Abstract

The prediction of bypass transition remains an important problem in many engineering applications. This is largely because there is no suitable theoretical model for bypass transition and predictions are made using empirical models. This paper presents numerical results for the receptivity of a zero pressure gradient boundary layer subjected to simple freestream waveforms which are the constituent parts of a turbulent flow field. Significant receptivities are only obtained for a minority of freestream waveforms and these lead to two types of flow structure in the boundary layer. The first type of flow structure is essentially two dimensional in nature and consists of two rows of counter rotating spanwise vortices and is induced by freestream waves of large normal and spanwise wavelength and streamwise wavelengths approximately equal to the boundary layer thickness. The second type of flow structure are the streamwise streaks frequently observed in flow visualisation experiments. These streaks are induced by freestream waves of long streamwise and normal wavelength and spanwise wavelengths in the range of 14.5 to 46 θ (1.7 to 5.4δ). The freestream waves can be formed of velocity components in any direction, however the boundary layer is most receptive to fluctuations that lie in a plane perpendicular to the streamwise direction. The overall receptivity to a full spectrum of waves typical of freestream turbulence is considered and is shown to have similar characteristics to those from experiments.

Keywords

Boundary layer, Transition, Receptivity
Nomenclature

**G**  Near wall gain (equation 31)

**K**  Freestream turbulent kinetic energy

**p**  static pressure

**R**  receptivity (equation 30)

\[ \text{Re}_{x, y, z} = \frac{U_x}{\nu}, \frac{U_y}{\nu}, \frac{U_z}{\nu} \]  position Reynolds numbers

\[ \text{Re}_{x0} = \frac{U_{x0}}{\nu} \]  leading edge Reynolds number

**t**  time

**u, v, w**  steady velocity components in x, y and z directions

**u', v', w'**  unsteady velocities in x, y and z directions

**u_f, v_f, w_f**  unsteady velocity amplitudes (equation 5)

**U**  steady freestream velocity

**U_f, V_f, W_f**  unsteady freestream velocity components

**x, y, z**  streamwise, normal and spanwise co-ordinates

\[ \delta \]  distance from turbulence generating grid to plate leading edge

\[ \beta \]  turbulence decay constant (equation 24)

\[ \delta \]  boundary layer thickness

**\omega**  temporal frequency

\[ \omega_x, \omega_y, \omega_z \]  spatial frequencies

\[ \Omega = \frac{\omega}{U}, \text{ dimensionless temporal frequency} \]

\[ \Omega_x, \Omega_y, \Omega_z = \frac{\omega_x}{U}, \frac{\omega_y}{U}, \frac{\omega_z}{U}, \text{ dimensionless spatial frequencies} \]

**\nu**  fluid dynamic viscosity

**\rho**  fluid density
1. Introduction

The transition of an attached boundary layer from a laminar to a turbulent state is usually classified as either being through a natural mode or a bypass mode. Natural transition is the dominant mode for flows where the freestream turbulence level is less than about 1% and therefore is the mode of relevance to aircraft structures, whereas bypass transition is the dominant mode for higher levels of freestream turbulence which occur within gas turbine engines, Morkovin [1969]. Natural and bypass transition are usually treated as independent modes of transition. However in natural transition, the development of the two dimensional T-S waves is only the first stage of the transition process, which is followed by the development of three dimensional structures similar in nature to the streaky structures (or Klebanoff modes) observed in bypass transition. Similarly in bypass transition, although only the streaky structures are observed through flow visualisation experiments, T-S activity is apparent from spectral analysis (Hughes and Walker [2001]). It may therefore be inappropriate to treat natural and bypass transition as independent modes. Tollmien Schlichting waves and the bypass fluctuations can be thought of as analogous to the free and forced vibrations of a mechanical system. Tollmien Schlichting waves result from instability and hence adopt the natural resonant frequency for the boundary layer and grow exponentially when damping is insufficient (i.e. once a critical Reynolds number is exceeded). On the other hand, bypass fluctuations within the boundary layer are forced by the freestream fluctuations and have an amplitude directly proportional to the amplitude of the freestream forcing (Johnson and Erkan [1999]).

The theory for Tollmien Schlichting waves which form in the first stage of natural transition was derived by Tollmien [1929], before the first experimental observations were made by Schubauer and Skramstad [1943]. No similar theoretical work exists however for the velocity fluctuations leading to bypass transition. This is probably largely due to the fact that Tollmien Schlichting waves are two dimensional in nature and generally exist as a single frequency. However the structures observed in laminar boundary layers upstream of bypass transition are three dimensional in nature and are composed of many frequencies. Previous researchers (Leib et al. [1999] and Ricco and Wu [2007]) have determined the response of a boundary layer to freestream vortical fluctuations. Their work has shown that the boundary layer is particularly sensitive to vortical disturbances of long streamwise wavelength and spanwise wavelengths of the order of the boundary layer thickness. Direct numerical simulations (e.g. Ovchinnikov et al. [2008], Wu and Moin [2009]) have considered the response of the boundary layer to freestream turbulence. Their results also exhibit the streaky structures observed in the experiments, but also show how transition inception is characterised by the formation of groups of hairpin vortices. Researchers in Sweden have also undertaken extensive theoretical studies into the receptivity of boundary layers to freestream disturbances. Schlatter et al. [2008] studied the breakdown of the streaky structures within the boundary layer which leads to the formation of hairpin vortices and ultimately turbulent spots. More recently, Schrader et al. [2010], considered the receptivity of the boundary layer on a
flat plate to freestream vortical disturbances. Different freestream vortex orientations were considered and it was found that streamwise freestream vortices led to the formation of the streaky structures which are a characteristic precursor to bypass transition.

The purpose of the current work was to establish a theory for the response of a laminar boundary layer to a general three dimensional freestream disturbance and hence to identify those disturbances which dominate the receptivity process and hence initiate transition. In contrast to previous published work the fluctuations are considered as a linear perturbation to the steady flow.

2. Theory

Receptivity is the process by which freestream velocity fluctuations induce fluctuations within the boundary layer. Previous work (Johnson[2002]) has analysed this problem by assuming that the fluctuations are a linear perturbation to the steady flow which is represented by a non-developing Pohlhausen boundary layer profile. This approach gives theoretical results which are in close agreement with experimental observations; however the method is only capable of predicting the velocity fluctuations close to the wall and not the complete fluctuating velocity boundary layer profiles. This method is therefore extended here to overcome this problem by considering a developing boundary layer.

2.1 Steady flow

The geometry used for both the steady and unsteady computations is shown in figure 1. This consists of a plate of finite length, but infinitesimal thickness. The Reynolds number based on the chord length of the plate is 525,000. The steady incompressible flow was determined using a fourth order finite volume code on a regular rectangular mesh with 550 (streamwise) by 400 (normal) cells. The code uses cell centre storage for all variables and uses the pressure correction method for solution. The development of the predicted boundary layer is shown in figure 2. The development rate is 2% less than predicted by the Blasius theory, which is attributed to the favourable pressure gradient in the region immediately downstream of the leading edge stagnation point. The shape factor for \( \text{Re}_{x}>100,000 \) is 2.594 ±0.001 which is very close to the Blasius value of 2.592.

Location of figure 1 and 2.

2.2 Unsteady flow

If each flow quantity consists of a steady component and an unsteady perturbation then the resulting terms in the equations of motion will be zeroth, first or second order in the perturbation. The second order terms consist only of the Reynolds stress terms and as only the laminar flow is of interest here these second order terms can be neglected. The unsteady flow is therefore
restricted to a linear perturbation of the steady flow. The governing equations are then the continuity equation
\[
\frac{\partial u'}{\partial x} + \frac{\partial v'}{\partial y} + \frac{\partial w'}{\partial z} = 0
\]
and the momentum equations in the streamwise x, normal y and spanwise z, directions
\[
\frac{\partial u'}{\partial t} + \frac{\partial}{\partial x}\left(\frac{p'}{\rho} + 2uu'\right) + \frac{\partial}{\partial y}(vu'+uv') + \frac{\partial}{\partial z}(uw') - \nu \nabla^2 u' = 0
\]
\[
\frac{\partial v'}{\partial t} + \frac{\partial}{\partial x}(vu'+uv') + \frac{\partial}{\partial y}\left(\frac{p'}{\rho} + 2vv'\right) + \frac{\partial}{\partial z}(vw') - \nu \nabla^2 v' = 0
\]
\[
\frac{\partial w'}{\partial t} + \frac{\partial}{\partial x}(uw') + \frac{\partial}{\partial y}(vw') + \frac{\partial}{\partial z}\left(\frac{p'}{\rho}\right) - \nu \nabla^2 w' = 0
\]
noting that, as the steady flow is two dimensional w and the z derivatives of u, v and p are zero. Furthermore, the variation in the flow perturbations can be considered as periodic in time and also in the spanwise direction as the steady flow is invariant in this direction and so
\[
u' = u_I(x,y)e^{i(\omega_z z + \omega_x x)}
\]
with similar expressions for v', w' and p'. The equations can then be written as
\[
\frac{\partial u_I}{\partial x} + \frac{\partial v_I}{\partial y} + i\omega_z w_I = 0
\]
\[
iou_I + \frac{\partial}{\partial x}\left(\frac{p_I}{\rho} + 2u_I u_I\right) + \frac{\partial}{\partial y}(v_I + u_I) + \omega_z u_I - \nu \nabla^2 u_I + \nu \omega_z^2 u_I = 0
\]
\[
iow_I + \frac{\partial}{\partial x}(v_I + u_I) + \frac{\partial}{\partial y}\left(\frac{p_I}{\rho} + 2v_I v_I\right) + \omega_z v_I - \nu \nabla^2 v_I + \nu \omega_z^2 v_I = 0
\]
\[
iow_I + \frac{\partial}{\partial x}(u_I + v_I) + \frac{\partial}{\partial z}\left(\frac{p_I}{\rho}\right) - \nu \nabla^2 w_I + \nu \omega_z^2 w_I = 0
\]

2.3 Boundary conditions

At the inlet, a boundary condition must be specified which represents the unsteady flow in the freestream. This boundary condition must be a solution of
the set of equations just derived, simplified for the freestream flow which is in
the streamwise direction and uniform such that u=U. These simplified
momentum equations are

\[ i \omega u_t + \frac{\partial}{\partial x} \left( \frac{p_t}{\rho} - U u_t \right) - \nu \nabla^2 u_t + \nu \omega_x^2 u_t = 0 \]  \hspace{1cm} (10)

\[ i \omega v_t + \frac{\partial}{\partial x} (U v_t) + \frac{\partial}{\partial y} \left( \frac{p_t}{\rho} \right) - \nu \nabla^2 v_t + \nu \omega_y^2 v_t = 0 \]  \hspace{1cm} (11)

\[ i \omega w_t + \frac{\partial}{\partial x} (U w_t) + \frac{\partial}{\partial z} \left( \frac{p_t}{\rho} \right) - \nu \nabla^2 w_t + \nu \omega_z^2 w_t = 0 \]  \hspace{1cm} (12)

Equations 10, 11 and 12 can be combined to eliminate the velocity
components \( u_t, v_t \) and \( w_t \) resulting in

\[ \nabla^2 p_t = 0 \]  \hspace{1cm} (13)

Freestream turbulence consists of spatially and temporally periodic
waveforms which decay in the streamwise direction and satisfy the flow
equations. The freestream solution of the current equations must therefore be
of the form

\[ u' = u_1 e^{i(\omega_x t + \omega_y y + \omega_z z) - \beta x} \]  \hspace{1cm} (14)

\[ v' = v_1 e^{i(\omega_x t + \omega_y y + \omega_z z) - \beta x} \]  \hspace{1cm} (15)

\[ w' = w_1 e^{i(\omega_x t + \omega_y y + \omega_z z) - \beta x} \]  \hspace{1cm} (16)

\[ p' = p_1 e^{i(\omega_x t + \omega_y y + \omega_z z) - \beta x} \]  \hspace{1cm} (17)

Substituting for \( p' \) in equation 13

\[ \beta^2 - \omega_x^2 - \omega_y^2 - \omega_z^2 - 2i\omega_x = 0 \]  \hspace{1cm} (18)

It therefore follows from equation 17 that if \( \omega_x \) and \( \beta \) are non zero, \( p_1=0 \).

The freestream solution of these equations is therefore

\[ u_t = u_1 e^{i(\omega_x t + \omega_y y) - \beta x} \]  \hspace{1cm} (19)

\[ v_t = v_1 e^{i(\omega_x t + \omega_y y) - \beta x} \]  \hspace{1cm} (20)
\[ w_i = w_i e^{i(\omega_x x + \omega_y y - \omega z z)} \]  
\[ p_i = 0 \]

This solution is therefore a travelling wave with spatial frequencies \( \omega_x \), \( \omega_y \) and \( \omega_z \) which decays in the streamwise direction at a rate \( \beta \).

Substituting these expressions into any one of the momentum equations and equating the real and imaginary parts also provides a relationship between the streamwise spatial and temporal frequencies

\[ \omega = -(U + 2\beta \nu)\omega_x \]  

and an expression for the streamwise decay rate

\[ \beta = -\frac{U}{2\nu} + \left( \frac{U}{2\nu} \right)^2 + \omega_x^2 + \omega_y^2 + \omega_z^2 \right)^{0.5} \]  

or

\[ \beta x = -\frac{Re_x}{2} + \frac{Re_x}{2} \left( 1 + 4(\Omega_x^2 + \Omega_y^2 + \Omega_z^2) \right)^{0.5} \]  

where \( \Omega_x = \frac{\nu \omega_x}{U} \), \( \Omega_y = \frac{\nu \omega_y}{U} \) and \( \Omega_z = \frac{\nu \omega_z}{U} \)

Typically in engineering problems

\[ \Omega_x^2 + \Omega_y^2 + \Omega_z^2 \ll 1 \]  

and so

\[ \beta x = Re_x \left( \Omega_x^2 + \Omega_y^2 + \Omega_z^2 \right) \]  

These relationships indicate that the freestream turbulence is convected at a velocity slightly greater than the freestream velocity and decays in proportion to \( \nu \) and approximately in proportion to the square of the overall spatial frequency

\[ \Omega_x^2 + \Omega_y^2 + \Omega_z^2 \]  

The amplitudes of the velocity fluctuations, \( u_1 \), \( v_1 \) and \( w_1 \) must also satisfy the continuity equation and so

\[ (i\omega_x - \beta)u_1 + i\omega_y v_1 + i\omega_z w_1 = 0 \]
The velocity vector must therefore lie in a plane which is perpendicular to the frequency vector \((\omega_x, \omega_y, \omega_z)\). In the current work, two solutions were found for each spatial frequency \(\Omega_x\), \(\Omega_y\) and \(\Omega_z\) for freestream flows where either \(u_t\) or \(v_t\) was zero. I.e. for waves containing velocities only in the x-y plane (spanwise planar) or y-z plane (streamwise planar). It should be noted that because of the linear nature of the equations, all valid solutions can be produced by combination of these spanwise planar and streamwise planar solutions and in particular the freestream wave solution with velocities only in the x-z plane (normal planar) can be obtained by eliminating \(v_1\) between the streamwise and spanwise planar solutions.

Equations 19, 20 and 21 are therefore used to determine the velocities on the inlet boundary where the values of \(u_t\), \(v_t\) and \(w_t\) satisfy equation 29.

The remaining boundary conditions are more straightforward. On the plate the usual no slip boundary condition is used. The fluctuating pressure is set to zero on the boundaries parallel to and downstream of the plate, which is consistent with the freestream fluctuating velocity condition used at the inlet and the freestream solution.

The equations were solved using the meshes used for the steady computation using a very similar fourth order accurate solution procedure.

Calculations were performed to obtain spanwise and streamwise planar solutions for 1377 (=17x9x9) different frequency combinations. The frequency ranges used were logarithmically distributed between \(10^{-6}\) and \(10^{-2}\) for \(\Omega_x\) and \(3\times10^{-5}\) and \(3\times10^{-3}\) for \(\Omega_y\) and \(\Omega_z\). The calculations were performed simultaneously for the boundary layers on the upper and lower sides of the plate (see figure 1). The waves therefore impacted the boundary layer at an acute angle on the upper surface and an oblique angle on the lower surface and hence the solutions for the two boundary layers were not identical. Solutions for the boundary layer on the lower surface are equivalent to those which would have been obtained on the upper surface if a negative \(\Omega_y\) had been used and so are identified as such in the results. The solutions were used to identify the frequencies of the most receptivity modes. This was assessed by determining the receptivity coefficient \(R\), which is here defined as the ratio between the near wall turbulence level and that at the turbulence generating location (or grid).

\[
R = \frac{u_t}{u} \frac{U}{U_{10}}
\]

The near wall turbulence level \(\frac{u_t}{u}\) is determined at a position in the boundary layer where \(\frac{y}{\delta} = 0.1\). This is within the near wall region, which extends from the
wall to approximately $\frac{V}{\delta} = 0.2$. Johnson and Ercan [1999] found that $\frac{u}{u_f}$ is approximately constant throughout the near wall region and so the exact position chosen when determining the receptivity is not critical. $U_{f0} = \left( \frac{u_f^2 + v_f^2 + w_f^2}{3} \right)^{0.5}$ is the freestream fluctuation velocity at the turbulence generating grid. In the current work the turbulence grid is assumed to be located at a streamwise distance $x_0$ upstream of the plate leading edge such that $Re_{x_0} = \frac{U_{x_0}}{v} = 5 \times 10^5$, which is typical of most laboratory wind tunnels. It should be noted that the gain $G$ used previously by Johnson and Ercan [1999] is related to $R$ by the equation

$$G = R e^{-\beta(x+x_0)} \quad (31)$$

For low frequencies $\beta = 0$ and so $R$ and $G$ have the same value, but for high frequencies the amplitude of the fluctuation will decay significantly between the grid and the measurement position and hence $R$ will be much smaller than $G$. $R$ therefore represents the amplitude of a particular frequency mode in the near wall spectrum if it is assumed that all frequency modes have an identical amplitude when they are created at the turbulence grid.

3. Results

The receptivity coefficients were compared for each of the 1377 (17x9x9) frequency modes for both spanwise and streamwise planar fluctuations at ten equally spaced streamwise positions along the plate. Only a minority of spatial frequency combinations led to significant receptivity values (greater than one) and so only these cases will be considered in detail here. For most cases, but not all, the receptivity increased with distance along the plate and so the majority of results are presented for the station at $Re_x = 468,500$ where $Re_\theta = 433$. The spanwise and streamwise planar fluctuations will be considered separately.

**Spanwise planar fluctuations**

It should be noted that freestream spanwise planar fluctuations only induce fluctuations within the boundary layer which are also in the spanwise plane and so the spanwise ($z$) velocity component ($w'$) is zero in both freestream and boundary layer. The maximum receptivity coefficients were obtained for $\Omega_x = 5.3 \times 10^{-5}$. Figure 3 shows the variation in the receptivity coefficient with $\Omega_y$ and $\Omega_z$ at $Re_x = 468,500$ and shows that maximum receptivity results for minimum $\Omega_y$ and $\Omega_z = 5.3 \times 10^{-4}$. The increase in receptivity with decreasing $\Omega_y$ is due to an increase in the amplitude of $v'$ in the freestream as required by the continuity equation 29, however as $\Omega_y$ approached zero $v'$ only increases by a further 15% and hence the receptivity only increases modestly for values of $\Omega_y$ below those shown in figure 3. Figure 4 shows the velocity variations on
the y-z plane which are typical of the streaky structures observed in pre-transitional boundary layers. The freestream fluctuations lead to strong streamwise fluctuations within the boundary layer which are 180° out of phase with those in the freestream. At this location the spanwise wavelength for the frequency is equal to 26 θ (approximately 3 δ) which is larger than reported elsewhere (e.g. Westin et al. [1994]), but it can be seen from figure 3 that similar receptivities are obtained at spanwise frequencies of 0.3 and 0.95 which are equivalent to 46 and 14.5 θ (5 and 1.7 δ) respectively. It can therefore be expected that streaks will be present with a significant range of spanwise spacings, but the spacing observed in an experiment will effectively be the closest spacing. Figure 5 shows the variation in the receptivity along the plate for \( \Omega_x = 5.3 \times 10^{-5} \) and \( \Omega_z = 5.3 \times 10^{-4} \). For low \( \Omega_y \) the receptivity increases approximately linearly with \( Re_y \), but the high decay rate in the freestream at high \( \Omega_y \) leads to a much lower and insignificant receptivity.

Location of figures 3, 4 and 5

A second high receptivity region occurs at the much higher streamwise frequency of \( \Omega_x = 3.2 \times 10^{-4} \). Figure 6 indicates that in this case the maximum receptivity occurs at low \( \Omega_y \) and \( \Omega_z \) although in the case of negative \( \Omega_y \) the receptivity is virtually constant up to values of \( 5.3 \times 10^{-4} \). The receptivity drops rapidly with \( \Omega_z \) indicating that the dominant flow structure is within the x, y plane. This flow structure is shown in figure 7 and consists of two rows of counter-rotating spanwise vortices. The lower row lies within the boundary layer whereas the upper row is primarily in the freestream. The transfer of momentum from the freestream into the boundary layer and vice versa induced by the vortices has the effect of increasing the strength of both the boundary layer and freestream vortices and thus the vortices strengthen as they move downstream. Figure 8 shows that the strength of this vortex structure does not increase monotonically along the length of the plate. It appears that once the boundary layer reaches a critical thickness at around \( Re_x = 300,000 \), the momentum transfer mechanism becomes less effective. It should be noted that these structures are not related to Tollmien Schlichting (T-S) waves as T-S waves travel at approximately 50% of freestream velocity whereas the current structures travel at approximately 100% freestream velocity.

Location of figures 6, 7 and 8

Streamwise planar fluctuations

Freestream streamwise planar fluctuations induce fluctuations in the boundary layer not only in the streamwise plane (v’ and w’ components) but also in the streamwise direction (u’ component). The maximum receptivity for the streamwise planar fluctuations occurs at the lowest values of \( \Omega_x \), although the receptivity values only decrease by about 50% even with an order of magnitude increase in \( \Omega_x \) from \( 10^{-6} \) to \( 10^{-5} \). Figure 9 shows the receptivities at \( Re_x = 468,500 \) for \( \Omega_x = 10^{-6} \). The maximum receptivity is about three times larger than observed for the spanwise planar waves, but again occurs for \( \Omega_z = \)
5.3\times 10^{-4}. The flow structure for \( \Omega_x = 10^{-6}, \ \Omega_y = 9.487\times 10^{-4} \) and \( \Omega_z = 9.487\times 10^{-4} \) is shown in figure 10. The streaky structure flow pattern within the boundary layer is very similar to that initiated by the spanwise planar fluctuations although a stronger response is observed here with the streamwise planar fluctuations. In both these cases the streaks do not lie in the streamwise direction, but rather at an angle \( \tan^{-1}\left(\frac{\Omega_z}{\Omega_x}\right) \) to it. The reason for the streaks being at an angle is a consequence of a single flow solution being considered in isolation. The real flow will also contain a ‘mirror’ waveform identical to the present one, but reflected in the spanwise direction and hence with a negative \( \Omega_z \) frequency where

\[
v' = v_2 e^{i(\Omega_x x + \Omega_y y - \Omega_z z + \Omega t) - \beta x}
\]

and

\[
w' = w_2 e^{i(\Omega_x x + \Omega_y y - \Omega_z z + \Omega t) - \beta x}
\] (32)

On average this waveform will have an equal amplitude to the original one, although it may be phase shifted such that \( v_2 = v_1 e^{i\phi} \) and hence \( w_2 = w_1 e^{i\phi} \).

Combining these two waveforms leads to

\[
v' = \frac{(v_1 + v_2)}{2} e^{i(\Omega_x x + \Omega_y y + \Omega t) - \beta x} \cos(\Omega_z z - \phi) \text{ and}
\]

\[
w' = \frac{(w_1 + w_2)}{2} e^{i(\Omega_x x + \Omega_y y + \Omega t) - \beta x} \cos(\Omega_z z - \phi)
\] (33)

which for a low \( \Omega_x \) is an array of streamwise vortices pitched at a small angle to the flow direction. These vortices with their axes close to the streamwise direction (low \( \Omega_x \)), convect streamwise momentum into the boundary layer over an extended streamwise length at the downwash positions between neighbouring vortices. Conversely momentum is removed at upwash locations. It is this momentum transfer which leads to the formation of the streamwise streaks.

**Location of figures 9 and 10**

Figure 11 shows the receptivity averaged over both normal and spanwise frequency for the streamwise planar waves. The streamwise planar waves are dominant in the receptivity process and hence this plot closely resembles the receptivity for isotropic freestream turbulence and so is qualitatively very similar to the measured values obtained by Johnson and Ercan [1999]. It should be noted that here the variation with the spatial streamwise frequency \( \Omega_x \) is considered whereas Johnson and Ercan’s measurements considered the variation with the temporal frequency \( \Omega \), however these are approximately equal for low frequency where the decay rate is low. The boundary layer is most receptive to low \( \Omega_x \) frequencies, however as \( \Omega_x \) is decreased the receptivity tends to a constant maximum value at each Reynolds number. As the Reynolds number is increased, the low frequency receptivity increases, but this maximum value is only sustained to a lower maximum frequency. An explanation for this can be deduced by
considering the mechanism for the streaky flow structure described earlier. The increase in receptivity with decreasing $\Omega_x$ is due to the fact that momentum is being added to the boundary layer by the downwash between adjacent freestream vortices over a long streamwise length. However, the maximum length over which momentum can be added is the available upstream length of the plate. The maximum receptivity is therefore achieved for all frequencies where the upstream length of the plate is less than approximately one quarter of the fluctuation streamwise wavelength (i.e. $\Omega_x < \frac{0.69}{Re_0}$).

Location of figure 11

Unsteady boundary layer profiles.

Figure 12 shows the streamwise fluctuation velocity profiles through the boundary layer for the three high receptivity cases considered. The two cases which led to streamwise streaks (see figures 12a and c) show similar characteristics. In the case of the streamwise planar waves (figure 12c), there is virtually no difference between the results for positive and negative $\Omega_y$, however for the spanwise planar waves (figure 12a) the negative $\Omega_y$ frequencies result in a $180^\circ$ phase change between the near wall and freestream fluctuations which occurs close to the edge of the boundary layer. The receptivities are also lower for the negative $\Omega_y$ frequencies.

The fluctuation profiles shown in figure 12b for the spanwise planar flow structure shown in figure 7 are quite different. Increased fluctuation levels are observed in a region extending to approximately two boundary layer thicknesses beyond the boundary layer edge whereas in the previous cases this region was restricted to about half a boundary layer thickness. The peak fluctuation level has also shifted to the outer part of the boundary layer.

Location of figure 12

4. Conclusions

1) The effect of freestream waveforms of varying streamwise, normal and spanwise frequency and consisting of velocity fluctuations in a plane normal to either the spanwise or the streamwise direction, on the velocity fluctuations within a zero pressure gradient boundary layer have been computed using a fourth order CFD approach. The boundary layer was found to be only highly receptive to a number of relatively narrow frequency bands.

2) Two flow structures which lead to high receptivity have been identified. Each flow structure achieves high receptivity by transferring streamwise momentum effectively from the freestream to the boundary layer. The first flow structure is essentially two dimensional in nature and induces spanwise vortices within the boundary layer which have a
streamwise wavelength of approximately $\delta$. The second type of flow structure results in streamwise velocity streaks in the boundary layer.

3) The two dimensional flow structure consists of two rows of counter rotating vortices. The inner row is contained within the boundary layer whereas the outer row lies within both the boundary layer and the freestream. This flow structure alters the unsteady flow field for a distance of about two boundary layer thicknesses beyond the boundary layer edge.

4) The streamwise velocity streaks are generated by waves which have low frequencies in the $x$ and $y$ directions and a spanwise wavelength in the range of 14.5 to 46 $\theta$ (1.7 to 5.4 $\delta$). The streaks are induced by either streamwise or spanwise planar waves and hence, by the principle of superposition, any freestream wave with these spatial frequencies. The receptivity is highest though for streamwise planar waves, which when combined in positive and negative spanwise frequency pairs will form vortices aligned in approximately the streamwise direction in the freestream.

5) The overall receptivity of the boundary layer to freestream turbulence is dominated by the streaky structures induced by the approximately streamwise vortices. The variation of the receptivity averaged over all normal and spanwise frequencies with streamwise frequency and Reynolds number is similar to that measured experimentally.

References


**Figures**

Figure 1. Geometry used for calculations on flat plate

Figure 2. Development of laminar boundary layer.

Figure 3. u', v' disturbance receptivity at Re_x = 468,500 for Ω_x = 5.3x10^{-5}. Black symbols: Ω_y>0, grey symbols: Ω_y<0.

Figure 4. u', v' disturbance velocity vectors on Re_x = 468,500 plane for Ω_x = 5.3x10^{-5}, Ω_y = 3x10^{-5} and Ω_z = 5.3x10^{-4}.

Figure 5. u', v' disturbance receptivity for Ω_x = 5.3x10^{-5} and Ω_z = 5.3x10^{-4}. Black symbols: Ω_y>0, grey symbols: Ω_y<0.

Figure 6. u', v' disturbance receptivity at Re_x = 468,500 for Ω_x = 3.2x10^{-3}. Black symbols: Ω_y>0, grey symbols: Ω_y<0.

Figure 7. u', v' disturbance velocity vectors on Re_x = 468,500 plane for Ω_x = 3.2x10^{-3}, Ω_y = 3x10^{-5} and Ω_z = 3x10^{-5}.

Figure 8. u', v' disturbance receptivity for Ω_x = 3.2x10^{-3} and Ω_z = 3x10^{-5}. Black symbols: Ω_y>0, grey symbols: Ω_y<0.

Figure 9. v', w' disturbance receptivity at Re_x = 468,500 for Ω_x = 1x10^{-6}. Black symbols: Ω_y>0, grey symbols: Ω_y<0.

Figure 10. v', w' disturbance velocity vectors on Re_x = 468,500 plane for Ω_x = 1x10^{-6}, Ω_y = 3x10^{-5} and Ω_z = 5.3x10^{-4}.

Figure 11. v', w' disturbance receptivity for Ω_x = 1x10^{-6} and Ω_z = 5.3x10^{-4}. Black symbols: Ω_y>0, grey symbols: Ω_y<0.

Figure 12. u' fluctuation profiles. Black symbols: Ω_y>0, grey symbols: Ω_y<0.

a) u', v' velocity perturbations. Ω_x = 5.3x10^{-5}, Ω_y = 3x10^{-5} and Ω_z = 5.3x10^{-4}.

b) u', v' velocity perturbations. Ω_x = 3.2x10^{-4}, Ω_y = 3x10^{-5} and Ω_z = 3x10^{-5}.

c) v', w' velocity perturbations. Ω_x = 1x10^{-6}, Ω_y = 3x10^{-5} and Ω_z = 5.3x10^{-4}.