Optimal investment and consumption in a market with random coefficients and different rates for lending and borrowing

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Problem formulation
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- Market with two different asset types (risky and risk-less assets).
- Bank account and stock with one dimensional standard Brownian motion.
- Short selling is not allowed, stock can be acquired by borrowing at lending rate $R(t)$, which is higher than the interest rate $r(t)$ i.e. $R(t) > r(t), \forall t \in [0, T]$.
- According to Fleming & Zariphopoulou (1991), the investor’s wealth is given by

$$
\begin{cases}
    dy(t) = [r(t)y(t) + (\mu(t) - r(t))\pi(t)]
    \quad -(R(t) - r(t))\phi(t) - c(t)] dt + \sigma(t)\pi(t) dW, \quad t \in [0, T], \\
    y(0) = y_0 > 0.
\end{cases}
$$

(1)
The objective is to maximise the expected utility from consumption and terminal wealth under the following control variables:

$$J\left(\pi(\cdot), c(\cdot), \phi(\cdot)\right) := -\mathbb{E}\left[\int_0^T c^\gamma(t)dt + y^\gamma(T)\right],$$  \hspace{1cm} (2)

where $\gamma \in (0, 1)$. Hence, we can write our optimization problem in the following form

$$\begin{align*}
\min_{(\pi, c, \phi) \in \mathcal{A}} J\left(\pi(\cdot), c(\cdot), \phi(\cdot)\right), \\
\text{s.t.} \quad (1),
\end{align*}$$  \hspace{1cm} (3)

Two types of utility functions are considered, the power utility and the logarithmic utility.
Thank You!