Investigating the Foundations of Physical Law
9 Particles

Particle structures from nilpotent quantum mechanics

Particle (i.e. fermion) structures have a simple origin in fundamental terms. They are a description of point charges, electric, strong and weak. Essentially, all charges may be present or absent, which we can symbolize by 1 or 0; they may be + or –; they may be *e*, *s* or *w*, and, if so, will be associated with the characteristics with which these are associated when they are packaged together in the fermionic nilpotent, respectively scalar, vector and pseudoscalar. There are also considerations related to *CPT* symmetry and the chirality of fermions under the weak interaction. The different possible combinations of structures within these rules are what make the particle structures that are possible. Everything else follows automatically, except the particle masses, though even some aspects of this difficult subject are consequences of the same rules. Most of what can be achieved is via pattern recognition on a large scale, and there are a number of different ways of creating the patterns. Interestingly, though they have quite different starting points, they all seem to be pointing in the same direction. As in several other areas, an appeal to fundamental ideas often leads to a particular twist on well-known results and entirely new consequences. We will show a calculation that, reinterpreted on fundamental grounds, leads to a very important consequence with directly testable predictions.

Ultimately, the charge structures, like the fermions themselves, involve some kind of combination of two 3-dimensionalities, one with symmetry broken and the other with symmetry preserved (see lecture 3, p. 13). This is, as we have previously shown, how the point structure is created. One way of doing this is to go straight to the nilpotent representation, in which the four components of the spinor represent the full potentiality of what any fermionic state could be transformed into, with the weak, strong and electric interactions as the means of making this transfer. According to the ideas postulated in the last lecture, the gravitational or inertial interaction is 'passive' in this respect, the vacuum reflection (expressible as 1ψ or scalar $\times$ ψ) leading to the state itself. ψ is taken to be the local, 'inertial', manifestation of the fermion, with $-\psi$ the nonlocal, gravitational, dual. Now, of the two types of fundamental fermion, only quarks incorporate the explicit vector behaviour (i.e. showing a structure made of 3 components) of the momentum operator in their spinor state vectors. That is, in our postulated nilpotent formalism for a baryon (lecture 6, p 4), each of the three nilpotents used to construct this state, which we now call (valence) quarks, only contain, at any instant, one component of the total momentum vector $\mathbf{p} = (\mathbf{p}_1, \mathbf{p}_2, \mathbf{p}_3)$ of the baryon, and that, in the allowed *phases* of the interaction, the total momentum is in *just one* of these components. However, as we saw in lecture 2, we can create a nilpotent without specifying that $\mathbf{p}$ is a vector divisible into components, and so can construct a nilpotent fermion without having an initial separation into component parts. The result is an overall final structure which looks the same, but the first process has more simultaneous options than the second. We can account for the distinction in terms of the vector phase, which is here an instantaneous choice of direction, remembering that, as we have shown in lecture 5, all the information about a fermion is contained in the direction of its momentum or spin vector. So a baryon state vector might have a form such as

$$\begin{matrix} inertial \\ strong \\ weak \\ electric \end{matrix} \begin{pmatrix} ikE \pm \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_1 + \boldsymbol{j}m \\ ikE \mp \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_1 + \boldsymbol{j}m \\ -ikE \pm \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_1 + \boldsymbol{j}m \\ -ikE \mp \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_3 + \boldsymbol{j}m \end{pmatrix} \begin{pmatrix} ikE \pm \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_2 + \boldsymbol{j}m \\ ikE \mp \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_2 + \boldsymbol{j}m \\ -ikE \pm \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_3 + \boldsymbol{j}m \\ -ikE \mp \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_2 + \boldsymbol{j}m \end{pmatrix} \begin{pmatrix} ikE \pm \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_3 + \boldsymbol{j}m \\ ikE \mp \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_3 + \boldsymbol{j}m \\ -ikE \pm \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_2 + \boldsymbol{j}m \\ -ikE \mp \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_1 + \boldsymbol{j}m \end{pmatrix}$$

or

$$\begin{matrix} inertial \\ strong \\ weak \\ electric \end{matrix} \begin{pmatrix} ikE \pm \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_1 + \boldsymbol{j}m \\ ikE \mp \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_1 + \boldsymbol{j}m \\ -ikE \pm \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_1 + \boldsymbol{j}m \\ -ikE \mp \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_3 + \boldsymbol{j}m \end{pmatrix} \begin{pmatrix} ikE \mp \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_2 + \boldsymbol{j}m \\ ikE \pm \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_2 + \boldsymbol{j}m \\ -ikE \mp \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_3 + \boldsymbol{j}m \\ -ikE \pm \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_2 + \boldsymbol{j}m \end{pmatrix} \begin{pmatrix} ikE \pm \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_3 + \boldsymbol{j}m \\ ikE \mp \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_3 + \boldsymbol{j}m \\ -ikE \pm \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_2 + \boldsymbol{j}m \\ -ikE \mp \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_1 + \boldsymbol{j}m \end{pmatrix}$$

The principle is that the **p** term determines which of the three components of a baryon carries an 'active' component of that kind. Let's say $\mathbf{p}_3$ determines 'activity' in the first representation at any given time. Then the first quark has an active electric component, the second an active weak component, and the third active inertial and strong components. This will determine whether these charges are present or absent (1 or 0). How will we decide which component determines 'activity' at any moment? This will be determined by the inertial phase, and this will be governed by the direction of **σ**, which is unique to each fermion. Clearly, the labels are arbitrary and can be changed to ensure that gauge invariance is preserved. With the two available signs of **p** in the two representations given, all six phases of the strong interaction are simultaneously possible.

The strong charge goes through all possible phases, while the weak and electric charges remain relatively (although not absolutely) fixed on single phases. That is, if we specify that one direction of the **p** vector instantaneously contains all the information about the system (here described as 'active'), then we can define this as one of the three axes along which, as we saw in lecture 5, the three quark momentum components could be aligned. The strong interaction will then take us through all three possible directions, with the 'active' one defined at any moment by coincidence with the one we have described as 'inertial'. However, the weak and electric **p** components will only be aligned along that of the 'inertial' (and therefore 'active) one in one of three cases.

In a baryon, the weak and electric phases must be on different quarks. The inertial element again goes through all possible phases in fixing the direction of spin **σ**. Mesons have the same structure as baryons, except that they are single fermions combined with the corresponding antifermions and the three phases ('colours') should be considered in a purely temporal (i.e. non-spatial) sequence. The weak and electric charges 'switch on / off' as the phase changes through the components 1, 2, 3.

For free fermions or leptons, the phases are purely the inertial phases. Only the direction of vector properties of **p**, of course, define a strong phase – the magnitude is determined by the combination of E and m. For free fermions, there is no strong charge because no information is carried about direction, and there is no $SU(3)$ symmetry. Leptons have weak and electric occupancy on the same phase, with a temporal cycle, 1-2-3, as the structure rotates through the three directions involved in **p**.

$$\begin{matrix} inertial \\ strong \\ weak \\ electric \end{matrix} \quad \begin{pmatrix} \boldsymbol{k}E \pm \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_1 + \boldsymbol{j}m \\ \boldsymbol{ik}E \mp \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_1 + \boldsymbol{j}m \\ -\boldsymbol{ik}E \pm \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_1 + \boldsymbol{j}m \\ -\boldsymbol{ik}E \mp \boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p}_1 + \boldsymbol{j}m \end{pmatrix}$$

We can consider iE, **σ.p** and m as the respective coefficients for the weak, strong and electric vacuum terms. There are two pseudoscalar terms $\pm\, iE$; six vector terms $\pm$ $\boldsymbol{\sigma}.\mathbf{p}_1$, $\pm\, \boldsymbol{\sigma}.\mathbf{p}_2$, $\pm\, \boldsymbol{\sigma}.\mathbf{p}_3$; and one scalar term m. The weak component switches in such a way as to make iE into $-iE$. The strong switches in such a way as to make $\boldsymbol{\sigma}.\mathbf{p}_1$ convert to $-\boldsymbol{\sigma}.\mathbf{p}_1$, and also to $\boldsymbol{\sigma}.\mathbf{p}_2$; $-\boldsymbol{\sigma}.\mathbf{p}_2$; $\boldsymbol{\sigma}.\mathbf{p}_3$; and $-\boldsymbol{\sigma}.\mathbf{p}_3$. The weak transition involves dipolarity. The strong transition requires a constant rate of change of **p**, which is equivalent to a linear potential. The electric component preserves m. The respective group structures are $SU(2)$, $SU(3)$ and $U(1)$.

Only iE and **σ.p** vary, and only **σ.p** varies in magnitude with phase. The E terms are, therefore, always global. There are two ways of setting up these transitions, both using covariant derivatives for the operators iE and **σ.p**. We can either set up a combination of (pseudo)scalar and vector group generators; or, using the idea that these groups represent the spherical symmetry of a point source, and are covariant, we can replace the scalar and vector parts by scalar potential functions of r, associated with E. In the first case, the scalar parts are scalar phase (Coulomb) terms; the vector parts are the generators that make the individual interactions have the $SU(3)$, $SU(2)$ or $U(1)$ symmetries associated with the **p**, E and m operators in the nilpotent, or with the direction, handedness or radial magnitude of the angular momentum. In the second case, we can group all terms related to a single particle under a single representation of E as a scalar function of r, which applies globally to the entire state.

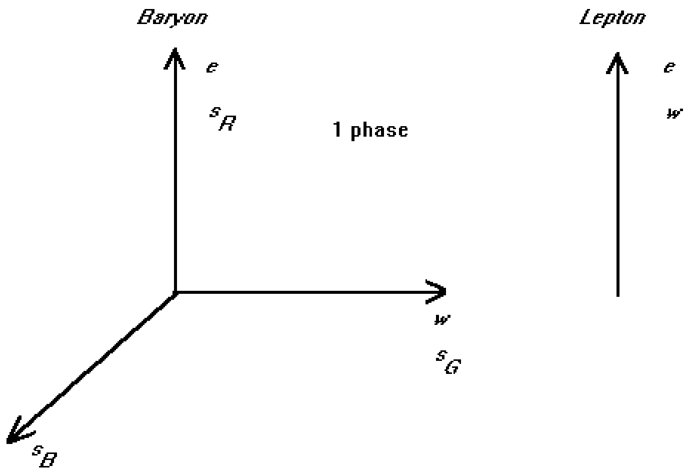
The two $SU(2)$ states – filled electric and empty electric background – being global, are automatically set with respect to E. That is, the background is incorporated as the potential producing a scalar or $U(1)$ phase in the E term. In the case of spherical symmetry, this becomes a Coulomb potential. It is possible to combine all the information into a single expression by using the fact that Lorentz invariance, in the case of a purely point source with spherical symmetry, allows us to transfer all the information contained in $\boldsymbol{\sigma}.\mathbf{p}$ to the E term by adding a potential function of r which reproduces the specific aspect of spherical symmetry ($SU(3)$, $SU(2)$ or $U(1)$) incorporated in the covariant part of $\boldsymbol{\sigma}.\mathbf{p}$, that is, the part responsible for the interaction. When the frame is chosen such that all this information is transferred to the E term, then all specific phase information is lost. The rotation of vector $\mathbf{p}$ terms ensures that the strong term is a linear function ($\propto r$). The scalar nature of m ensures that the electric term is a scalar phase (Coulombic) ($\propto 1 / r$). The dipolarity of $\pm iE$ ensures that the weak term is a dipolar equivalent of the scalar phase ($\propto 1 / r^3$). These options are also evident as a result of directly applying the condition of spherical symmetry to the fermionic state.

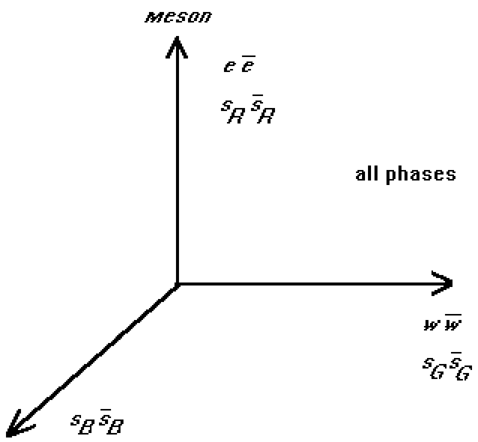
Phase diagrams

We can picture lepton, baryon and meson charge structures using phase diagrams. In the case of the strong interaction, only one component of angular momentum is well-defined at any moment, and the strong charge appears to act in such a way that the well-defined direction manifests itself by 'privileging' one out of three independent phases making up the complete phase cycle. In a truly gauge invariant system, this can only be accomplished in relative terms. If the weak and electric charges are also related to angular momentum, then the same must apply to them, and the relative 'privileging' of phase can only be defined between the different interactions. We have, here, two options. If the 'privileged' or 'active' phases of E and m (or w and e) coincide with each other, then this also determines the 'privileged' phase of $\mathbf{p}$; the result is no 'privileged' relative phase. Since the strong charge is defined only through the directional variation of $\mathbf{p}$, via a 'privileged' relative phase, a system in which the phases coincide cannot be strongly bound. If, however, they are different, then this information can only be carried through $\mathbf{p}$ (or s), and the strong interaction must be present.

We can imagine the arrangements diagrammatically using a rotating vector to represent the 'privileged' direction states for the charges. Each charge has only one 'active' phase out of three at any one time to fix the angular momentum direction; the symbols e, s, and w here refer to these states, not the actual charges. The vectors may be thought of as rotating over a complete spherical surface. In the case of the quark-based states – baryons and mesons – the total information about the angular momentum state is split between three axes, whereas the lepton states carry all the information on a single axis.

The axes in the diagrams represent both charge states and angular momentum states for leptons, mesons and baryons. As we have seen, each type of charge carries a different aspect of angular momentum (or helicity) conservation; *s* carries the directional information (linked to **p**); *w* carries the sign information (+ or – helicity) (linked to *iE*); *e* carries information about magnitude (linked to *m*). Another way of looking at this is to associate these properties, respectively, with the symmetries of rotation, inversion, and translation, which can also be mapped onto the dihedral symmetries (or rotations around *x*, *y* and *z* axes) of the fundamental parameter group.





The significance of these diagrams arises from the fact that all the information about the states is contained in the instantaneous direction of the momentum or spin operator **p**. The absence of a strong interaction for the leptons means that the splitting of this operator into 3 separable parts never takes place, and all the information is along a single direction. In the case of the hadronic or strongly-interacting structures, the information is split between components along 3 orthogonal axes.

Dirac equation for charge

The ***k***, ***i***, ***j*** operators correspond both to charge (*w*-*s*-*e*) and aspects of angular momentum (*E*-**p**-*m*) (see chapter 4, pp. 17-18). So can we use a Dirac-type equation to specifically describe the conservation of charge, rather than of angular momentum? Here we will use one of the more standard forms of the Dirac equation,

$$(\boldsymbol{\alpha}.\mathbf{p} + \beta m - E)\ \psi = 0,$$

which can be expanded, using a 4 × 4 matrix, to

$$(\boldsymbol{\alpha}.\mathbf{p} + \beta m - E)\psi = \begin{pmatrix} -E & 0 & im & -ip \\ 0 & -E & ip & im \\ -im & -ip & -E & 0 \\ ip & -im & 0 & -E \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \\ \psi_3 \\ \psi_4 \end{pmatrix} = 0 .$$

The column vector, here, is the usual 4-component spinor, and the terms *E* and **p** represent the quantum differential operators rather than their eigenvalues.

If we replace the *E*-**p**-*m* terms by the corresponding *w*-*s*-*e*, we can derive an expression for conserved charge:

$$\begin{pmatrix} \boldsymbol{k}w & 0 & -\boldsymbol{ij}e\uparrow & -\boldsymbol{i}i\mathbf{s} \\ 0 & \boldsymbol{k}w & -\boldsymbol{i}i\mathbf{s} & \boldsymbol{ij}e\downarrow \\ -\boldsymbol{ij}e\downarrow & \boldsymbol{i}i\mathbf{s} & -\boldsymbol{k}w & 0 \\ \boldsymbol{i}i\mathbf{s} & \boldsymbol{ij}e\uparrow & 0 & -\boldsymbol{k}w \end{pmatrix} \begin{pmatrix} \boldsymbol{k}w + \boldsymbol{i}i\mathbf{s} + \boldsymbol{ij}e\uparrow \\ \boldsymbol{k}w + \boldsymbol{i}i\mathbf{s} - \boldsymbol{ij}e\downarrow \\ -\boldsymbol{k}w - \boldsymbol{i}i\mathbf{s} + \boldsymbol{ij}e\downarrow \\ -\boldsymbol{k}w - \boldsymbol{i}i\mathbf{s} - \boldsymbol{ij}e\uparrow \end{pmatrix} = 0$$

The 4 × 4 matrix used here is almost identical in form to the matrix for the Dirac differential operator, although the + and – signs are in different places. The *s* term effectively takes up the vector-type properties of **p**, and can be represented as a vector with a single well-defined direction. The electric charge is like mass in the conventional version of the Dirac equation, a passive term. There it is an expression of *zitterbewegung*, here it is weak isospin. The sign applied to *e* is that of the charge itself, but *e* has the added property of weak isospin, so that the *e*'s on the first and fourth rows of the matrix and on the first and fourth rows of the column vector can be considered as isospin 'up' (↑) and the others as isospin 'down' (↓). The opposite states of isospin are not + and – but 1 and 0. So, we should apply to these *e* terms the matrices:

$$\begin{pmatrix} 1 \\ 0 \end{pmatrix} ; \begin{pmatrix} 0 \\ 1 \end{pmatrix} ; \begin{pmatrix} -1 \\ 0 \end{pmatrix} ; \begin{pmatrix} 0 \\ -1 \end{pmatrix} .$$

The result of this is that all terms involving *e* disappear on multiplication.

If we now create an exponential term $e^{-i(wt - \mathbf{s.r})}$, to produce a state vector for charge, and define $i\partial/\partial t = -iw$ and $-i\nabla = i\mathbf{s}$, we obtain:

$$\begin{pmatrix} \boldsymbol{ik}\partial/\partial t & 0 & -\boldsymbol{ij}e\uparrow & -\boldsymbol{i}\nabla \\ 0 & \boldsymbol{ik}\partial/\partial t & -\boldsymbol{i}\nabla & \boldsymbol{ij}e\downarrow \\ -\boldsymbol{ij}e\downarrow & \boldsymbol{i}\nabla & -\boldsymbol{ik}\partial/\partial t & 0 \\ \boldsymbol{i}\nabla & \boldsymbol{ij}e\uparrow & 0 & -\boldsymbol{ik}\partial/\partial t \end{pmatrix} \begin{pmatrix} \boldsymbol{k}w + i\boldsymbol{i}\mathbf{s} + \boldsymbol{ij}e\uparrow \\ \boldsymbol{k}w + i\boldsymbol{i}\mathbf{s} - \boldsymbol{ij}e\downarrow \\ -\boldsymbol{k}w - i\boldsymbol{i}\mathbf{s} + \boldsymbol{ij}e\downarrow \\ -\boldsymbol{k}w - i\boldsymbol{i}\mathbf{s} - \boldsymbol{ij}e\uparrow \end{pmatrix} e^{-i(wt - \mathbf{s.r})} = 0 .$$

The weak isospin terms cancel, suggesting why this becomes the scalar phase term. Without these 'phase' terms, this equation becomes:

$$\begin{pmatrix} \boldsymbol{ik}\partial/\partial t & 0 & 0 & -\boldsymbol{i}\nabla \\ 0 & \boldsymbol{ik}\partial/\partial t & -\boldsymbol{i}\nabla & 0 \\ 0 & \boldsymbol{i}\nabla & -\boldsymbol{ik}\partial/\partial t & 0 \\ \boldsymbol{i}\nabla & 0 & 0 & -\boldsymbol{ik}\partial/\partial t \end{pmatrix} \begin{pmatrix} \boldsymbol{k}w + i\boldsymbol{i}\mathbf{s} \\ \boldsymbol{k}w + i\boldsymbol{i}\mathbf{s} \\ -\boldsymbol{k}w - i\boldsymbol{i}\mathbf{s} \\ -\boldsymbol{k}w - i\boldsymbol{i}\mathbf{s} \end{pmatrix} e^{-i(wt - \mathbf{s.r})} = 0 .$$

Here, each term of the resultant column vector becomes a pseudo-Dirac or Dirac-type equation for charge:

$$(\boldsymbol{ik}\partial/\partial t + \boldsymbol{i}\nabla)(\boldsymbol{k}w + i\boldsymbol{i}\mathbf{s})e^{-i(wt - \mathbf{s.r})} = 0 ,$$

in the same way as each term of the resultant column matrix becomes a Dirac equation for the *E*-**p**-*m* combination.

In producing a 'Dirac'-type equation for charge, we are finding *equivalent* properties for the charges to those for the *E*-**p**-*m* terms in a similar way to the one in which equivalent *local* properties reproduced nonlocal ones in lecture 6. In this interpretation, starting with the real Dirac equation (for *E*-**p**-*m*), we introduce a filled fermion vacuum to create the two-sign degree of freedom required for *E*. We also define a particular status for antifermions beyond the original requirement that each charge-type has two possible signs. We assume, therefore, that a particular type of charge, say *s*, can only be unit in one of the three 'colours' needed to make up an observed state. This excludes charges of the opposite sign, so we take the concept of antistates from the Dirac equation, and assign –*s* to the antifermions. We cannot, however, repeat the same procedure for, say, *e*, which must have both signs in both states and antistates. So, we preserve the rule that a charge (–*e* in this case) can be unit in only one of the three 'colours', but make the 'default' position (*e*, *e*, *e*) as opposed to (0, 0, 0) for *s*, and so produce two signs by creating 'weak isospin', with alternatives (*e*, *e*, 0) and (0, 0, –*e*). Subsequently, we find that using 'weak isospin' actually gives us a suitable zero for the matrix equation for charge. Finally, to accommodate two signs of *w*, we have to refer to the fact that a filled vacuum, with antiparticles nonexistent in the ground state, violates charge conjugation symmetry for the charge (*w*) which specifies the fermion state.

Fermion states from the algebra

Of the total of 64 generators in the Dirac nilpotent algebra, 60 can be arranged into 12 nilpotent pentads, and each of these can be used to represent a fermion.

generation		isospin	
1	electron neutrino	up	$\boldsymbol{i}\mathbf{i}$ $\boldsymbol{i}\mathbf{j}$ $\boldsymbol{i}\mathbf{k}$ $i\boldsymbol{k}$ $\boldsymbol{j}$
	electron	down	$i\mathbf{i}\boldsymbol{i}$ $i\mathbf{i}\boldsymbol{j}$ $i\mathbf{i}\boldsymbol{k}$ $i\mathbf{k}$ $\mathbf{j}$
2	muon neutrino	up	$\boldsymbol{j}\mathbf{i}$ $\boldsymbol{j}\mathbf{j}$ $\boldsymbol{j}\mathbf{k}$ $i\boldsymbol{i}$ $\boldsymbol{k}$
	muon	down	$i\mathbf{j}\boldsymbol{i}$ $i\mathbf{j}\boldsymbol{j}$ $i\mathbf{j}\boldsymbol{k}$ $i\mathbf{i}$ $\mathbf{k}$
3	tau neutrino	up	$\boldsymbol{k}\mathbf{i}$ $\boldsymbol{k}\mathbf{j}$ $\boldsymbol{k}\mathbf{k}$ $i\boldsymbol{j}$ $\boldsymbol{i}$
	tau	down	$i\mathbf{k}\boldsymbol{i}$ $i\mathbf{k}\boldsymbol{j}$ $i\mathbf{k}\boldsymbol{k}$ $i\mathbf{j}$ $\mathbf{i}$

generation		isospin	
1	antielectron-neutrino	up	$-\boldsymbol{i}\mathbf{i}$ $-\boldsymbol{i}\mathbf{j}$ $-\boldsymbol{i}\mathbf{k}$ $-i\boldsymbol{k}$ $-\boldsymbol{j}$
	antielectron	down	$-i\mathbf{i}\boldsymbol{i}$ $-i\mathbf{i}\boldsymbol{j}$ $-i\mathbf{i}\boldsymbol{k}$ $-i\mathbf{k}$ $-\mathbf{j}$
2	antimuon-neutrino	up	$-\boldsymbol{j}\mathbf{i}$ $-\boldsymbol{j}\mathbf{j}$ $-\boldsymbol{j}\mathbf{k}$ $-i\boldsymbol{i}$ $-\boldsymbol{k}$
	antimuon	down	$-i\mathbf{j}\boldsymbol{i}$ $-i\mathbf{j}\boldsymbol{j}$ $-i\mathbf{j}\boldsymbol{k}$ $-i\mathbf{i}$ $-\mathbf{k}$
3	antitau-neutrino	up	$-\boldsymbol{k}\mathbf{i}$ $-\boldsymbol{k}\mathbf{j}$ $-\boldsymbol{k}\mathbf{k}$ $-i\boldsymbol{j}$ $-\boldsymbol{i}$
	antitau	down	$-i\mathbf{k}\boldsymbol{i}$ $-i\mathbf{k}\boldsymbol{j}$ $-i\mathbf{k}\boldsymbol{k}$ $-i\mathbf{j}$ $-\mathbf{i}$

generation		isospin	
1	up quark	up	$\boldsymbol{i}\mathbf{i}$ $\boldsymbol{i}\mathbf{j}$ $\boldsymbol{i}\mathbf{k}$ $i\boldsymbol{k}$ $\boldsymbol{j}$
	down quark	down	$i\mathbf{i}\boldsymbol{i}$ $i\mathbf{i}\boldsymbol{j}$ $i\mathbf{i}\boldsymbol{k}$ $i\mathbf{k}$ $\mathbf{j}$
2	charmed quark	up	$\boldsymbol{j}\mathbf{i}$ $\boldsymbol{j}\mathbf{j}$ $\boldsymbol{j}\mathbf{k}$ $i\boldsymbol{i}$ $\boldsymbol{k}$
	strange quark	down	$i\mathbf{j}\boldsymbol{i}$ $i\mathbf{j}\boldsymbol{j}$ $i\mathbf{j}\boldsymbol{k}$ $i\mathbf{i}$ $\mathbf{k}$
3	top quark	up	$\boldsymbol{k}\mathbf{i}$ $\boldsymbol{k}\mathbf{j}$ $\boldsymbol{k}\mathbf{k}$ $i\boldsymbol{j}$ $\boldsymbol{i}$
	bottom quark	down	$i\mathbf{k}\boldsymbol{i}$ $i\mathbf{k}\boldsymbol{j}$ $i\mathbf{k}\boldsymbol{k}$ $i\mathbf{j}$ $\mathbf{i}$

generation		isospin	
1	antiup-quark	up	$-\boldsymbol{i}\mathbf{i}$ $-\boldsymbol{i}\mathbf{j}$ $-\boldsymbol{i}\mathbf{k}$ $-i\boldsymbol{k}$ $-\boldsymbol{j}$
	antidown-quark	down	$-i\mathbf{i}\boldsymbol{i}$ $-i\mathbf{i}\boldsymbol{j}$ $-i\mathbf{i}\boldsymbol{k}$ $-i\mathbf{k}$ $-\mathbf{j}$
2	anticharmed-quark	up	$-\boldsymbol{j}\mathbf{i}$ $-\boldsymbol{j}\mathbf{j}$ $-\boldsymbol{j}\mathbf{k}$ $-i\boldsymbol{i}$ $-\boldsymbol{k}$
	antistrange-quark	down	$-i\mathbf{j}\boldsymbol{i}$ $-i\mathbf{j}\boldsymbol{j}$ $-i\mathbf{j}\boldsymbol{k}$ $-i\mathbf{i}$ $-\mathbf{k}$
3	antitop-quark	up	$-\boldsymbol{k}\mathbf{i}$ $-\boldsymbol{k}\mathbf{j}$ $-\boldsymbol{k}\mathbf{k}$ $-i\boldsymbol{j}$ $-\boldsymbol{i}$
	antibottom-quark	down	$-i\mathbf{k}\boldsymbol{i}$ $-i\mathbf{k}\boldsymbol{j}$ $-i\mathbf{k}\boldsymbol{k}$ $-i\mathbf{j}$ $-\mathbf{i}$

The first term in each pentad represents the energy operator, the next 3 the momentum operator, and the last term the mass operator. If we treat these structures symbolically, they can be seen as representing 12 fermions, say, 6 quarks and 6 leptons, or 6 quarks / leptons and 6 antiquarks / antileptons. The total becomes $2 \times 12 = 24$ if we include left- and right-handed states (the parity P duality); and $2 \times 2 \times 12 = 48$ if we include fermion and antifermion states (the charge conjugation *C* duality), in addition to quarks and leptons.

Equation for specifying particle states

It is possible to write down a single equation to generate the entire set of charge structures for quarks and leptons (and their antistates):

$$\boldsymbol{\sigma}_z.(\boldsymbol{i}\,\hat{\mathbf{p}}_a\,(\delta_{bc}-1)+\boldsymbol{j}\,(\hat{\mathbf{p}}_b-\mathbf{1}\delta_{0m})+\boldsymbol{k}\,\hat{\mathbf{p}}_c\,(-1)^{\delta_{1g}}\,g) \qquad \text{(A)}$$

The quaternion operators ***i***, ***j***, ***k*** are respectively strong, electric and weak charge units; $\boldsymbol{\sigma}_z$ is the spin pseudovector component defined in the *z* direction (here used as a reference); $\hat{\mathbf{p}}_a$, $\hat{\mathbf{p}}_b$, $\hat{\mathbf{p}}_c$ are each units of quantized angular momentum, selected *randomly* and *independently* from the three orthogonal components $\hat{\mathbf{p}}_x$, $\hat{\mathbf{p}}_y$, $\hat{\mathbf{p}}_z$. $\boldsymbol{\sigma}_z$ and the remaining terms are logical operators representing existence conditions, and defining four fundamental divisions in fermionic states. Each of the operators creates one of these fundamental divisions – fermion / antifermion; quark / lepton (colour); weak up isospin / weak down isospin; and the three generations – which are identified, respectively, by the weak, strong, electromagnetic and gravitational interactions.

(1) $\boldsymbol{\sigma}_z = -\mathbf{1}$ defines left-handed states; $\boldsymbol{\sigma}_z = \mathbf{1}$ defines right-handed. For a filled weak vacuum, left-handed states are predominantly fermionic, right-handed states become antifermionic 'holes' in the vacuum (which is 0 in this representation).

(2) $b = c$ produces leptons; $b \neq c$ produces quarks. If $b \neq c$ we are obliged to take into account the three directions of **p** at once. If $b = c$, we can define a single direction. Taking into account all three directions at once, we define baryons composed of three quarks (and mesons composed of quark and antiquark), in which each of *a*, *b*, *c* cycle through the directions *x*, *y*, *z*.

(3) *m* is an electromagnetic mass unit, which selects the state of weak isospin. It becomes 1 when present and 0 when absent. So $m = 1$ is the weak isospin up state; and $m = 0$ weak isospin down. The unit condition can be taken as an empty electromagnetic vacuum; the zero condition a filled one.

(4) *g* represents a conjugation of weak charge units, with $g = -1$ representing maximal conjugation. If conjugation fails maximally, then $g = 1$. *g* can also be thought of as a composite term, containing a parity element (*P*) and a time-reversal element (*T*). So, there are two ways in which the conjugated *PT* may remain at the unconjugated value (1). $g = -1$ produces the generation *u*, *d*, ν_e, *e*; $g = 1$, with *P* responsible, produces *c*, *s*, ν_μ, μ; $g = 1$, with *T* responsible, produces *t*, *b*, ν_τ, τ.

The weak interaction can only identify (1). This occupies the ***ik****E* site in the anticommuting Dirac pentad (***ik****E* + ***i*****p** + ***j****m*), with the *i* term being responsible for the fermion / antifermion distinction. Because it is attached to a complex operator, the sign of ***k*** has two possible values even when those of ***i*** and ***j*** are fixed; the sign of the weak charge associated with ***k*** can therefore only be determined physically by the sign of $\boldsymbol{\sigma}_z$. The filled weak vacuum is an expression of the fact that the 'ground state of the universe' can be specified in terms of positive, but not negative, energy (*E*), because, physically, this term represents a continuum state.

The strong interaction identifies (2). This occupies the ***i*****p** (or ***i*****σ.p**) site and it is the three-dimensional aspect of the **p** (or **σ.p**) term which is responsible for the three-dimensionality of quark 'colour'. A separate 'colour' cannot be identified any more successfully than a separate dimension, and the quarks become part of a system, the three parts of which have $\hat{\mathbf{p}}_a$ values taking on one each of the orthogonal components $\hat{\mathbf{p}}_x$, $\hat{\mathbf{p}}_y$, $\hat{\mathbf{p}}_z$. Meson states have corresponding values of $\hat{\mathbf{p}}_a$, $\hat{\mathbf{p}}_b$ and $\hat{\mathbf{p}}_c$ in the fermion and antifermion components, although the logical operators δ_{0m} and $(-1)^{\delta_{1g}}g$ may take up different values for the fermion and the antifermion, and the respective signs of $\boldsymbol{\sigma}_z$ are opposite.

The electromagnetic interaction identifies (3). This occupies the ***j****m* site in the Dirac pentad. Respectively, the three interactions ensure that the orientation, direction and magnitude of angular momentum are separately conserved. Gravity (mass), finally, identifies (4).

There is a charge conjugation from $-w$ to w, in the second and third generations, with corresponding violations of *P* and *T* symmetries. This is brought about by the filled weak vacuum needed to avoid negative energy states. The two weak isospin states are associated with this idea in (3), the **1** in ($\hat{\mathbf{p}}_b$ – $\mathbf{1}\delta_{0m}$) being a 'filled' state, with its absence an unfilled state, and the weak interaction acts by annihilating and creating *e*, either filling the vacuum or emptying it – which is why, unlike the strong interaction, it always involves the equivalent of particle + antiparticle = particle + antiparticle, and involves a massive intermediate boson. We thus create two possible vacuum states to allow variation of the sign of electric charge by weak isospin, and this variation is linked to the filling of the vacuum which occurs in the weak interaction, and could be connected with a mass-related 'bosonic' spin 0 linking of the two isospin states (in addition to the spin 1 gauge bosons involved in the interaction).

The weak and electric interactions are linked by this filled vacuum in the $SU(2)_L \times U(1)$ model, as they are in the description of weak isospin, and we can regard these as alternative formalisms for representing the same physical truth. It is significant that the Higgs mechanism for generating masses of intermediate weak bosons and fermions requires the same Higgs vacuum field both for $SU(2)_L$ and for $U(1)$. In addition, the combination of scalar and pseudoscalar phases in the mathematical description of the combined electric and weak interactions clearly relates to the use of a complex scalar field in the conventional derivation of the Higgs mechanism.

The formalism actually explains easily how mass is generated when an element of partial right-handedness is introduced into an intrinsically left-handed system. In principle, anything which alters the signs of the terms in the expression $(\boldsymbol{i}\ \hat{\mathbf{p}}_a\ (\delta_{bc} - 1) + \boldsymbol{j}\ (\hat{\mathbf{p}}_b - \mathbf{1}\delta_{0m}) + \boldsymbol{k}\ \hat{\mathbf{p}}_c\ (-1)^{\delta_{1g}}\ g)$, or reduces any of these terms to zero, is a mass generator, because it is equivalent to introducing the opposite sign of $\boldsymbol{\sigma}_z$ or a partially right-handed state. Thus mass can be produced separately by weak isospin, by quark confinement, and by weak charge conjugation violation. The degree of right-handedness is a direct measure of the value of mass, as it determines the *zitterbewegung* frequency. The second and third generations, by successively violating parity and time-reversal symmetry, effectively bring about 'step functions' in this introduction of a right-handed component.

The expression (A) is remarkable in leading to exactly twelve fermionic structures, created by discrete operations with differing degrees of right-handedness introduced.

down	$-\boldsymbol{\sigma}.\ (-\boldsymbol{j}\hat{\mathbf{p}}_a + \boldsymbol{i}\hat{\mathbf{p}}_b + \boldsymbol{k}\hat{\mathbf{p}}_c)$
up	$-\boldsymbol{\sigma}.\ (-\boldsymbol{j}(\hat{\mathbf{p}}_a - \mathbf{1}) + \boldsymbol{i}\hat{\mathbf{p}}_b + \boldsymbol{k}\hat{\mathbf{p}}_c)$
strange	$-\boldsymbol{\sigma}.\ (-\boldsymbol{j}\hat{\mathbf{p}}_a + \boldsymbol{i}\hat{\mathbf{p}}_b - z_P\boldsymbol{k}\hat{\mathbf{p}}_c)$
charmed	$-\boldsymbol{\sigma}.\ (-\boldsymbol{j}(\hat{\mathbf{p}}_a - \mathbf{1}) + \boldsymbol{i}\hat{\mathbf{p}}_b - z_P\boldsymbol{k}\hat{\mathbf{p}}_c)$
bottom	$-\boldsymbol{\sigma}.\ (-\boldsymbol{j}\hat{\mathbf{p}}_a + \boldsymbol{i}\hat{\mathbf{p}}_b - z_T\boldsymbol{k}\hat{\mathbf{p}}_c)$
top	$-\boldsymbol{\sigma}.\ (-\boldsymbol{j}(\hat{\mathbf{p}}_a - \mathbf{1}) + \boldsymbol{i}\hat{\mathbf{p}}_b - z_T\boldsymbol{k}\hat{\mathbf{p}}_c)$
electron	$-\boldsymbol{\sigma}.\ (-\boldsymbol{j}\hat{\mathbf{p}}_a + \boldsymbol{k}\hat{\mathbf{p}}_a)$
e neutrino	$-\boldsymbol{\sigma}.\ (-\boldsymbol{j}(\hat{\mathbf{p}}_a - \mathbf{1}) + \boldsymbol{k}\hat{\mathbf{p}}_a)$
muon	$-\boldsymbol{\sigma}.\ (-\boldsymbol{j}\hat{\mathbf{p}}_a - z_P\boldsymbol{k}\hat{\mathbf{p}}_a)$
μ neutrino	$-\boldsymbol{\sigma}.\ (-\boldsymbol{j}(\hat{\mathbf{p}}_a - \mathbf{1}) - z_P\boldsymbol{k}\hat{\mathbf{p}}_a)$
tau	$-\boldsymbol{\sigma}.\ (-\boldsymbol{j}\hat{\mathbf{p}}_a - z_T\boldsymbol{k}\hat{\mathbf{p}}_a)$
τ neutrino	$-\boldsymbol{\sigma}.\ (-\boldsymbol{j}(\hat{\mathbf{p}}_a - \mathbf{1}) - z_T\boldsymbol{k}\hat{\mathbf{p}}_a)$

Both antiquarks and antileptons simply replace $-\boldsymbol{\sigma}$ with $\boldsymbol{\sigma}$.

Tables of charge structures

The outcome of all the processes determining the 0 and 1 *e*-*s*-*w* charge structures of the fundamental fermions may be expressed in terms of a set of three 'quark' tables, *A*-*C*, with an extra table *L* for the left-handed leptons and antileptons (the unlabelled columns in *L* represent left-handed antineutrinos). Applying these to the known fermions, *A*-*C* would appear to have all the properties of the coloured quark system, with *s* pictured as being 'exchanged' between the three states (although in reality, of course, all the states exist simultaneously), in the same way as the operator $\mathbf{p}$ in the nilpotent baryon wavefunction. In relation to these tables, we can look on symmetry-breaking, in general, as a consequence of the setting up of the algebraic model for charges. When we map time, space and mass onto the charges *w*-*s*-*e*, to create the anticommuting Dirac pentad, only one charge (*s*) has the full range of vector options. ‘Fixing’ one of the others (say *e*) for *s* to vary against, gives us only 2 remaining options for *w*, unit on the same colour as *e* or unit on a different one. Putting both *w* and *e* on the same colour denies the necessary three degrees of freedom in the direction of angular momentum, so this is forbidden in a quark system. (It is assumed that a violation of parity, symbolised by z_P, or a violation of time-reversal symmetry symbolised by z_P, acts in the second and third generations to restore the sign of the weak charge.)

A

		B	**G**	**R**
u	$+e$	$1\mathbf{j}$	$1\mathbf{j}$	$0\mathbf{i}$
	$+s$	$1\mathbf{i}$	$0\mathbf{k}$	$0\mathbf{j}$
	$+w$	$1\mathbf{k}$	$0\mathbf{i}$	$0\mathbf{k}$
d	$-e$	$0\mathbf{j}$	$0\mathbf{k}$	$1\mathbf{j}$
	$+s$	$1\mathbf{i}$	$0\mathbf{i}$	$0\mathbf{k}$
	$+w$	$1\mathbf{k}$	$0\mathbf{j}$	$0\mathbf{i}$
c	$+e$	$1\mathbf{j}$	$1\mathbf{j}$	$0\mathbf{i}$
	$+s$	$1\mathbf{i}$	$0\mathbf{k}$	$0\mathbf{j}$
	$-w$	$z_P\mathbf{k}$	$0\mathbf{i}$	$0\mathbf{k}$
s	$-e$	$0\mathbf{j}$	$0\mathbf{k}$	$1\mathbf{j}$
	$+s$	$1\mathbf{i}$	$0\mathbf{i}$	$0\mathbf{k}$
	$-w$	$z_P\mathbf{k}$	$0\mathbf{j}$	$0\mathbf{i}$
t	$+e$	$1\mathbf{j}$	$1\mathbf{j}$	$0\mathbf{i}$
	$+s$	$1\mathbf{i}$	$0\mathbf{k}$	$0\mathbf{j}$
	$-w$	$z_T\mathbf{k}$	$0\mathbf{i}$	$0\mathbf{k}$
b	$-e$	$0\mathbf{j}$	$0\mathbf{k}$	$1\mathbf{j}$
	$+s$	$1\mathbf{i}$	$0\mathbf{i}$	$0\mathbf{k}$
	$-w$	$z_T\mathbf{k}$	$0\mathbf{j}$	$0\mathbf{i}$

B

		B	**G**	**R**
u	$+e$	$1\mathbf{j}$	$1\mathbf{j}$	$0\mathbf{k}$
	$+s$	$0\mathbf{i}$	$0\mathbf{k}$	$1\mathbf{i}$
	$+w$	$1\mathbf{k}$	$0\mathbf{i}$	$0\mathbf{j}$
d	$-e$	$0\mathbf{i}$	$0\mathbf{k}$	$1\mathbf{j}$
	$+s$	$0\mathbf{j}$	$0\mathbf{i}$	$1\mathbf{i}$
	$+w$	$1\mathbf{k}$	$0\mathbf{j}$	$0\mathbf{k}$
c	$+e$	$1\mathbf{j}$	$1\mathbf{j}$	$0\mathbf{k}$
	$+s$	$0\mathbf{i}$	$0\mathbf{k}$	$1\mathbf{i}$
	$-w$	$z_P\mathbf{k}$	$0\mathbf{i}$	$0\mathbf{j}$
s	$-e$	$0\mathbf{i}$	$0\mathbf{k}$	$1\mathbf{j}$
	$+s$	$0\mathbf{j}$	$0\mathbf{i}$	$1\mathbf{i}$
	$-w$	$z_P\mathbf{k}$	$0\mathbf{j}$	$0\mathbf{k}$
t	$+e$	$1\mathbf{j}$	$1\mathbf{j}$	$0\mathbf{k}$
	$+s$	$0\mathbf{i}$	$0\mathbf{k}$	$1\mathbf{i}$
	$-w$	$z_T\mathbf{k}$	$0\mathbf{i}$	$0\mathbf{j}$
b	$-e$	$0\mathbf{i}$	$0\mathbf{k}$	$1\mathbf{j}$
	$+s$	$0\mathbf{j}$	$0\mathbf{i}$	$1\mathbf{i}$
	$-w$	$z_T\mathbf{k}$	$0\mathbf{j}$	$0\mathbf{k}$

C

		B	**G**	**R**
u	$+e$	$1\boldsymbol{j}$	$1\boldsymbol{j}$	$0\boldsymbol{k}$
	$+s$	$0\boldsymbol{i}$	$1\boldsymbol{i}$	$0\boldsymbol{j}$
	$+w$	$1\boldsymbol{k}$	$0\boldsymbol{k}$	$0\boldsymbol{i}$
d	$-e$	$0\boldsymbol{j}$	$0\boldsymbol{k}$	$1\boldsymbol{j}$
	$+s$	$0\boldsymbol{i}$	$1\boldsymbol{i}$	$0\boldsymbol{k}$
	$+w$	$1\boldsymbol{k}$	$0\boldsymbol{j}$	$0\boldsymbol{i}$
	$+e$	$1\boldsymbol{j}$	$1\boldsymbol{j}$	$0\boldsymbol{k}$
	$+s$	$0\boldsymbol{i}$	$1\boldsymbol{i}$	$0\boldsymbol{j}$
	$-w$	$z_P\boldsymbol{k}$	$0\boldsymbol{k}$	$0\boldsymbol{i}$
s	$-e$	$0\boldsymbol{j}$	$0\boldsymbol{k}$	$1\boldsymbol{j}$
	$+s$	$0\boldsymbol{i}$	$1\boldsymbol{i}$	$0\boldsymbol{k}$
	$-w$	$z_P\boldsymbol{k}$	$0\boldsymbol{j}$	$0\boldsymbol{i}$
t	$+e$	$1\boldsymbol{j}$	$1\boldsymbol{j}$	$0\boldsymbol{k}$
	$+s$	$0\boldsymbol{i}$	$1\boldsymbol{i}$	$0\boldsymbol{j}$
	$-w$	$z_T\boldsymbol{k}$	$0\boldsymbol{k}$	$0\boldsymbol{i}$
b	$-e$	$0\boldsymbol{j}$	$0\boldsymbol{k}$	$1\boldsymbol{j}$
	$+s$	$0\boldsymbol{i}$	$1\boldsymbol{i}$	$0\boldsymbol{k}$
	$-w$	$z_T\boldsymbol{k}$	$0\boldsymbol{j}$	$0\boldsymbol{i}$

L

		$\bar{e}$	$\bar{e}$	ν_e
	$+e$	$1\boldsymbol{j}$	$1\boldsymbol{j}$	$0\boldsymbol{j}$
	$+s$	$0\boldsymbol{k}$	$0\boldsymbol{i}$	$0\boldsymbol{i}$
	$+w$	$0\boldsymbol{i}$	$0\boldsymbol{k}$	$1\boldsymbol{k}$
				e
	$-e$	$0\boldsymbol{i}$	$0\boldsymbol{k}$	$1\boldsymbol{j}$
	$+s$	$0\boldsymbol{j}$	$0\boldsymbol{i}$	$0\boldsymbol{i}$
	$+w$	$0\boldsymbol{k}$	$0\boldsymbol{j}$	$1\boldsymbol{k}$
		$\bar{\mu}$	$\bar{\mu}$	ν_μ
	$+e$	$1\boldsymbol{j}$	$1\boldsymbol{j}$	$0\boldsymbol{j}$
	$+s$	$0\boldsymbol{k}$	$0\boldsymbol{i}$	$0\boldsymbol{i}$
	$-w$	$0\boldsymbol{i}$	$0\boldsymbol{k}$	$z_P\boldsymbol{k}$
				μ
	$-e$	$0\boldsymbol{i}$	$0\boldsymbol{k}$	$1\boldsymbol{j}$
	$+s$	$0\boldsymbol{j}$	$0\boldsymbol{i}$	$0\boldsymbol{i}$
	$-w$	$0\boldsymbol{k}$	$0\boldsymbol{j}$	$z_P\boldsymbol{k}$
		$\bar{\tau}$	$\bar{\tau}$	ν_τ
	$+e$	$1\boldsymbol{j}$	$1\boldsymbol{j}$	$0\boldsymbol{j}$
	$+s$	$0\boldsymbol{k}$	$0\boldsymbol{i}$	$0\boldsymbol{i}$
	$-w$	$0\boldsymbol{i}$	$0\boldsymbol{k}$	$z_T\boldsymbol{k}$
				τ
	$-e$	$0\boldsymbol{i}$	$0\boldsymbol{k}$	$1\boldsymbol{j}$
	$+s$	$0\boldsymbol{j}$	$0\boldsymbol{i}$	$0\boldsymbol{i}$
	$-w$	$0\boldsymbol{k}$	$0\boldsymbol{j}$	$z_T\boldsymbol{k}$

SU(5)

The idea that the 5-fold Dirac algebra is responsible for the symmetry breaking which leads to the $SU(3) \times SU(2)_L \times U(1)$ splitting in the interactions between fundamental particles, suggests that Grand Unification between the three local interactions may involve the $SU(5)$ group, or even $U(5)$, or something containing $SU(5)$ and extending it to right-handed states, such as $SO(10)$. In principle, we effectively have five units expressed in different forms:

E	p_x	p_y	p_z	m
w	s_G	s_R	s_B	e
$i\boldsymbol{k}$	$\mathbf{ii}$	$\mathbf{ij}$	$i\mathbf{k}$	$\boldsymbol{j}$
γ^0	γ^1	γ^2	γ^3	γ^5

The mapping of the strong terms is always exact, but the electroweak terms are so closely linked physically that transposition to equivalent representations may be necessary to reflect the physical manifestations of these interactions. The five charge units (e, s_G, s_B, s_R, w, taking into account the vector nature of s) map directly onto the five Dirac operators (***ik***; **i*i***; **j*i***; **k*i***; ***j***), and the five quantities (m, p_x, p_y, p_z, E) involved in the Dirac equation, and generate both an overall $SU(5)$ and its breakdown to $SU(3) \times SU(2)_L \times U(1)$. The 24 $SU(5)$ generators can be represented in terms of any of these units. For example:

	$\bar{s}_G$	$\bar{s}_B$	$\bar{s}_R$	$\bar{w}$	$\bar{e}$
s_G					
s_B		gluons		Y	X
s_R					
w		Y		Z^0, γ	W^-
e		X		W^+	Z^0, γ

or:

	$\bar{p}_x$	$\bar{p}_y$	$\bar{p}_z$	$\bar{E}$	$\bar{m}$
p_x					
p_y		gluons		Y	X
p_z					
E		Y		Z^0, γ	W^-
m		X		W^+	Z^0, γ

The only unobserved generators here are the strong-electroweak bosons X and Y, which earlier $SU(5)$ schemes have taken to imply direct proton decay. However, such decay would be forbidden by separate charge conservation rules, as it involves the complete elimination of a strong charge unit, and it also disregards the necessary dipolarity of the weak charge. Here, the X and Y generators remain linked to the dipolar particle + antiparticle mechanism of the ordinary weak interaction. There is, in fact, one mechanism that unites strong and electroweak interactions in the way that would be expected of X and Y, and this is ordinary beta decay. We can propose that, though this process is not mediated by X or Y at the energies now observable, there will be an amplitude for X or Y intervention at higher energies.

$SU(5)$, however, is not the full story. If we had a 25th generator (which the Standard Model disregards on the grounds that it is not observed), the group would become $U(5)$, and all the generators would be entirely equivalent to scalar phases. Such a particle, if it existed, would couple to all matter in proportion to the amount, and, as a colour singlet, would be ubiquitous. This is a precise description of the gravitational-inertial force of gravity, and, if we can show that Grand Unification of the electromagnetic, strong and weak forces occurs at the Planck mass, the energy usually taken as characteristic of quantum gravity, then it is possible that $U(5)$, or something containing it, might be the true Grand Unification group, and that it also incorporates a gravitational-inertial generator, most probably a spin 1 pseudo-boson for the inertial reaction. This generator would then link the 2 colourless gluons with the Z^0 and γ, along the diagonal of the group table, suggesting a link between all four interactions. Reduction of the generators to scalar phases would mean that, at grand unification, all interactions would be identical in effect, and all non-Coulombic structure would disappear. The unification would be exact.

Quarks

The non-Abelian gauge theory of quantum chromodynamics (QCD) for the strong interaction between coloured quarks has turned out to be one of the most successful aspects of the Standard Model. In the model as normally applied, the up quark has electric charge $2e / 3$, where e is the fundamental electronic charge, while the down quark has a charge of $-e / 3$. The 'strong' property manifested in the baryon number (B), equivalent to our strong charge, is shared out between the 'coloured' quarks.

	Blue	**Green**	**Red**
up	$2e / 3$	$2e / 3$	$2e / 3$
	$B / 3$	$B / 3$	$B / 3$
down	$-e / 3$	$-e / 3$	$-e / 3$
	$B / 3$	$B / 3$	$B / 3$

QED phenomenology of quantum electrodynamics has shown over many experiments that quarks behave as though constituted in exactly this way, irrespective of the energy of the interaction. The question is, why does nature choose a structure like this? We seem to have 3 separate 'units' of charge ($e / 3$, $2e / 3$ and e), with an implication that the electron, with e, and even the up quark (with $2e / 3$), may not actually be 'elementary'. What we seem to see here is evidence of a *broken symmetry*. Now, broken symmetries, according to our fundamental thinking, can't exist at the most fundamental level. They are *emergent*. So what could be more fundamental? To my way of thinking, it has to be something that puts quarks and leptons on something like the same level, and shows up their common origins.

In fact, there is such a possibility, almost as old as Gell-Mann and Zweig's original quark theory of 1963, and that is the first coloured quark theory by Han and Nambu of 1964, which proposed that exactly the same results could be obtained using *integral* and zero charges and assigning an integral baryon number to a single quark. Now this proposal seems to fit particularly well with the structures which seem to emerge from the nilpotent wavefunction, with the baryon number taking the place of the momentum operator. Here, colour is not an additional property, but an integral part of the structure, and the quarks now look significantly like leptons.

	Blue	Green	Red
up	e	e	0
	B	0	0
down	0	0	$-e$
	B	0	0

Broken symmetries seem to emerge where the situation introduces complexity. Here, the complexity for the electric charge distribution is the strong interaction which is the only reason for the quarks' existence, and which demands perfect gauge invariance. This cannot be broken and the quarks in either representation will necessarily be seen as fractional for QED, so showing the correctness of the first version *for that purpose*. The difference between the two representations can now be clarified. In the first the fractional nature of electric charges is due to the electric interaction itself; in the second it is externally imposed by the strong interaction. Because the masslessness of gluons shows that the gauge invariance of this interaction is exact, there will never be a high energy regime where the integral nature of the charges will be revealed. The fractional electric charges reflect the perfect equivalence between the different coloured states or phases of the interaction. They are QED or electroweak eigenstates.

We now know how the process works in the parallel case of the fractional quantum Hall effect, where electrons appear with effective charges of e / 3, e / 5 and other fractional values, because an electron or other fermion can form a pseudobosonic combination with an odd number of magnetic flux lines and so effectively share itself out between them. Interestingly, in this parallel case, it is the *weak* interaction, rather than the strong, acting as the external agent determining QED phenomenology. The question is not whether QED should use fractional charges – clearly it should – but whether these are *fundamental* or stem from a broken symmetry. QED phenomenology clearly doesn't decide the question of which basic structure of charges to use in other areas such as Grand Unification, or the *gauge relations between the interactions*. Using our fundamental methodology would suggest that we go for the best fit with the symmetry, and, if that leads to modifications in our usual practice, see if this has any interesting consequences. In fact, as we shall see, there is one of exceptional interest.

Grand unification: a prediction

A Grand Unified Theory (GUT), to unite electric, weak and strong interactions, based on the $SU(5)$ group, was first proposed by Georgi and Glashow in 1974, following on from the Glashow-Weinberg-Salam $SU(2) \times U(1)$ unification of the electric and weak interactions. The electroweak unification is governed by the weak mixing angle parameter $\sin^2\theta_W$, which is effectively the ratio (α / α_2) between the weak and electric couplings (α_2 and α). Georgi and Glashow showed that, in any GU scheme determined by a single GU gauge group, $\sin^2\theta_W$ would be given by the ratio of the sum of all the squared units of weak isospin (t_3) for the fermions of the Standard Model to the sum of all their squared units of electric charge (Q).

$$\sin^2\theta_W = \frac{Tr\left(t_3^2\right)}{Tr\left(Q^2\right)}.$$

Taking the weak components with only left-handed contributions to weak isospin, for the first generation of quarks and leptons, that is, for 3 colours of u, 3 colours of d, and the leptons e and ν, we obtain:

$$Tr\left(t_3^2\right) = \frac{1}{4} \times 8 = 2.$$

Quarks and leptons have identical units of weak isospin, and so this summation will be the same for both quark theories, and will also be the result expected for phenomenology. But for the electric charge structure, the summations of the two representations is different. For, fractional charges, with both left- and right-handed contributions in the first generation, we obtain

$$Tr\left(Q^2\right) = 2 \times \left(\frac{4}{9} \times 3 \times \frac{1}{9} \times 3 + 1 + 0\right) = \frac{16}{3}$$

from which $\sin^2\theta_W = 0.375$.

For integral charges, however, we have

$$Tr\,(Q^2) = 2 \times (1 + 1 + 0 + 0 + 0 + 1 + 1 + 0) = 8,$$

leading to $\sin^2\theta_W = 0.25$.

The value of 0.375 is, as Steven Weinberg has said, in ‘gross disagreement’ with the experimental value for $\sin^2\theta_W$ of 0.231 at around the mass-energy of the Z particle (M_Z = 91 GeV), but 0.25 is relatively close to this value and would be even closer (with some small second order corrections) if the effect of the direct production of W and Z bosons at their mass-energies M_W and M_Z is taken into account (or if the 0.25 occurs at the vacuum expectation energy (246 GeV) rather than at M_W or M_Z). 0.25 is also the value that would be obtained purely from the leptonic contribution, and it is rather curious that the value for a purely electroweak parameter should be different in the quark and lepton sectors.

Going back to the original ‘minimal $SU(5)$’ GUT of Georgi and Glashow, we find that it doesn’t actually unify the pure interactions at all, for, though the theory begins with the equations for the running weak and strong coupling constants, derived by quantum field theory from their respective $SU(2)$ and $SU(3)$ structures:

$$\frac{1}{\alpha_2(\mu)} = \frac{1}{\alpha_G} - \frac{5}{6\pi} ln \frac{M_X^2}{\mu^2}$$

and

$$\frac{1}{\alpha_3(\mu)} = \frac{1}{\alpha_G} - \frac{7}{4\pi} ln \frac{M_X^2}{\mu^2} ,$$

(where M_X is the GU energy scale, α_G is the fine structure constant at this energy and μ is the energy scale of measurement) it proposes that the grand unified gauge group structure will modify the equivalent $U(1)$ equation for the electromagnetic coupling (1 / α), assumed (in this theory) to be

$$\frac{1}{\alpha(\mu)} = \frac{1}{\alpha_G} + \frac{5}{3\pi} ln \frac{M_X^2}{\mu^2} ,$$

to one in which it is mixed with the weak value, based on $SU(2) \times U(1)$. So, now we have

$$\frac{1}{\alpha_1(\mu)} = \frac{1}{\alpha_G} + \frac{1}{6\pi} ln \frac{M_X^2}{\mu^2} ,$$

where

$$\frac{5}{3\alpha_1(\mu)} + \frac{1}{\alpha_2} = \frac{1}{\alpha} .$$

From these equations, we derive a grand unified mass scale (M_X) of order 10^{15} GeV, and from

$$\sin^2 \theta_W = \frac{\alpha(\mu)}{\alpha_2(\mu)},$$

we find 'renormalized' values of $\sin^2\theta_W$ at the measurement scale of order 0.19 to 0.21.

As is well known, the curves representing the variations of the parameters α_1, α_2 and α_3 at different energy scales (μ) don't actually cross at anything very close to a point, leading to the somewhat *ad hoc* proposal that a supersymmetric model may be the only solution. In addition we are forced to use a combined electroweak parameter which makes assumptions about group structure, and relies on a particular value for the squared 'Clebsch-Gordan coefficient' of the group, $C^2 = 1 / \sin^2\theta_W - 1 = 5 / 3$, that has, as yet, no experimental or theoretical justification. Unifying *electroweak*, weak and strong parameters doesn't seem as convincing as using the original electric, weak and strong values, while the assumed value of $\sin^2\theta_W = 0.375$ at GU suggests that the electroweak unification is not even then complete, as the two forces are not an equal footing.

The 'convergence' is also three or four orders of magnitude below the scale of the Planck energy at which quantum gravity is assumed to operate, suggesting that another principle will be needed to include gravitation. But, in any case, compensating errors in the combination tend to disguise the massive inconsistencies between the separate equations for the coupling constants. In particular, recalculation of the value of $\sin^2\theta_W$ at $\mu = 10^{15}$ GeV gives 0.6 rather than the 0.375 which was initially assumed in setting up the equations!

Now, one advantage of using integral quark charges is that it gives us an *independent* value for $\sin^2\theta_W = \alpha / \alpha_2$ of the right order, and we can perform a much simpler calculation for M_X without making assumptions about the group structure, by avoiding the problematic running coupling constant equation for $1 / \alpha_1$, using only the more secure equations for $1 / \alpha_2$ and $1 / \alpha_3$. In addition, the hypercharge numbers for the $U(1)$ electromagnetic running coupling equation will now be no longer identical to those for a quark model based purely on QED phenomenology. The fermionic contribution to QED vacuum polarization is, for *fundamental* fractional charges,

$$\frac{4}{3}\times\frac{1}{2}\times\left(\frac{1}{36}\times3+\frac{1}{36}\times3+\frac{1}{9}\times3+\frac{4}{9}\times3+\frac{1}{4}\times1+\frac{1}{4}\times1+1\right)\frac{n_g}{4\pi}=\frac{5}{3\pi},$$

where $n_g = 3$ is the number of fermion generations, and the terms in the bracket represent, respectively, the squared average charge in the isospin quark doublet, the squared charges of the quarks, the squared average charge of the isospin lepton doublet, and the squared charges of the leptons, all for both left- and right-handed states. Modifying this for fundamental integral charges, we obtain:

$$\frac{4}{3}\times\frac{1}{2}\times\left(\frac{1}{4}\times3+\frac{1}{4}\times3+1+1+0+0+0+1+\frac{1}{4}\times1+\frac{1}{4}\times1+1\right)\frac{n_g}{4\pi}=\frac{3}{\pi}.$$

This result corresponds to a change in the squared Clebsch-Gordan coefficient from $C^2 = 5 / 3$ to $C^2 = 3$, when $\sin^2\theta_W = 1 / (1 + C^2)$ changes from 0.375 to 0.25. With the new values we have obtained for the hypercharge numbers, the running coupling of the pure electromagnetic interaction, will be:

$$\frac{1}{\alpha(\mu)}=\frac{1}{\alpha_G}+\frac{3}{\pi}ln\frac{M_X^2}{\mu^2}.$$

Leaving out the speculative equation for $1 / \alpha_1$, and, for the moment, this new one for $1 / \alpha$, but using the well-established ones for $1 / \alpha_2$ and $1 / \alpha_3$, and $\sin^2\theta_W = \alpha / \alpha_2$, we obtain

$$\sin^2\theta_W(\mu)=\alpha(\mu)\left(\frac{1}{\alpha_3(\mu)}+\frac{11}{6\pi}ln\frac{M_X}{\mu}\right)$$

Taking typical values for $\mu = M_Z = 91.2$ GeV, $\alpha_3(M_Z^2) = 0.118$ (or 0.12), $\alpha(M_Z^2) = 1 / 128$, and $\sin^2\theta_W = 0.25$, we obtain a value for the GU energy scale ($M_X = 2.8 \times 10^{19}$

GeV) which is extraordinarily close to the Planck value (1.22×10^{19} GeV), and may well be exactly so, as purely first-order calculations overestimate the value of M_X. Assuming that M_X *is* the Planck mass, we obtain α_G (the GU value for all interactions) = 1 / 52.4, and $\alpha_2(M_Z^2)$ = 1 / 31.5, which is exactly the kind of value we would expect for the weak coupling with $\sin^2\theta_W = 0.25$ close to M_Z.

To provide an independent check on the validity of the procedure, we can *now* make direct use of the equation we have derived for 1 / α, with the new hypercharge numbers and GU at the Planck mass, to obtain 1 / $\alpha\,(M_Z^2)$ = 128, which is, of course, exactly the value obtained experimentally at energies corresponding to $\mu = M_Z$. This appears to be a striking confirmation of the assumptions made in the first calculation, leading to M_X, as coincidental agreements are most unlikely for equations involving logarithmic terms, and it is also potentially very significant, for it would now appear that the unification which occurs at M_X might well involve a direct numerical equalization of the strengths of the three, or even four, physical force manifestations, without reference to the exact unification structure.

The analysis suggests that, at grand unification, $C^2 = 0$ and $\sin^2\theta_W = 1$, creating an exact symmetry in every respect between weak and electric interactions, as well as between weak and strong, which is completely different from the only partial unification achieved using the fundamental fractional charges, and linking this with the scale associated with quantum gravity. The mixing parameter, $\sin^2\theta_W$, as normally understood, may then be interpretable as the electroweak constant for a specifically *broken* symmetry, taking the value of 0.25 at the energy range where the symmetry breaking occurs (presumably at M_W-M_Z, or, alternatively, the expectation value of the Higgs field, 246 GeV), and gradually decreasing from the maximum ($\sin^2\theta_W = 1$) to this value at intermediate energies. At GU, we may suppose, all four forces are reduced to scalar phases, with $U(1)$ symmetry and purely Coulombic interaction, all distinguishing aspects of the weak and strong interactions having diminished to zero.

One of the most significant aspects of the calculation is that it leads to completely testable predictions, as the values of the three coupling constants can be calculated for any energy with relative precision from the known values of α_G and M_X. In particular, the value of α changes rapidly in a way that can be determined at energies now available to us experimentally. At 14 TeV, for instance, it would have the value of 1 / 118, compared to 1 / 125 from the minimal $SU(5)$ theory of Georgi and Glashow.

Relatively simple considerations based on results from the extensive quantum field theories of the electric, weak and strong interactions, which require only a small amount of arithmetical and algebraic manipulation, thus suggest that, if the fundamental representation behind the fractional broken symmetry requires integral charges, then it has major consequences for GU, which are accessible by experiment, and it would also have another advantage in making quark-lepton unification (in symmetry terms) much more likely, as both sectors would now be characterized by integral charges, and quarks would have the same electroweak structure as leptons.

The Higgs mechanism and fermion masses

There is another area where this application of fundamental methodology may have a significant consequence. This is in the problematic application of the Higgs mechanism to the generation of fermion masses. Here, to generate separate masses for the two isospin states in each generation, we require two different hypercharge (or 2 × average charge) units of 1 and –1, yet, if the fractional charges are fundamental, there is only one hypercharge value for all quarks, and that is the fractional value, 2/3. The only expedient then is to 'invent' two hypercharges not justified by the assumed charge structure. If integral charges are fundamental, however, the different colours of quark automatically produce the two hypercharge values, 1 and –1, which we require for both isospin states and which would be repeated in each generation.

The leptons, of course, are not fractionally charged, but there is a separate area of difficulty with them. In the past, the lepton mass mechanism could be accommodated by assigning the single hypercharge value in the first generation to electrons, but the discovery of neutrino masses means that the opposite hypercharge value is now required for neutrinos. It is possible that this difficulty can be resolved in both representations if the neutrino is a Majorana particle, with a low mass resulting from the low probability of the neutrino transforming to its antistate with the opposite hypercharge.

Larger group structures for fermions and bosons

One of the major questions in particle physics is: can the fundamental particles be linked in a single group representation? The particles represent many broken symmetries, and, according to our methodology, broken symmetries are a result of complexity or synthesis, and not from some unknown 'symmetry-breaking principle' which applies to large-scale structures. There appear to be two major symmetries in the parameter group based on the numbers 2 (for duality) and 3 (for anticommutativity) and versions of these link up in the creation of fermion point particles in a broken symmetry based on the number 5.

If we look at the fundamental particles, all the symmetries which apply to them seem to be constructed from smaller symmetries based on these units. The same also applies to many of the groups thought to be of significance in this area, particularly those based on the octonion symmetries, such as the exceptional groups E_6, E_7 and E_8. Because the symmetry-breaking is ultimately 3-dimensional in origin (and manifested, for example, in quarks and 3 particle generations), the symmetries involved in particle groupings tend to map naturally onto geometries in 3-dimensional space. However, the higher groupings which collect together particles such as quarks and leptons, or fermions and bosons, also show relationships with structures in higher-dimensional spaces and groups connected with them. Here, as we would expect, the symmetries become less broken, perhaps culminating in an unbroken root vector structure in E_8, the highest group symmetry to emerge from this type of mathematics. Generally, the structures in the higher-dimensional figures carry with them the numbers associated with those from the lower dimensions.

The group E_8 has long been suspected of being a possible unifying group for the fundamental particles, and was discussed as such, among other places, in *Zero to Infinity*. In 2007, Garrett Lisi proposed that all known fermions and gauge bosons could be fitted into the 240 root vectors of the E_8 group. The model has been heavily criticized, and doesn't look right as it stands. Its particles don't add up to 240, leading to a completely *ad hoc* speculation about particles needed to make up the numbers, the gravity theory is very speculative, the generations don't arise naturally, etc. Some of the assignments seem very difficult to understand. Though I don't think that the model is correct, the idea may be, and it fits in with previous ideas on the significance of E_8. One of the things that was criticized was the inclusion of fermions and bosons in the same representation. Lisi argued that this was possible, but only through the exceptional groups E_6, E_7, E_8. If this is correct (and it has gained some support), it may be Lisi's most significant contribution, along with the emphasis on root vectors.

In the Standard Model, the particles are divided into fermions (spin ½) and gauge bosons (spin 1), and the fermions are divided into quarks and leptons. There are 6 quarks arranged in 3 generations, each of which has 2 weak isospin states (up / down; charm / strange; top / bottom), and each of which comes in 3 varieties of 'colour'. Corresponding to these are 6 leptons, again in 3 generations, each with 2 weak isospin states (electron neutrino / electron; muon neutrino / muon; tau neutrino / tau). There are no colours associated with the leptons, so each set of 3 coloured quarks and 1 lepton, in each isospin state in each generation, represents a kind of 4-dimensional structure, parallel to that of space and time. The total of real fermions in the Standard Model is therefore 24 (18 coloured quarks + 6 leptons).

In addition, each fermion has 2 possible spin states, and to every fermion state there is a corresponding antifermion state, making a total of 96 real fermionic + antifermionic states. The 2 spin states and fermion / antifermion options are an intrinsic aspect of the fermion's spinor structure, $(\pm i\boldsymbol{k}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$. Now, spin 1 gauge bosons can be represented by $(\pm i\boldsymbol{k}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\ (\mp i\boldsymbol{k}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$. In effect, 4 fermionic states are required to produce a boson, and the nilpotent formalism shows that 1 spin 1 vacuum boson (never seen, but still mathematically necessary) is created for every 4 real fermionic states, and 4 vacuum fermionic states (again never seen, but still necessary) are created for every real spin 1 boson.

Now, the number of real spin 1 gauge bosons in the Standard Model is 12 (8 gluons, $W+$, W^-, Z^0, γ), but virtually all Grand Unified theories (and certainly $SU(5)$) predict the existence of another 12 (6 *X* and 6 *Y*) to unify strong and electroweak interactions. These are equivalent in number but are distinct from the 24 vacuum bosons which accompany the 96 real fermionic / antifermionic states in the nilpotent formalism, in the same way as the real fermionic / antifermionic states are distinct from the 96 vacuum fermion / antifermion states which accompany the 24 real bosons. A combination of 96 fermions / antifermions and 24 spin 1 gauge bosons creates a total of 120 real particle states. If there are an equal number of vacuum states, then the total becomes 240, the kissing number in 8 dimensions and also the number of root vectors in E_8. Perhaps we can justify something like the following:

	quarks	leptons	bosons		fermions	bosons							
1	3	1	1	=	4	1	=	5					
2	6	2	2	=	8	2	=	10	S				
3	9	3	3	=	12	3	=	15					G
4	12	4	4	=	16	4	=	20	S	I			
5	18	6	6	=	24	6	=	30	S				G
6	24	8	8	=	32	8	=	40	S	I	A		
7	36	12	12	=	48	12	=	60	S	I			G
8	48	16	16	=	64	16	=	80	S	I	A	V	
9	72	24	24	=	96	24	=	120	S	I	A		G
10	144	48	48	=	192	48	=	240	S	I	A	V	G

1	1 generation with 1 isospin and 1 spin
2	1 generation with 1 isospin and 2 spins
3	3 generations with 1 isospin and 1 spin
4	1 generation with 2 isospins and 2 spins
5	3 generations with 1 isospin and 2 spins
6	1 generation with 2 isospins and 2 spins + antiparticles
7	3 generations with 2 isospins and 2 spins
8	1 generation with 2 isospins and 2 spins + antiparticles + vacuum
9	3 generations with 2 isospins and 2 spins + antiparticles
10	3 generations with 2 isospins and 2 spins + antiparticles + vacuum

Notably absent from this structure are the spin 0 Higgs boson (which is not a gauge boson), the spin 2 graviton (which may not exist), and the spin 1 inertial pseudoboson (which is not really a separate particle from the photon, but a special realisation of it at the Planck energy).

Particles constitute a 5, of 3 quarks + lepton + boson, which multiplies by 4 factor 2 dualities (spin up / down, isospin up / down, fermion / antifermion, particle / vacuum, symbolised respectively by S, I, A, V, and relating, respectively, to space, charge, time and mass) and 1 factor 3 triplet (generations, symbolised by G), and so (depending on the order in which the 4 factors of 2 and 1 of 3 are multiplied) generates number series up to a maximum number of 240. All products of 5 are equally artificial constructs, for example, linking fermions with bosons in the exceptional groups E_6 to E_8 through the fact that the last term in the 5 can be a scalar.

In effect, we invert the derivation of 12 structures from a 5-unit pentad, and map the fermions and bosons onto a new pentad structure, of which the pseudoscalar component (the *iE* term) is 24 leptons / antileptons, and the vector component (the **p** term) 72 quarks / antiquarks. Bosons are scalar particles, and scalars are the squared products of pseudoscalars and vectors, just as bosons are the squared products of fermions / antifermions. So if the 24 bosons occupy the *scalar* part of the pentad (the *m* term), then we can use nilpotency to group the 96 fermions (24 leptons and 72 quarks) with the 24 bosons into a single structure with 120 fermions plus bosons, and these would seem to be represented by the stages 48 + 12 = 60, 96 + 24 = 120, 192 + 48 = 240. So, in a nilpotent system, we have a physical as well as mathematical reason for combining fermions and bosons in the same representation, and, in the nilpotent structure, a fermion is necessarily, in some sense, its own vacuum boson, and vice versa. The 12 bosons we would put with the 48 are the 8 gluons, the 2 *W* and 1 *Z* boson, and the photon, that is, all except *X* and *Y*.

The interesting thing about the whole idea is that we can follow through many of the fundamental algebras derived from real and complex numbers, quaternions and octonions, geometries in spaces from 3 to 8 dimensions, and many associated groups, and the key numbers seem all to appear in this table. In it we see the complexity building up from the simplest symmetries in a way that suggest why the higher symmetries have physical meaning and why they are always broken.

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