theta = \cos^2\theta - \sin^2\theta = 2\cos^2\theta - 1 = 1 - 2\sin^2\theta \tag{10}$$

$$\sin 2\theta = 2\sin\theta\cos\theta \tag{11}$$

$$\tan 2\theta = \frac{2\tan\theta}{1-\tan^2\theta} \tag{12}$$

Note that you can get (5) from (4) by replacing B with $-B$, and using the fact that $\cos(-B) = \cos B$ (cos is even) and $\sin(-B) = -\sin B$ (sin is odd). Similarly (7) comes from (6). (8) is obtained by dividing (6) by (4) and dividing top and bottom by $\cos A\cos B$, while (9) is obtained by dividing (7) by (5) and dividing top and bottom by $\cos A\cos B$. (10), (11), and (12) are special cases of (4), (6), and (8) obtained by putting $A = B = \theta$.

Sum and product formulae

$$\cos A + \cos B = 2\cos\frac{A+B}{2}\cos\frac{A-B}{2} \tag{13}$$

$$\cos A - \cos B = -2\sin\frac{A+B}{2}\sin\frac{A-B}{2} \tag{14}$$

$$\sin A + \sin B = 2\sin\frac{A+B}{2}\cos\frac{A-B}{2} \tag{15}$$

$$\sin A - \sin B = 2\cos\frac{A+B}{2}\sin\frac{A-B}{2} \tag{16}$$

Note that (13) and (14) come from (4) and (5) (to get (13), use (4) to expand $\cos A = \cos(\frac{A+B}{2} + \frac{A-B}{2})$ and (5) to expand $\cos B = \cos(\frac{A+B}{2} - \frac{A-B}{2})$, and add the results). Similarly (15) and (16) come from (6) and (7).

Thus **you only need to remember (1), (4), and (6)**: the other identities can be derived from these.