Mathematics for Physicists: Complex Functions I

Exercise 2 of MATH281

Review

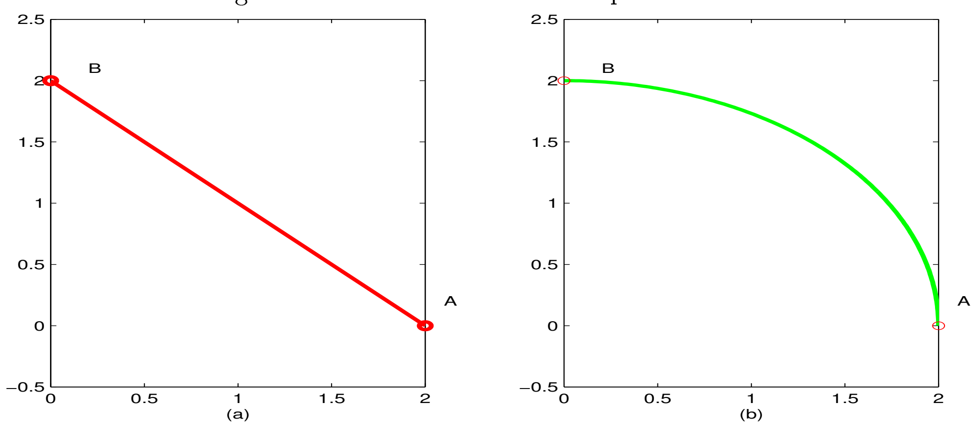
- Line and contour integrals of a real function $f = f(x, y)$
- Green Theorem $\int_\Gamma P dx + Q dy$ in the plane and variants
- Line and contour integrals of a complex function $f = f(z)$

1. Integrate the real function $G(x) = e^x \sin(x)$ and evaluate $I = \int_0^{2\pi} G(x)dx$. *Hint. Recursion.*

2. Evaluate the following integrals (for $\Gamma = AB$: a straight line as shown below in Figure 1(a))

$$I_1 = \int_\Gamma (3x^2 + 2y^3)dx; \qquad I_2 = \int_\Gamma (3x^2 + 2y^3)dy; \qquad I_3 = \int_\Gamma (3x^2 + 2y^3)ds.$$

Figure 1: Two boundaries Γ from point A to B



3. Repeat the previous question using $\Gamma = AB$: a quarter of the circle as shown above in Figure 1(b).

4. Use the Green theorem to compute the area of an ellipse domain $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ i.e. $\iint_\Omega dxdy$.

5. Compute the complex integrals

$$(a)\ I_1 = \int_\Gamma (3x^2 + 2y^3)dz; \qquad (b)\ I_2 = \int_\Gamma \frac{1}{z|z|}dz; \qquad (c)\ I_3 = \int_\Gamma z dz,$$

where Γ is the closed boundary of the unit circle centred at $(0, 0)$.