ABSTRACT

Boundary layers on concave surfaces differ from those on flat plates due to the presence of Taylor Goertler (T-G) vortices. These vortices cause momentum transfer normal to the blade's surface and hence result in a more rapid development of the laminar boundary layer and a fuller profile than is typical of a flat plate. Transition of boundary layers on concave surfaces also occurs at a lower Re than on a flat plate. Concave surface transition correlations have been formulated previously from experimental data, but they are not comprehensive and are based on relatively sparse data. The purpose of the current work was to attempt to model the physics of both the laminar boundary layer development and transition process in order to produce a transition model suitable for concave surface boundary layers. The development of the laminar boundary layer on a concave surface was modeled by considering the profiles at the upwash and downwash locations separately. The profiles of the boundary layers at these two locations were then combined to successfully approximate the spanwise averaged profile. The profiles of the boundary layers at these two locations were then combined to successfully approximate the spanwise averaged profile. The ratio of the boundary layer thicknesses at the two locations was found to be as great as 50 and this leads to laminar boundary layer shape factors as low as 1.3 and skin friction coefficients up to 12 times the value for a flat plate laminar boundary layer. Boundary layers therefore grow much more rapidly on concave surfaces than on flat plates. The transition model assumed that transition commenced in the upwash location boundary layer at the same transition inception Reθ observed on a flat plate. Transition at the downwash location then results from the growth of turbulent spots from the upwash location rather than through the initiation of spots. The model showed that initially curvature promotes transition because of the thickened upwash boundary layer, but for strong curvature the T-G vortices effectively stabilise the boundary layer and transition then occurs at a higher Reθ than on a flat plate. Results from the transition model were in broad agreement with experimental observations. The current work therefore provides a basis for the modeling of transition on concave surfaces.

INTRODUCTION

Traditional transition prediction models developed from flat plate data fail to predict the transitional behaviour of boundary layers on the pressure surface of gas turbine blades. This is because of the destabilising effect of the centrifugal pressure gradient induced by the blade curvature which leads to the formation of Taylor Goertler (T-G) vortices. These vortices can only be observed in the quiescent conditions of a laboratory wind tunnel, but the consequences of their presence in terms of increased skin friction coefficient and premature transition is observed on gas turbine blades. Many researchers e.g. [1-10] have studied the transition of boundary layers on concave surfaces over several decades through experiment, but to date no reliable predictive technique has been developed for either the enhanced development of the laminar boundary layer or its transition. The objective of the current work is to address these issues.

TAYLOR GOERTLER VORTICES

T-G vortices have been observed in several experimental studies e.g. Shigemi et al. [2]. These vortices have fixed spanwise locations for Goertler numbers G ≤ 5 and distinct upwash and downwash boundary layer profiles can be measured at fixed spanwise locations. At higher G, e.g. Hachem and Johnson [4] the vortices start to meander in the spanwise direction and hence time averaged boundary layer profiles are invariant with spanwise position. Although the T-G vortices can not be observed at high G, their influence is apparent through the distorted boundary layer profiles and enhanced skin friction.

LAMINAR BOUNDARY LAYER PROFILES

It should be noted that all the boundary layer profiles discussed in this paper are flat plate equivalent boundary layer FPEBL profiles. The FPEBL is derived from the actual boundary layer profile using the procedure of Riley et al (5) given in the
Shigemi et al. [2] took detailed measurements of laminar boundary layers on a concave surface at low $G_o$, where the T-G vortices had stable positions. Their measurements showed that the boundary layer thickness varied by at least a factor of two between the upwash (U) and downwash (D) locations. The boundary layer profiles at each location were however undistorted from the flat plate profile with shape factors $H$ of approximately 2.5. For boundary layers at a higher $G_o$, the spanwise averaged boundary layer profiles have a very characteristic shape as shown in figure 1. The velocity gradient at the wall is high, but once the velocity reaches 60 to 80% of the freestream velocity a distinct kink is seen in the profile and a much lower velocity gradient exists in the outer part of the layer. The reason for this sudden change in velocity gradient is the large boundary layer thickness variation in the spanwise direction. To illustrate this consider the boundary layer thickness to vary in the spanwise direction according to the square wave shown in figure 2. The upwash and downwash profiles are also assumed to be zero pressure gradient Pohlhausen profiles such that

$$u/U = 2 \left( \frac{y}{\delta} \right)^2 - 2 \left( \frac{y}{\delta} \right)^3 + \left( \frac{y}{\delta} \right)^4$$

(1)

Figure 1. Typical laminar boundary layer profile on a concave surface.

Now considering the profile in figure 1, we can determine the values of the three quantities shown in figure 2, i.e. the upwash boundary layer thickness $\delta_U$, the proportion of the span occupied by the upwash profile $s_U$ and the upwash to downwash boundary layer thickness ratio $r$, using a least r.m.s. error technique. This leads to the upwash and downwash profiles and overall fitted profile shown in figure 1. The figure shows that although a square wave is a very crude approximation to the actual spanwise variation an acceptable fit to the measured data is obtained. The figure also clearly shows that the kink in the overall profile is due to the rapid change in velocity gradient associated with the edge of the downwash boundary layer.

The square wave model is used throughout this paper to represent the spanwise variation in boundary layer thickness resulting from the action of the T-G vortices. However the model can only be used to predict the growth of laminar boundary layers on concave surfaces, if a relationship between $s_U$ and $r$ and the flow conditions can be established. The experimental results of Hachem and Johnson [3] were used for this purpose. Hachem and Johnson measured laminar boundary layer development on two constant radius concave surfaces at two different tunnel speeds and subjected to nominally zero freestream pressure gradients. In the present work, their profiles are defined by two parameters, the shape factor $H$ and $C_f \cdot Re$, which characterises the velocity gradient at the wall. For a flat plate zero pressure gradient boundary layer these parameters have the values 2.54 and 0.47 respectively, but it can be seen from figure 3 that Hachem and Johnson measured shape factors as low as 1.3 and $C_f \cdot Re$ values as high as 6 on their concave surfaces.

Figure 2. Square wave model for spanwise variation in boundary layer thickness.

Figure 3. Skin friction coefficient and shape factors for laminar boundary layers on concave surfaces.

The boundary layer profile depends on the blade radius $R$, the freestream velocity $U$, the fluid density $\rho$ and viscosity $\mu$ and
the distance $x$ along the blade at which the profile is measured. In terms of dimensionless parameters this reduces to

$$C_f \frac{Re}{R} = f\left(Re_R, \frac{x}{R}\right)$$

and

$$H = f\left(Re_R, \frac{x}{R}\right)$$

Hence

$$s_{U} = f\left(Re_R, \frac{x}{R}\right)$$

and

$$r_d = f\left(Re_R, \frac{x}{R}\right)$$

$s_{U}$ and $r_d$ were evaluated for the data in figure 3 and are shown as functions of $\frac{x}{R}$ in figures 4 and 5.

Figure 4 suggests that the dependence of $s_{U}$ on $\frac{x}{R}$ is much stronger than on $Re_R$ and hence the equation

$$s_{U} = 0.1 + 0.9 \left(1 - \exp\left(-0.8 \frac{x}{R}\right)\right)$$

is a reasonable fit to the data, $r_d$ on the other hand does show dependence on both $Re_R$ and $\frac{x}{R}$. The discontinuity in Figure 5 at $\frac{x}{R} = 0.85$ suggests that $r_d$ scales with $\frac{1}{Re_R}$. A least r.m.s. curve fit to the data replotted against $\frac{x}{R} \frac{1}{Re_R}$ (see figure 6) provides the correlation equation

$$r_d = 1 + 2.5 \times 10^6 \frac{x}{R} \frac{1}{Re_R}$$

Figure 6 therefore provides a model for the boundary layer profile on a constant curvature concave surface with a zero streamwise pressure gradient.

LAMINAR BOUNDARY LAYER DEVELOPMENT

The overall skin friction coefficient $C_f$ and displacement thickness $\delta^*$ are given by simple weighted averages of the upwash and downwash values

$$C_f = s_{U} C_{fU} + (1 - s_{U}) C_{fD}$$

and

$$\delta^* = s_{U} \delta_U^* + (1 - s_{U}) \delta_D^*$$

The velocity product within the momentum thickness integral gives rise to a $s_{U}(1 - s_{U})$ term

$$\frac{\frac{1}{\delta}}{\frac{1}{\delta}} = s_{U} \frac{37}{315} + (1 - s_{U}) \frac{37}{315}$$

$$s_{U} \left(1 - s_{U}\right) \left(115r_d^5 - 263r_d^4 + 168r_d^3 - 27r_d^2 + 7\right)$$

$$\frac{330r_d^3}{630r_d^3}$$
Equations 8, 9 and 10 can therefore be used to determine the laminar boundary layer development from the boundary layer momentum equation

\[ \frac{d\theta}{dx} = C_f \frac{\theta}{2} \]  

Figure 7 shows the predicted laminar boundary layer development for a range of blade radius Reynolds numbers. The flat plate boundary layer development is shown for comparison. For low Re_x, before the T-G vortices have developed significantly, the boundary layer growth is identical to that on a flat plate. The T-G vortices develop most rapidly on the blades with low and hence the boundary layer growth Re_x increases above the flat plate value earliest for the lowest Re_R values.

Figure 8 shows the development of the laminar boundary layer for a blade where Re_R = 800,000. The T-G vortices begin to influence the flow at about Re_x = 30,000 when the upwash boundary layer begins to develop faster than the flat plate profile and the downwash boundary layer more slowly. By Re_x = 250,000 the T-G vortices are sufficiently strong to remove low energy fluid from the downwash location faster than it is formed and hence the boundary layer at this location begins to thin. At Re_x = 1,000,000 the upwash boundary layer is more than four times the thickness of the downwash boundary layer. The pre-transitional measurements of Zhang et al. [6], which were taken on a constant curvature blade also with Re_R = 800,000, are shown in the figure. The measured Re_\theta values are lower than the predictions, but the ratio between the values at up and downwash is reliably predicted. The discrepancy in the absolute values could be due to a slightly favourable pressure gradient upstream of the first measurement point or because of the limitations of the square wave model.

**TRANSITION INCEPTION**

Shigemi et al determined transition inception at both the upwash and downwash spanwise locations and found that transition occurred slightly earlier at the upwash location. The transition inception Re_\theta at the upwash location was very similar to that for a flat plate boundary layer transition, whereas the value at the downwash location was much lower than the flat plate value. If it is assumed that transition commences at the upwash location in a similar manner to transition on a flat plate and then the spots spread laterally into the neighboring downwash regions, this would lead to the observed premature transition at the downwash locations. Shigemi et al observed T-G vortices at a regular spacing of 25 mm across the span of their blade. If their spots spread at a typical angle of 10 degrees then they would spread laterally by a quarter vortex wavelength in a streamwise distance of about 35 mm which is consistent with their experimental observations. A transition model where transition commences when the upwash Re_\theta reaches the value required for transition on a flat plate will therefore be used in the current work. Mayle’s [10] criterion for transition on a flat plate

\[ Re_\theta = 400Tu^{0.625} \]  

is used as the upwash Re_\theta value at the transition inception.

Transition inception Re_\theta results for concave surfaces have been correlated by previous researchers, against a number of different dimensionless parameters including the Goertler number G_o, the boundary layer momentum thickness to radius ratio 1/Re and the blade radius Reynolds number Re_R. As G_o, 1/Re and Re_R are all related through Re_\theta, any correlation can be formulated in terms of any one of these parameters. In the current paper, the predicted transition inception Re_\theta values are plotted against the inverse of the blade radius Reynolds number Re_R in figure 9. This figure shows how curvature can reduce the transition Re_\theta by up to 50%. This effect occurs at low curvature for low freestream turbulence levels as the T-G vortices are able to develop to a greater extent prior to transition occurring. Stronger curvature (higher 1/Re) does however continue to promote transition. This is because ultimately the T-G vortices mix out the differences between the upwash and downwash locations(\Sigma u = 1) and transition is predicted at the same Reynolds number as on a flat plate. The empirical concave curvature transition correlation of Mayle [10] is also shown in the figure. Mayle’s correlation for Tu = 0.7%
Figure 9. Predicted transition inception on concave surfaces.

shows good agreement with the current prediction, however for Tu = 2.6% the correlation suggests that transition is delayed by increasing curvature whereas the prediction indicates the opposite. Although the square wave model used in this paper can lead to a modest increase in the transition inception Reynolds number from the flat plate value, this increase is limited because the up and downwash boundary layers are assumed to retain zero pressure gradient Pohlhausen profiles. It must be concluded therefore that when the curvature effects are strong, the boundary layer profiles at the up and downwash locations are distorted by the TG vortices. Profiles measured by e.g. Winoto and Low [3] show distortions which can not be approximated by the current simple square wave model.

TRANSITION PATH

Johnson and Ercan [11] developed a transition prediction method for flat plate boundary layers and showed that it was capable of accurately predicting the transition process for the ERCOFTAC T3A, B and C test cases. This predictive procedure is used in the current paper to predict transition on concave surfaces. The only modification which has been made is to make the same assumption which was made to obtain figure 9, namely that transition will commence in the laminar boundary layer at the upwash location.

Calculations were performed for four constant radius concave surfaces and a flat surface. The freestream velocity and freestream turbulence level were U = 6m/s and Tu = 1% respectively. The Johnson and Ercan transition model is also sensitive to the freestream turbulence length scale. This was set at 6 mm which results in a transition inception location of Re₂ = 404 on the flat plate which is consistent with the Mayle transition inception correlation (equation 12).

Figure 10 shows the variation of the shape factor H through transition on each of the blades. The points in the transition process where the intermittency was 1%, 25%, 50%, 75% and 99% are also marked. On the flat plate transition follows the classic path commencing at Reₚ = 345,000 when H = 2.54 and completing at Reₚ = 708,000 when H = 1.46. Curvature has the effect of reducing the Reₚ at which transition commences, although the transition length remains more or less constant. The shape factor value at the transition inception also drops with increasing curvature, until for Reₚ = 200,000 the value is only 1.65, which is lower than for a turbulent boundary layer with the same Reₚ and so the shape factor actually increases during the early part of transition up to an intermittency of 25%. The measurements from Zhang et al. [6] in the figure for Reₚ = 800,000 show transition inception close to the predicted value, however the measurements indicate a shorter transition length.

Figure 11. Skin friction coefficient Cₚ variation through transition.

The variation in the skin friction coefficient Cₚ along each of the blades is plotted in figure 11. Increased curvature increases the skin friction in the laminar boundary layer due to the action of the T-G vortices. The effect is most dramatic for the Reₚ = 200,000 blade where Cₚ is approximately three or four times the flat plate value up to Reₚ = 500,000. The implications of this are important in gas turbine blade design where the Reₚ
values are commonly in the range 200,000 to 500,000. The current results clearly show that if flat plate boundary layer theory is used the skin friction and hence heat transfer rate will be grossly under predicted.

6) The present study suggests that transition commences at the upwash location, where the boundary layer has its maximum thickness, when $Re_\theta$ at that location reaches the value which would initiate transition on a flat plate at the same freestream turbulence level.

7) The present study suggests that spreading of turbulent spots from the upwash location rather than the initiation of turbulent spots causes transition at the downwash location.

8) Curvature promotes transition initially, because the T-G vortices produce a thickened boundary layer at the upwash location. For strong curvature however the transition inception $Re_\theta$ can exceed the flat plate value.

9) The current work implies that if the laminar boundary layer development on a concave surface can be predicted using a RANS calculation then transition can be predicted successfully using a flat plate correlation and the upwash $Re_\theta$ value.

FURTHER WORK

1) The model used in the current paper for the development of the laminar boundary layer is crude. The model can be improved by using more detailed experimental or numerical results from laminar boundary layers on concave surfaces.

2) The current work has only been undertaken for zero streamwise pressure gradient flows. The work suggests however that with a streamwise pressure gradient the boundary layer development could be modelled by assuming that the boundary layer profiles at the upwash and downwash locations are the appropriate non-zero pressure gradient flat plate profiles. This needs to be verified from experimental or numerical results.

3) The current work only considers constant curvature blades and therefore has limited application for turbomachinery blades which have variable curvature. The method can be extended to variable curvature blades if a suitable laminar boundary layer model can be developed.

NOMENCLATURE

$C_f$ Skin friction coefficient  
$G$ Goertler number  
$H$ Shape factor  
$r_s$ Ratio of upwash to downwash boundary layer thickness  
$R$ Blade radius  
$Re_R$ Blade radius Reynolds number  
$Re_s$ Surface length Reynolds number  
$Re_\theta$ Momentum thickness Reynolds number  
$s_u(1-s_u)$ Proportion of span occupied by upwash profile  
$T_u$ Freestream turbulence level  
$u$ Local velocity  
$u'$ Freestream velocity  
$x$ Distance measured along blade from leading edge  
$y$ Wall normal co-ordinate  
$\delta$ Boundary layer thickness  
$\theta$ Boundary layer momentum thickness  
$\lambda$ Taylor Goertler vortex spanwise wavelength  
$\nu$ Kinematic viscosity

Subscripts

D Downwash location  
U Upwash location
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REFERENCES


APPENDIX

On a concave surface with a constant radius of curvature R, the freestream velocity U varies with the distance y from the wall according to the free vortex condition $U(R - y) = \text{constant}$. The fact that the velocity continues to increase beyond the boundary layer edge means that this point can not be identified as simply as for a flat plate boundary layer. To overcome this problem the flat plate equivalent boundary layer (FPEBL) was determined from the experimental velocities for each boundary layer using the technique due to Riley et al. [9]. The FPEBL is the boundary layer which would be formed if the boundary layer on the concave surface were to flow on to a flat plate with no further viscous dissipation such that the normal pressure gradient is relieved. The Bernoulli and continuity equations together with the free vortex condition lead to

\[ u_e^2 = u_m^2 + 2 \int_0^y \frac{u_m^2}{R-y} \, dy_m \]  \hspace{1cm} (A1)

and

\[ \int_0^y u_e \, dy_e = \int_0^y u_m \, dy_m \]  \hspace{1cm} (A2)

where the m and e subscripts refer to the measured and FPEBL values respectively. These equations are integrated numerically to obtain the FPEBL. The boundary layer integral parameters were evaluated from the FPEBL profile.