

Class Test MATH102 Monday 25 February 2008

1. Write down the Taylor series at 0 of $f(x) = e^{-x}$. [3 marks]

2. Write down the quadratic Taylor polynomial $P_2(x) = P_2(x, 4)$ near $x = 4$ for $f(x) = x^{1/2}$, and work out $P_2(5)$. Write down an expression for the remainder term $R_2(x)$. Now find an upper bound on $|R_2(5)|$ and hence show that

$$|\sqrt{5} - P_2(5)| \leq 2^{-9} = 0.001953125.$$

Confirm this by using your calculator to compute $\sqrt{5}$.

[9 marks]

3. Solve

$$\frac{dy}{dx} + xy = 0, \quad y(0) = 2.$$

[5 marks]

4. Solve

$$x \frac{dy}{dx} + 4y = x^3, \quad y(1) = 0.$$

[7 marks]

5. Find the general solution to

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = 0.$$

[3 marks]

6. Find the general solution to

$$\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = x.$$

[7 marks]

7. Show that

$$\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 + 2xy}{x^2 + y^2}$$

does not exist, by calculating limits along two different directions at $(0, 0)$.

[4 marks]