# Investigating the Foundations of Physical Law
## 6 Nilpotent quantum mechanics II

### Bosons

Here, we take our quantum mechanics to the next level of complexity, and look at interacting fermions. This lecture will follow three stages. First of all, we will consider the *nonlocal* structures that emerge from the nilpotent formalism, in particular the combination states and superpositions which are required to describe bosons and baryons. Then we will see how these nonlocal structures have *local* consequences. In effect, we will see how local changes *inside* nilpotent brackets can produce the same results as nonlocal changes *outside* them. The local changes involve the creation of local potentials added to the energy and momentum operators. Following these local changes, we will then show how the nilpotent operators as now defined lead to just three possible solutions.

Fermion interactions require bosons, and these differ from fermions in having scalar wavefunctions and not being Pauli exclusive. These facts, while fundamental, have never been explained in fundamental terms. The nilpotent 4-spinor, as we have seen, is composed of the lead term, defining the 'real' particle state and three terms, which are, effectively, the *P*-, *T*- and *C*-transformed versions of this state. They are the possible states into which it could transform without changing the magnitude of its energy or momentum. We can also see them as vacuum 'reflections' of the real particle state, and we will later show how they arise from vacuum operations that can be mathematically defined, through a partitioning of the continuous vacuum into a 3-dimensional vacuum space, with each reflection being in one 'dimension' of the space. Now, Pauli exclusion prevents a fermion from forming a combination state with itself, but we can imagine it forming a combination state with each of these vacuum 'reflections', and, if the 'reflection' exists or materialises as a 'real' state, then the combined state can form one of the three classes of bosons or boson-like objects: spin 1 boson, spin 0 boson, or fermion-fermion combination.

A combination of fermion and antifermion with the same spins but opposite helicities will produce a state equivalent to a spin 1 boson. Suppose we take the product of a row vector fermion and a column vector antifermion, both written as columns for convenience:

$$\begin{array}{ll} \textit{row} & \textit{column} \\ (ikE + i\mathbf{p} + jm) & (-ikE + i\mathbf{p} + jm) \\ (ikE - i\mathbf{p} + jm) & (-ikE - i\mathbf{p} + jm) \\ (-ikE + i\mathbf{p} + jm) & (ikE + i\mathbf{p} + jm) \\ (-ikE - i\mathbf{p} + jm) & (ikE - i\mathbf{p} + jm) \end{array}$$

The antifermion structure reverses the signs of $E$ throughout, and spin reversal changes the sign of $\mathbf{p}$, but the phase factor of both fermion and antifermion components will be, according to our original construction of the nilpotent formalism, the same, dependent on the values of $E$ and $\mathbf{p}$ but not on their signs. Sign variations ensure cancellation of all the terms with quaternion coefficients, so the product is a nonzero scalar. The same result will apply if the spin 1 boson is massless (as is the case with such gauge bosons as photons and gluons). Then we have:

$$\begin{array}{ll} (i\boldsymbol{k}E + \boldsymbol{i}\mathbf{p}) & (i\boldsymbol{k}E + \boldsymbol{i}\mathbf{p}) \\ (i\boldsymbol{k}E - \boldsymbol{i}\mathbf{p}) & (-i\boldsymbol{k}E - \boldsymbol{i}\mathbf{p}) \\ (-i\boldsymbol{k}E + \boldsymbol{i}\mathbf{p}) & (i\boldsymbol{k}E + \boldsymbol{i}\mathbf{p}) \\ (-i\boldsymbol{k}E - \boldsymbol{i}\mathbf{p}) & (i\boldsymbol{k}E - \boldsymbol{i}\mathbf{p}) \end{array}$$

To construct a spin 0 boson structure we reverse the $\mathbf{p}$ signs in either fermion or antifermion, so that the components have the opposite spins but the same helicities:

$$\begin{array}{ll} (i\boldsymbol{k}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m) & (-i\boldsymbol{k}E - \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m) \\ (i\boldsymbol{k}E - \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m) & (-i\boldsymbol{k}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m) \\ (-i\boldsymbol{k}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m) & (i\boldsymbol{k}E - \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m) \\ (-i\boldsymbol{k}E - \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m) & (i\boldsymbol{k}E + \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m) \end{array}$$

The product is a nonzero scalar, but, this time, if we reduce the mass to zero, we will zero the product as well.

$$\begin{array}{ll} (i\boldsymbol{k}E + \boldsymbol{i}\mathbf{p}) & (-i\boldsymbol{k}E - \boldsymbol{i}\mathbf{p}) = 0 \\ (i\boldsymbol{k}E - \boldsymbol{i}\mathbf{p}) & (-i\boldsymbol{k}E + \boldsymbol{i}\mathbf{p}) = 0 \\ (-i\boldsymbol{k}E + \boldsymbol{i}\mathbf{p}) & (i\boldsymbol{k}E - \boldsymbol{i}\mathbf{p}) = 0 \\ (-i\boldsymbol{k}E - \boldsymbol{i}\mathbf{p}) & (i\boldsymbol{k}E + \boldsymbol{i}\mathbf{p}) = 0 \end{array}$$

The implication of this is that a spin 0 boson, defined in this way, cannot be massless. So Goldstone bosons cannot exist, and the Higgs boson must have a mass. The mass works out, additionally, as will become apparent, as a measure of the degree of right-handedness in the fermion component and left-handedness in the antifermion component. In this context, we could say that a massless spin 0 boson must be made up of a left-handed fermion (weak-allowed) and left-handed antifermion (weak-prohibited), $(\pm\, i\boldsymbol{k}E \pm \boldsymbol{i}\mathbf{p})\ (\mp\, i\boldsymbol{k}E \mp \boldsymbol{i}\mathbf{p})$, or a right-handed fermion (weak-prohibited) and right-handed antifermion (weak-allowed), $(\pm\, i\boldsymbol{k}E \mp \boldsymbol{i}\mathbf{p})\ (\mp\, i\boldsymbol{k}E \pm \boldsymbol{i}\mathbf{p})$. To overcome the prohibited conditions, we must generate mass. Before the discovery of the Higgs boson, it was a matter of guesswork what its mass might be. On the basis of these structures, my own *final* guess was that the two weak-prohibited structures might each require half of the Higgs field, leading to a mass of 123 GeV. At 126 GeV, it looks closer to half of the mass of the boson structures ($W^+$, $W^-$, $Z^0$) which carry the weak interaction, and are directly generated from the field.

A third type of boson-like state can be formed by combining two fermions with opposite spins and opposite helicities:

$$
\begin{array}{ll}
(i\mathbf{k}E + \mathbf{i}\mathbf{p} + \mathbf{j}m) & (i\mathbf{k}E - \mathbf{i}\mathbf{p} + \mathbf{j}m) \\
(i\mathbf{k}E - \mathbf{i}\mathbf{p} + \mathbf{j}m) & (i\mathbf{k}E + \mathbf{i}\mathbf{p} + \mathbf{j}m) \\
(-i\mathbf{k}E + \mathbf{i}\mathbf{p} + \mathbf{j}m) & (-i\mathbf{k}E - \mathbf{i}\mathbf{p} + \mathbf{j}m) \\
(-i\mathbf{k}E - \mathbf{i}\mathbf{p} + \mathbf{j}m) & (-i\mathbf{k}E + \mathbf{i}\mathbf{p} + \mathbf{j}m)
\end{array}
$$

Again the product is a pure scalar. States of this kind can be imagined to occur in Cooper pairing in superconductors, in $He^4$ and Bose-Einstein condensates, in spin 0 nuclei, in the Jahn-Teller effect, the Aharonov-Bohm effect, the quantum Hall effect, and, in general, in states where there is a nonzero Berry phase to make fermions become single-valued in terms of spin.

In general, these will be spin 0 states, but a spin 1 fermion-fermion combination is known in the case of $He^3$. Here, the two components move with respect to each other with components of motion in opposite directions, presumably in some kind of harmonic oscillator fashion, meaning that they could have the same spin states but opposite helicities.

In the case of the fermion-fermion spin 0 state, we can make a prediction which can be tested by experiment. Because the formation of this state necessarily requires intrinsically massive components, even in those cases where it assumes nonzero effective mass through a Fermi velocity less than $c$, time reversal symmetry (the one applicable to the transition) must be broken in the weak formation or decay of such states. The most likely opportunity of observing such a process might be in one of the physical manifestations of the nonzero Berry phase, say the quantum Hall effect, in some special type of condensed matter such as graphene. Here, the conduction electrons have zero effective mass and a Hamiltonian that can be written in the form $\pm v_F\mathbf{i}(\mathbf{i}p_x + \mathbf{j}p_y)$, where $v_F$ is the Fermi velocity. We can imagine creating a boson-like state with single-valued spin by the quantum Hall effect, Aharonov-Bohm effect, or Bose-Einstein condensation, and then observing, perhaps through a change in the Fermi velocity during its decay, the violation of both $P$ and $CP = T$ symmetries.

Bosons, we have said, cannot be seen as independent of fermions. If a fermion could combine with its own vacuum, it would annihilate automatically, but this is, of course, impossible because this vacuum represents the entire universe outside of the fermion; however, we can imagine it combining with a *component* of vacuum, or the fermion state equivalent to this. The various boson states may be seen as simultaneous realisations of the two vector spaces involved in the creation of the fermionic state, though at the expense of making only one component of the vacuum space well defined, just as only one component of angular momentum is well defined in real space.

Spin 1 bosons are involved in all local interactions, but they occur in a particular way in weak interactions. Here, fermions and antifermions are annihilated while bosons are created, or bosons are annihilated while fermions and antifermions are created, and, frequently, both processes (or equivalent) occur. We recognise the creation and annihilation of states as the action of a harmonic oscillator. It can also be thought of as the creation and annihilation of the weak source (or weak 'charge'). A very important difference between fermions and bosons is that fermions are sources for weak interactions, while bosons are not. The $W$ and $Z$ bosons are carriers of the weak force, but are not sources of it. Bosons, considered as created at fermion-antifermion vertices, are the products of weak interactions. Even in examples such as electron-positron collisions, where the predominant interaction is electric at low energies, there is an amplitude for a weak interaction.

**Baryons**

We have postulated that fermions are created as singularities through a dual space structures. Baryons complicate this structure by introducing an explicit 3-dimensionality into the real space part of the structure. They are an expression of the fact that singularities are incompatible with a pure 3-dimensional space and are only possible where we have a dual space. The strong interaction begins in a combination state, which reflects the *vector* nature of the $\mathbf{p}$ term in the nilpotent wavefunction. Essentially, the 'quarks' in a baryon are like components of a vector; they can no more be separated than the directions in space. Effectively, the vector aspect of the strong charge requires a source term and corresponding vacuum with three components. Though we clearly cannot combine three components in the form:

$$(i\boldsymbol{k}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\ (i\boldsymbol{k}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\ (i\boldsymbol{k}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$$

as this will automatically reduce to zero, we can imagine a three-component structure in which the vector nature of $\mathbf{p}$ plays an explicit role

$$(i\boldsymbol{k}E \pm \boldsymbol{i}\ \mathbf{i}p_x + \boldsymbol{j}m)\ (i\boldsymbol{k}E \pm \boldsymbol{i}\ \mathbf{j}p_y + \boldsymbol{j}m)\ (i\boldsymbol{k}E \pm \boldsymbol{i}\ \mathbf{k}p_z + \boldsymbol{j}m)$$

This has nilpotent solutions when $\mathbf{p} = \pm \boldsymbol{i}\ \mathbf{i}p_x$, $\mathbf{p} = \pm \boldsymbol{i}\ \mathbf{j}p_y$, or $\mathbf{p} = \pm \boldsymbol{i}\ \mathbf{k}p_z$, or when the momentum is directed entirely along the $x$, $y$, or $z$ axes, in either direction, though these, of course, are arbitrarily defined. For convenience, we have written only the first term of the 4-component spinors, but we have retained the two spin states, as these will be needed explicitly.

The complete wavefunction will, in effect, contain information from the equivalent of six allowed independent nonlocally gauge invariant phases, all existing simultaneously and subject to continual transitions at a constant rate:

| | |
|---|---|
| $(i\mathbf{k}E + i\,\mathbf{i}p_x + jm)\ (i\mathbf{k}E + \ \ldots\ + jm)\ \ (i\mathbf{k}E + \ \ldots\ + jm)$ | $+RGB$ |
| $(i\mathbf{k}E - i\,\mathbf{i}p_x + jm)\ (i\mathbf{k}E - \ \ldots\ + jm)\ (i\mathbf{k}E - \ \ldots\ + jm)$ | $-RBG$ |
| $(i\mathbf{k}E + \ \ldots\ + jm)\ (i\mathbf{k}E + i\,\mathbf{j}p_y + jm)\ (i\mathbf{k}E + \ \ldots\ + jm)$ | $+BRG$ |
| $(i\mathbf{k}E - \ \ldots\ + jm)\ (i\mathbf{k}E - i\,\mathbf{j}p_y + jm)\ (i\mathbf{k}E - \ \ldots\ + jm)$ | $-GRB$ |
| $(i\mathbf{k}E + \ \ldots\ + jm)\ (i\mathbf{k}E + \ \ldots\ + jm)\ \ (i\mathbf{k}E + i\,\mathbf{k}p_z + jm)$ | $+GBR$ |
| $(i\mathbf{k}E - \ \ldots\ + jm)\ (i\mathbf{k}E - \ \ldots\ + jm)\ (i\mathbf{k}E - i\,\mathbf{k}p_z + jm)$ | $-BGR$ |

Any other phases can be written as a superposition of these. Using the appropriate normalization, these reduce to

| | |
|---|---|
| $(i\mathbf{k}E + i\,\mathbf{i}p_x + jm)$ | $+RGB$ |
| $(i\mathbf{k}E - i\,\mathbf{i}p_x + jm)$ | $-RBG$ |
| $(i\mathbf{k}E - i\,\mathbf{j}p_y + jm)$ | $+BRG$ |
| $(i\mathbf{k}E + i\,\mathbf{j}p_y + jm)$ | $-GRB$ |
| $(i\mathbf{k}E + i\,\mathbf{k}p_z + jm)$ | $+GBR$ |
| $(i\mathbf{k}E - i\,\mathbf{k}p_z + jm)$ | $-BGR$ |

with the third and fourth changing, very significantly, the sign of the **p** component. Because of this, there has to be a maximal superposition of left- and right-handed components, thus explaining the zero observed chirality in the interaction.

The group structure required to maintain these phases is an $SU(3)$ structure, with eight generators and a wavefunction, exactly as in the conventional model using coloured quarks,

$$\psi \sim (BGR - BRG + GRB - GBR + RBG - RGB).$$

'Colour' transitions in the 3-component structures are produced either by an exchange of the components of **p** between the individual quarks or baryon components, or by a relative switching of the component positions, independently of any real distance between the components. No direction can be privileged, so the transition must be gauge invariant, and the mediators must be massless, exactly as with the eight massless gluons of the gluon structure. Here, six gluons can be constructed from:

$(i\mathbf{k}E + \mathbf{i}\,\mathbf{i}p_x)\,(-i\mathbf{k}E + \mathbf{i}\,\mathbf{j}p_y) \quad (i\mathbf{k}E + \mathbf{i}\,\mathbf{j}p_y)\,(-i\mathbf{k}E + \mathbf{i}\,\mathbf{i}p_x)$
$(i\mathbf{k}E + \mathbf{i}\,\mathbf{j}p_y)\,(-i\mathbf{k}E + \mathbf{i}\,\mathbf{k}p_z) \quad (i\mathbf{k}E + \mathbf{i}\,\mathbf{k}p_z)\,(-i\mathbf{k}E + \mathbf{i}\,\mathbf{j}p_y)$
$(i\mathbf{k}E + \mathbf{i}\,\mathbf{k}p_z)\,(-i\mathbf{k}E + \mathbf{i}\,\mathbf{i}p_x) \quad (i\mathbf{k}E + \mathbf{i}\,\mathbf{i}p_x)\,(-i\mathbf{k}E + \mathbf{i}\,\mathbf{k}p_z)$

and two from combinations of

$(i\mathbf{k}E + \mathbf{i}\,\mathbf{i}p_x)\,(-i\mathbf{k}E + \mathbf{i}\,\mathbf{i}p_x) \quad (i\mathbf{k}E + \mathbf{i}\,\mathbf{j}p_y)\,(-i\mathbf{k}E + \mathbf{i}\,\mathbf{j}p_y)$
$(i\mathbf{k}E + \mathbf{i}\,\mathbf{k}p_z)\,(-i\mathbf{k}E + \mathbf{i}\,\mathbf{k}p_z)$

Alternatively, we can switch the signs, producing an equivalent set:

$(i\mathbf{k}E - \mathbf{i}\,\mathbf{i}p_x)\,(-i\mathbf{k}E - \mathbf{i}\,\mathbf{j}p_y) \quad (i\mathbf{k}E - \mathbf{i}\,\mathbf{j}p_y)\,(-i\mathbf{k}E - \mathbf{i}\,\mathbf{i}p_x)$
$(i\mathbf{k}E - \mathbf{i}\,\mathbf{j}p_y)\,(-i\mathbf{k}E - \mathbf{i}\,\mathbf{k}p_z) \quad (i\mathbf{k}E - \mathbf{i}\,\mathbf{k}p_z)\,(-i\mathbf{k}E - \mathbf{i}\,\mathbf{j}p_y)$
$(i\mathbf{k}E - \mathbf{i}\,\mathbf{k}p_z)\,(-i\mathbf{k}E - \mathbf{i}\,\mathbf{i}p_x) \quad (i\mathbf{k}E - \mathbf{i}\,\mathbf{i}p_x)\,(-i\mathbf{k}E - \mathbf{i}\,\mathbf{k}p_z)$

with two combinations from

$(i\mathbf{k}E - \mathbf{i}\,\mathbf{i}p_x)\,(-i\mathbf{k}E - \mathbf{i}\,\mathbf{i}p_x) \quad (i\mathbf{k}E - \mathbf{i}\,\mathbf{j}p_y)\,(-i\mathbf{k}E - \mathbf{i}\,\mathbf{j}p_y)$
$(i\mathbf{k}E - \mathbf{i}\,\mathbf{k}p_z)\,(-i\mathbf{k}E - \mathbf{i}\,\mathbf{k}p_z)$

where, as with the baryons, only the lead term is shown for each 4-component spinor.

A representation such as the 3-component baryon above, showing only one 'quark' active at any time in contributing to the angular momentum operator, seems to indicates why only 1/3 of baryon spin has been found to be due to the valence quarks. The rest of the spin then becomes a 'vacuum' contribution, split approximately 3 to 1 in favour of the gluons over the sea quarks, the gluons thus taking half the overall total. The simultaneous existence of all phases further means that *individual* quarks, and such identifying characteristics as electric charges, are not identifiable by their spatial positions (unlike, say, the proton and electron constituting a hydrogen atom), thus explaining, for example, why the neutron has no electric dipole moment. As we established in lecture 4, just as $U(1)$ establishes that spherical symmetry of a point source requires the rotation to be performed independently of the length of the radius vector, so $SU(3)$ requires the rotation to be independent of the coordinate system used. In terms of Noether's theorem, while $U(1)$ conserves the magnitude of angular momentum, $SU(3)$ conserves the direction.

The structures derived here produce insights into at least two fundamental physical problems. The first is the mass-gap problem for baryons, which is one of the Clay Institute's Millennium Prize Problems. We are confronted with the fact that baryons have nonzero mass and yet this mass is thought to be produced by the action of massless gluons. And, although the Higgs mechanism appears to be the main process by which mass is delivered to fermions, the gluon exchange is generally considered to be a non-Higgs process. In fact, the 3-component structures clearly require the simultaneous existence of two states of helicity for the symmetry to remain unbroken (because the placing of the middle component ensures that there is a sign switch in $\mathbf{p}$), and this can only be possible if the baryon has nonzero mass.

In addition, this process is the signature of the Higgs mechanism, and so, contrary to much current supposition, the generation of the masses of baryons follows exactly the same process as that of all other fermions. However, this does not contradict the fact, established by much calculation using QCD, that the bulk of the mass of a baryon is due to the exchange of massless gluons, as the exchange of gluons structured as above will necessarily lead to a sign change in the $\mathbf{p}$ operator, and hence of helicity, the exact mechanism which is responsible for the production of all known particle masses. In fact, the same will be true of all fermions involved in spin 1 boson exchange, and so, once again, all fermions must have nonzero masses.

The second problem is the specific nature and mechanism of the strong interaction between quarks. Again, we see that a solution is suggested by the exact structure of the nilpotent operator. Here, we already know that there must be a Coulomb component or inverse linear potential ($\propto 1 / r$), just to accommodate spherical symmetry. This has a known physical manifestation in the one-gluon exchange. But there is also at least one other component, which is responsible for quark confinement, for infrared slavery and for asymptotic freedom, and a linear potential ($\propto r$) has long been hypothesized and used in calculations. Here, we see that an exchange of $\mathbf{p}$ components at a constant rate would, in principle, require a constant rate of change of momentum, which is the signature of a linear potential.

**Partitioning the vacuum**

The nilpotent formalism defines a continuous vacuum $-(\pm i\boldsymbol{k}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$ to each fermion state $(\pm i\boldsymbol{k}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$, and this vacuum expresses the nonlocal aspect of the state. However, the use of the operators $\boldsymbol{k}$, $\boldsymbol{i}$, $\boldsymbol{j}$ suggests that we can partition this state into discrete components with a dimensional structure. In fact, this is where the idempotents become relevant. If we postmultiply $(\pm i\boldsymbol{k}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$ by the idempotent $\boldsymbol{k}(\pm i\boldsymbol{k}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$ any number of times, the only change is to introduce a scalar multiple, which can be normalized away.

$$(\pm i\boldsymbol{k}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\ \boldsymbol{k}(\pm i\boldsymbol{k}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)\ \boldsymbol{k}(\pm i\boldsymbol{k}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m) \ldots \rightarrow (\pm i\boldsymbol{k}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$$

The idempotent acts as a vacuum operator. The same applies to postmultiplication by $\boldsymbol{i}(\pm i\boldsymbol{k}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$ or $\boldsymbol{j}(\pm i\boldsymbol{k}E \pm \boldsymbol{i}\mathbf{p} + \boldsymbol{j}m)$, except that the latter also produces a unit vector which disappears on every alternate postmultiplication and has no effect on the nilpotent $(\pm ikE \pm i\mathbf{p} + jm)$. Now, the multiplication by $k(\pm ikE \pm i\mathbf{p} + jm)$ is also equivalent to applying a time-reversal transformation to every even $(\pm ikE \pm i\mathbf{p} + jm)$. So we have

$$(\pm ikE \pm i\mathbf{p} + jm)\,(\mp ikE \pm i\mathbf{p} + jm)\,(\pm ikE \pm i\mathbf{p} + jm)\,\ldots \rightarrow (\pm ikE \pm i\mathbf{p} + jm)$$

with every alternate state becoming an antifermion, which combines with the original fermion state to become a spin 1 boson $(\pm ikE \pm i\mathbf{p} + jm)\,(\mp ikE \pm i\mathbf{p} + jm)$.

Similar results are obtained with $i(\pm ikE \pm i\mathbf{p} + jm)$ or $j(\pm ikE \pm i\mathbf{p} + jm)$, which produce spin 0 bosons and bosonic paired fermions, via a parity transformation and a charge conjugation. It looks like that, from an initial fermion state, we can generate either three vacuum reflections, via respective *T*, *P* and *C* transformations, representing antifermion with the same spin, fermion with opposite spin, and antifermion with opposite spin, or combined particle-vacuum states which have the respective structures of spin 1 bosons, spin 0 bosons, or boson-like paired fermion (PF) combinations of the same kind as constitute Cooper pairs and the elements of Bose-Einstein condensates. Using just the lead terms of the nilpotents, and assuming that we can complete the spinor structures using the 3 conventional sign variations, we could represent these as:

| | |
|---|---|
| $(ikE + i\mathbf{p} + jm)\, k\, (ikE + i\mathbf{p} + jm)\, k\, (ikE + i\mathbf{p} + jm)\, k\, (ikE + i\mathbf{p} + jm)\ \ldots$ | *T* |
| $(ikE + i\mathbf{p} + jm)\,(-ikE + i\mathbf{p} + jm)\,(ikE + i\mathbf{p} + jm)\,(-ikE + i\mathbf{p} + jm)\ \ldots$ | spin 1 |
| $(ikE + i\mathbf{p} + jm)\, j\, (ikE + i\mathbf{p} + jm)\, j\, (ikE + i\mathbf{p} + jm)\, j\, (ikE + i\mathbf{p} + jm)\ \ldots$ | *P* |
| $(ikE + i\mathbf{p} + jm)\,(-ikE - i\mathbf{p} + jm)\,(ikE + i\mathbf{p} + jm)\,(-ikE - i\mathbf{p} + jm)\ \ldots$ | spin 0 |
| $(ikE + i\mathbf{p} + jm)\, i\, (ikE + i\mathbf{p} + jm)\, i\, (ikE + i\mathbf{p} + jm)\, i\, (ikE + i\mathbf{p} + jm)\ \ldots$ | *C* |
| $(ikE + i\mathbf{p} + jm)\, i\, (ikE - i\mathbf{p} + jm)\, i\, (ikE + i\mathbf{p} + jm)\, i\, (ikE - i\mathbf{p} + jm)\ \ldots$ | PF |

These processes indicate that repeated post-multiplication of a fermion operator by any of the discrete idempotent vacuum operators creates an alternate series of antifermion and fermion vacuum states, or, equivalently, an alternate series of boson and fermion states without changing the character of the real particle state. A fermion produces a boson state by combining with its own vacuum image, and the two states form a *supersymmetric* partnership. Nilpotent operators are thus intrinsically supersymmetric, with supersymmetry operators typically of the form:

| | |
|---|---|
| Boson to fermion: | $Q = (\pm ikE \pm i\mathbf{p} + jm)$ |
| Fermion to boson: | $Q^{\dagger} = (\mp ikE \pm i\mathbf{p} + jm)$ |

A fermion converts to a boson by multiplication by an antifermionic operator $Q^\dagger$; a boson converts to a fermion by multiplication by a fermionic operator $Q$, and we can represent the sequence $(ikE + i\mathbf{p} + jm)\ k\ (ikE + i\mathbf{p} + jm)$ ... by the supersymmetric

$$Q\, Q^\dagger\, Q\, Q^\dagger\, Q\, Q^\dagger\, Q\, Q^\dagger\, Q \ldots$$

We will see in the next lecture that we can interpret this as the series of boson and fermion loops, of the same energy and momentum, required by the exact supersymmetry which would eliminate the need for renormalization, and remove the hierarchy problem altogether. Fermions and bosons (with the same values $E$, $\mathbf{p}$ and $m$) then become their own supersymmetric partners through the creation of vacuum states, making the hypothesis of a set of real supersymmetric particles to solve the hierarchy problem potentially superfluous.

The identification of $i(ikE + i\mathbf{p} + jm)$, $k(ikE + i\mathbf{p} + jm)$ and $j(ikE + i\mathbf{p} + jm)$ as vacuum operators and $(ikE - i\mathbf{p} + jm)$, $(-ikE + i\mathbf{p} + jm)$ and $(-ikE - i\mathbf{p} + jm)$ as their respective vacuum ‘reflections’ at interfaces provided by $P$, $T$ and $C$ transformations suggests a new insight into the meaning of the Dirac 4-spinor. We can now interpret the three terms other than the lead term *in the spinor* as the vacuum ‘reflections’ that are created with the particle. We can regard the existence of three vacuum operators as a result of a partitioning of the vacuum as a result of quantization and as a consequence of the 3-part structure observed in the nilpotent fermionic state, while the *zitterbewegung* can be taken as an indication that the vacuum is active in defining the fermionic state.

The four components of the spinor cancel exactly when the operator or amplitude is written in nilpotent mode. This is even more apparent if the components are represented as operators using discrete calculus, when the cancellation is a zero algebraic sum. (In idempotent mode, the summed amplitudes would normalize to 1.) Though annihilation and creation operators are ‘black box’ devices in standard quantum field theory, here, like the wavefunctions, they have exact and explicit mathematical and physical representations. We can thus represent the four components of the nilpotent spinor as creation operators for

| | |
|---|---|
| fermion spin up | $(ikE + i\mathbf{p} + jm)$ |
| fermion spin down | $(ikE - i\mathbf{p} + jm)$ |
| antifermion spin down | $(-ikE + i\mathbf{p} + jm)$ |
| antifermion spin up | $(-ikE - i\mathbf{p} + jm)$ |

or annihilation operators for

| | |
|---|---|
| antifermion spin down | $(ikE + i\mathbf{p} + jm)$ |
| antifermion spin up | $(ikE - i\mathbf{p} + jm)$ |
| fermion spin up | $(-ikE + i\mathbf{p} + jm)$ |
| fermion spin down | $(-ikE - i\mathbf{p} + jm)$ |

They can equally well be regarded as two operators for creation and two for annihilation, for example:

| | |
|---|---|
| fermion spin up creation | $(ikE + i\mathbf{p} + jm)$ |
| fermion spin down creation | $(ikE - i\mathbf{p} + jm)$ |
| fermion spin up annihilation | $(-ikE + i\mathbf{p} + jm)$ |
| fermion spin down annihilation | $(-ikE - i\mathbf{p} + jm)$ |

In all cases, the cancellation is exact, both physically, and algebraically (when we use the discrete operators and leave out the passive mass component). It is interesting that the cancellation requires *four* components, rather than two, for, while the transitions:

$$(ikE + i\mathbf{p} + jm) \to (ikE - i\mathbf{p} + jm)$$

and

$$(ikE + i\mathbf{p} + jm) \to (-ikE + i\mathbf{p} + jm)$$

can occur through spin 1 boson and spin 0 paired fermion exchange, and the active space and time components, there is no process in nature for the *direct* transition:

$$(ikE + i\mathbf{p} + jm) \to (-ikE + i\mathbf{p} + jm)$$

with no active component as agent. In this context, it might be worth noting that the spin 0 fermion-fermion state

$$(ikE + i\mathbf{p} + jm)\ (ikE - i\mathbf{p} + jm)$$

is such as would be required in a pure weak transition from $-ikE$ to $+\ ikE$, or its inverse.

A special form of the generation of bosons by the process of fermion annihilation and creation occurs with *zitterbewegung*, where there is a continually switching between the four fermionic states, leading to the annihilation of one state and the creation of another, so fulfilling the conditions for a weak interaction. This is, of course, a vacuum process, and the ‘supersymmetric’ bosons it creates can be thought of as the result of every fermion interacting weakly with its own vacuum reflection. In a sense, every weak source (or fermion) acts as a weak dipole, the second ‘pole’ being the vacuum reflection, which, in a gauge invariant system, exists simultaneously with the real ‘particle’ state. Of the two allowed ‘direct’ transitions, the one that switches between a fermion in real space and an antifermion in vacuum space, with the opposite helicity (a result of the pseudoscalar nature of the coefficient of $i\mathbf{k}E$), requires a spin 0 bosonic state, and so is a natural mass generator. (The other direct transition is between spin up and spin down, which both exist in the real state of a fermion with nonzero mass.)

The weak interaction is clearly related to the nature of the pseudoscalar $iE$ operator, whose sign uniquely determines the helicity of a weakly interacting particle, or more specifically its weakly interacting component. It also has a unique feature, in that its fermionic source cannot be separated from its vacuum partner. A fermion or antifermion cannot be created or annihilated, even with an antifermionic or fermionic partner, unless its vacuum is simultaneously annihilated or created. In this sense, the weak source has a manifestly dipolar nature, whose immediate manifestation is the fermion's ½-integral spin. It is the most direct evidence we have of the duality of the vector space structure which underlies quantum physics.

Now, we know that the three vacuum coefficients $\mathbf{k}$, $\mathbf{i}$, $\mathbf{j}$ originate in (or are responsible for) the concept of discrete (point-like) charge. However, the operators, $\mathbf{k}$, $\mathbf{i}$ and $\mathbf{j}$, as we are using them here, perform another function of weak, strong and electric 'charges' or sources, in acting to partition the *continuous* vacuum represented by $-(i\mathbf{k}E + \mathbf{i}\mathbf{p} + \mathbf{j}m)$, and responsible for zero-point energy, into discrete components, whose special characteristics are determined by the respective pseudoscalar, vector and scalar natures of their associated terms $iE$, $\mathbf{p}$ and $m$.

In this way, they become are related to the 'real' weak, strong and electric localized charges, though they are delocalized. We can describe the partitions as strong, weak and electric 'vacua', and assign to them particular roles within existing physics:

| | | |
|---|---|---|
| $\mathbf{k}(i\mathbf{k}E + \mathbf{i}\mathbf{p} + \mathbf{j}m)$ | weak vacuum | fermion creation |
| $\mathbf{i}(i\mathbf{k}E + \mathbf{i}\mathbf{p} + \mathbf{j}m)$ | strong vacuum | gluon plasma |
| $\mathbf{j}(i\mathbf{k}E + \mathbf{i}\mathbf{p} + \mathbf{j}m)$ | electric vacuum | isospin / hypercharge |

These three vacua retain the characteristics of the generating charge structures, respectively pseudoscalar, vector and scalar, which explain also the special characteristics and group structures of the forces with which they are associated. It is the vector characteristic of the strong vacuum that makes baryon structure possible, and it is the pseudoscalar characteristic of the weak vacuum that makes the link between particle structure and vacuum possible at all. The 'electric vacuum' – empty or filled – can be seen as responsible for the transition between weak isospin up and down states.

**Local and nonlocal**

Many people think that nonlocality is a problem for quantum mechanics, but, in fact, it is a necessary component of any physics whose operations are truly universal. It is also an essential step towards understanding how *local* interactions actually work. Locality and nonlocality are not opposed concepts. They are two aspects of the description of any process in nature. They are parts of a dual system in which each aspect determines the behaviour of the other. The reason for the duality is simple. If we are describing the behaviour of a point source, we can start by specifying what the point is, or by what it isn't, i.e. everything else. The first is the local description, the second the nonlocal. It also represents the duality at the heart of quantum physics: real space requires its dual in vacuum space.

The holistic nature of physics means that the local cannot, in fact, be separated from the nonlocal. Even the terms can be misleading, because 'local' refers to the entire universe as much as nonlocal does. The nilpotent version of quantum mechanics shows that there is no such thing as an isolated system, and so a complete local description, which originates in the individual particle, will still require knowledge of the contents and disposition of the whole universe. In principle, the difference between local and nonlocal is not in the phenomena they describe, but in the method of description, essentially whether we use an iterative or recursive computational paradigm. Local interactions are determined by the collective nonlocal effect of the rest of the universe.

When we write down an operator or amplitude in the form $(\pm ikE \pm i\mathbf{p} + jm)$, the brackets may suggest that we have created a closed system, but in fact the $E$ and $\mathbf{p}$ terms may contain an unlimited number of potentials. We have created a system but it is open. Closure or energy conservation is only maintained over the entire universe, and requires the second law of thermodynamics as well as the first. So, though the bracket may define locality, locality does not imply a closed system. The creation of the fermion state is the creation of a local region in phase space, to which everything else becomes nonlocal; the creation of the two regions is simultaneous. Any subsequent change inside the bracket (a rewriting of the structures of $E$ and $\mathbf{p}$) also affects everything else outside it, and vice versa.

We can now show that the exact characteristics of the different local interactions (electric, strong and weak) are completely determined by the nonlocal vacuum structures with which they are associated. Beginning with the nonlocal characteristics that emerge from the way that the fermionic state is structured as a nilpotent operator, we can derive the *exact form* of the potentials that would produce the same result by a *local* process. We can then find analytic nilpotent solutions in all these cases which provide exact matches to the results found for the electric, strong and weak interactions from experiment, and from their descriptions in terms of the *U*(1), *SU*(3) and *SU*(2) symmetry groups. We will also show that these solutions are unique. No others are permitted within the nilpotent algebra. In other words, we can present a completely integrated description of local interactions and nonlocal vacuum structures, based on nilpotent quantum mechanics and its unique algebraic structure. The complete set of structures depends entirely on the algebraic symmetry-breaking that emerges from the creation of point sources from two vector spaces, which are dual to each other. In effect, we propose to show in a completely analytical form that the two-space algebra leads to the entire basis of the interactions required by the Standard Model.

We have three interactions to consider, and we have three nonlocal aspects of the nilpotent wavefunction from which we believe they can be derived. By nonlocal, we mean any operation that occurs outside a nilpotent bracket. The two principal examples are superposition, where brackets are added, and combination, where brackets are multiplied. Nonlocal operations are instantaneous and occur across the entire universe. Local operations are Lorentz-invariant and limited by the maximum speed *c*. They are determined entirely by what happens inside the bracket. However, nonlocal operations have local manifestations and *vice versa*. Decoherence of a quantum system (the so-called 'collapse of the wavefunction') is a clear example of the reverse transformation, as it uses local potentials to remove the nonlocal quantum coherence. This is how we propose to solve the 'measurement problem'.

The first nonlocal operation is nilpotency itself, or Pauli exclusion, with each fermion instantaneously ensuring that it has a different phase and amplitude to any other. We have seen that this is the result of the creation of a fixed point source, with spherical symmetry, and that it demands a Coulomb term (potential energy $\propto 1 / r$) in the energy operator, and that it is required in all possible local interactions. It is determined solely by scale, as it is the scalar values of *E*, *p* and *m* that fix nilpotency, and so is related to the sources associated with ***k***, ***i*** and ***j***. The addition of the inverse linear potential to the *E* term changes the nonlocal *U*(1) (nilpotent or Pauli exclusion) condition to a local one, since the potential is added *inside* the bracket; and this becomes the pattern for all the interactions. The second is the combination state formed between three component wavefunctions, expressing the vector nature of the operator **p**, and we have seen that this requires an additional linear potential energy ($\propto r$) to add to the Coulomb term in its *local* formulation. Because of its origin in the **p** term it originates in the source associated with ***i***. The third is the superposition state created by the four terms in the fermion spinor, which manifests itself as two fermion terms in real space (with *i****k****E*) and two antifermion terms in vacuum space (with –*i****k****E*), which results from the fact that the energy term *iE* is a pseudoscalar. Locally, this requires the source to act as a dipole (at least – it may be more complicated) as well as monopole, which requires an additional potential energy term proportional to

$1 / r^2$, $1 / r^3$, or some other $1 / r^n$. This clearly originates in the source connected to ***k***. Of course, these interactions also have manifestations which extend beyond the individual fermion, but these will be of exactly the same character. In fact, all weak interactions operate according to the same mechanism, and the mediator is a spin 1 boson (± ***ik****E* ± ***i*p** + ***j****m*) ***k*** (± ***ik****E* ± ***i*p** + ***j****m*) ***k***, equivalent to the state produced by a fermion acting on a 'weak' vacuum.

It would appear, then, that the nonlocal characteristics also require local manifestations appearing in the *E* or **p** terms of the nilpotent wavefunction. Here, we can restrict them to *E* by choice of frame. Having made no prior assumption about the nature of these interactions, we can assign the name of 'strong' to the one whose source is associated with ***i*p**, and the name of 'weak' to the one whose source is associated with ***ik****E*, leaving the name 'electric' for the one whose source is associated with ***j****m*, and has a purely Coulomb manifestation. We will now proceed to find the nilpotent structure corresponding to these three cases.

**The Coulomb (electric) interaction**

We have already produced the nilpotent operator with the required Coulomb term, we will find that it can be solved, using the known procedures, but eliminating many unnecessary ones, in only six lines of calculation. This may seem surprising in view of the complexities of calculations already available, but calculations are notably easier and more efficient in the nilpotent structure than in alternative formalisms, largely because *dual* information, concerning both fermion and vacuum, is available, as is completely new *physical* information. So we begin with:

$$\left( \pm i\mathbf{k}\left( E - \frac{A}{r} \right) \mp \mathbf{ii}\left( \frac{\partial}{\partial r} + \frac{1}{r} \pm i\,\frac{j + ½}{r} \right) + \mathbf{j}m \right) .$$

The basic requirement is to find the phase factor $\phi$ which will make the amplitude nilpotent. So, we try the standard solution:

$$\phi = e^{-ar} r^{\gamma} \sum_{\nu = 0} a_\nu r^\nu .$$

We then apply the operator we have defined to $\phi$, and square the result to 0 to obtain:

$$4\left( E - \frac{A}{r} \right)^2 = -2\left( -a + \frac{\gamma}{r} + \frac{\nu}{r} + \ldots \frac{1}{r} + i\,\frac{j + ½}{r} \right)^2 - 2\left( -a + \frac{\gamma}{r} + \frac{\nu}{r} + \ldots \frac{1}{r} - i\,\frac{j + ½}{r} \right)^2 + 4m^2 .$$

Equating constant terms leads to

$$a = \sqrt{m^2 - E^2} .$$

Equating terms in $1/r^2$, following standard procedure, with $\nu = 0$, we obtain:

$$\left(\frac{A}{r}\right)^2 = -\left(\frac{\gamma+1}{r}\right)^2 + \left(\frac{j+½}{r}\right)^2 .$$

Assuming the power series terminates at $n'$, following another standard procedure, and equating coefficients of $1/r$ for $\nu = n'$,

$$2EA = -2\sqrt{m^2 - E^2}\,(\gamma + 1 + n'),$$

the terms in ($j$ + ½) cancelling over the summation of the four multiplications, with two positive and two negative. Algebraic rearrangement of the last three equations then yields

$$\frac{E}{m} = \frac{1}{\sqrt{1 + \frac{A^2}{(\gamma+1+n')^2}}} = \frac{1}{\sqrt{1 + \frac{A^2}{\left(\sqrt{(j+½)^2 - A^2} + n'\right)^2}}} ,$$

With $A = Ze^2$, this is recognisable as the hyperfine or fine structure formula for a one-electron nuclear atom or ion (e.g. the hydrogen atom, where $Z = 1$).

**The strong interaction**

The strong interaction has never been explained analytically, though there is a general understanding, on empirical grounds, that it involves a linear potential. We have represented it as a nonlocal interaction between the parts of the wavefunction or 'quarks' incorporating $p_x$, $p_y$, and $p_z$. This interaction is independent of the physical separation of the components (and they must be spatially or temporally separated to have different local and nonlocal manifestations). The local effect of this is a transfer of **p** between the three brackets of each wavefunction. As it is local, with **p** changing within each bracket, this is not instantaneous, unlike the actual combinations and superpositions, and so there will be a nonzero local force or rate of change of momentum. However, because it does not depend on separation, this force will be constant. The local manifestation of a constant force is a potential that varies linearly with distance. In principle, it will be the same whether we are referring to a quark-antiquark combination, with a temporal cycle of the components $p_x$, $p_y$, and $p_z$, and a separation of quark and antiquark, or to a combination of three quarks, with a separation of each from the centre of the system. Using this argument based on fundamental principles, we create a differential operator incorporating Coulomb and linear potentials from a source with spherical symmetry, which is, in physical terms, either the centre of a 3-quark system or one component of a quark-antiquark pairing.

$$\left(\pm \boldsymbol{k}\left(E + \frac{A}{r} + Br\right) \mp \boldsymbol{i}\left(\frac{\partial}{\partial r} + \frac{1}{r} \pm i\frac{j + ½}{r}\right) + \boldsymbol{ij}m\right).$$

The question is: can it be solved analytically using nilpotent methods? Again, we need to identify the phase factor to which the operator applies, to yield nilpotent solutions. The pure Coulomb interaction gives us the template for solving more complicated systems, so, by analogy with this, we might propose that the phase factor is of the form:

$$\phi = \exp\left(-ar - br^2\right) r^{\gamma} \sum_{\nu=0} a_{\nu} r^{\nu} \ .$$

Applying the operator to this, and then the nilpotent condition, we obtain:

$$E^2 + 2AB + \frac{A^2}{r^2} + B^2 r^2 + \frac{2AE}{r} + 2BEr = m^2$$
$$-\left(a^2 + \frac{(\gamma + \nu + ... + 1)^2}{r^2} - \frac{(j + ½)^2}{r^2} + 4b^2 r^2 + 4abr - 4b(\gamma + \nu + ... + 1) - \frac{2a}{r}(\gamma + \nu + ... + 1)\right)$$

with the positive and negative *i*(*j* + ½) terms cancelling out over the four solutions, as previously. Then, assuming a termination in the power series (as with the Coulomb solution), we can equate:

| | |
|---|---|
| coefficients of $r^2$ to give | $B^2 = -4b^2$ |
| coefficients of $r$ to give | $2BE = -4ab$ |
| coefficients of $1 / r$ to give | $2AE = 2a(\gamma + \nu + 1)$ |

These equations immediately lead to:

$$b = \pm \frac{iB}{2}$$
$$a = \mp iE$$
$$\gamma + \nu + 1 = \mp iA \ .$$

The ground state case (where $\nu = 0$) then requires a phase factor of the form:

$$\phi = \exp\left(\pm iEr \mp iBr^2 / 2\right) r^{\mp iA - 1} \ .$$

The imaginary exponential terms in $\phi$ can be seen as representing asymptotic freedom, the exp ($\mp$ *iEr*) being typical for a free fermion. The complex $r^{\gamma}$ term can be structured as a component phase, $\chi(r)$ = exp ($\pm$ *iA* ln ($r$)), which varies less rapidly with $r$ than the rest of $\phi$. We can therefore write $\phi$ as

$$\phi = \frac{\exp(kr + \chi(r))}{r},$$

where $$k = \pm iE \mp iBr / 2 \ .$$

The first term in $k$ dominates at high energies, where $r$ is small, approximating to a free fermion solution, which can be interpreted as asymptotic freedom, while the second term, with its confining potential $Br$, dominates, at low energies, when $r$ is large, and this can be interpreted as infrared slavery. These are the established characteristics of the strong interaction and it seems that we have, for the first time, an explanation, derived on an analytic basis, for a force with these characteristics. The Coulomb term, which is required to maintain spherical symmetry, is the component which defines the strong interaction phase, $\chi(r)$, and this can be related to the directional status of **p** in the state vector. Once again, a nonlocal symmetry, related to the nilpotent structure, determines the known characteristics of a local interaction.

**The weak interaction**

There does not seem to be any coherent idea of how the weak interaction should be expressed in terms of a distance-related potential, although its symmetry is known to be that of the $SU(2)$ group. We, however, have presented evidence for a dipole component, which, in more complicated situations, might be expected to become multipole. This time we will approach the problem more generally to show that the nilpotent solutions are exclusive. We will begin with a nilpotent operator with a form such as

$$\left( \boldsymbol{k}\left( E - \frac{A}{r} - Cr^n \right) + \boldsymbol{i}\left( \frac{\partial}{\partial r} + \frac{1}{r} \pm i\frac{j+½}{r} \right) + \boldsymbol{ij}m \right) .$$

where $n$ is an integer greater than 1 or less than –1 (so including the dipole-monopole and dipole-dipole cases of $n = –2$ and $n = –3$). As usual, we look for a phase factor which will make the amplitude nilpotent. Again, we turn to the Coulomb solution, with the additional information that polynomial potential terms which are multiples of $r^n$ require the incorporation into the exponential of terms which are multiples of $r^{n+1}$. So, extending our work on the Coulomb solution, we may suppose that the phase factor is of the form:

$$\phi = \exp\left(-ar - br^{n+1}\right) r^{\gamma} \sum_{\nu=0} a_\nu r^\nu$$

Applying the operator and squaring to zero, with a termination in the series, we obtain

$$4\left( E - \frac{A}{r} - Cr^n \right)^2 = -2\left( -a + (n+1)br^n + \frac{\gamma}{r} + \frac{\nu}{r} + \frac{1}{r} + i\frac{j+½}{r} \right)^2$$
$$-2\left( -a + (n+1)br^n + \frac{\gamma}{r} + \frac{\nu}{r} + \frac{1}{r} - i\frac{j+½}{r} \right)^2$$

Equating constant terms, we find

$$a = \sqrt{m^2 - E^2}$$

Equating terms in $r^{2n}$, with $\nu = 0$:

$$C^2 = -(n+1)^2 b^2$$

$$b = \pm \frac{iC}{(n+1)} \ .$$

Equating coefficients of $r$, where $\nu = 0$:

$$AC = -(n+1)\, b\, (1+\gamma) \ ,$$

$$(1+\gamma) = \pm\, iA \ .$$

Equating coefficients of $1 / r^2$ and coefficients of $1 / r$, for a power series terminating in $\nu = n'$, we obtain

$$A^2 = -(1+\gamma+n')^2 + (j+½)^2$$

and

$$-EA = a\,(1+\gamma+n').$$

Combining the last three equations produces:

$$\left(\frac{m^2 - E^2}{E^2}\right)(1+\gamma+n')^2 = -(1+\gamma+n')^2 + (j+½)^2$$

$$E = -\frac{m}{j+½}\left(\pm\, iA + n'\right).$$

This equation has the form of a harmonic oscillator, with evenly spaced energy levels deriving from integral values of $n'$. Though it does not immediately suggest the value for the term $iA$, we can make the additional assumption, based on our interpretation of *zitterbewegung*, that $A$, the phase term required for spherical symmetry, has some connection with the random directionality of the fermion spin. We, therefore, assign to it a half-unit value (± ½ $i$), or (± ½ $i\hbar c$), using explicit values for the constants, and obtain the complete formula for the fermionic simple harmonic oscillator:

$$E = -\frac{m}{j+½}\left(½ + n'\right).$$

The dimensions of $A$ are those of charge ($q$) squared or interaction energy × range, and an $A$ numerically equal to ± ½ $\hbar c$ would be exactly that required by the uncertainty principle, allowing the value of the range of an interaction mediated by the $Z$ boson to be calculated as $\hbar \,/\, 2M_Z c = 2.166 \times 10^{-18}$ m, as observed. The ½ $\hbar c$ term is also significant in the expressions for zero-point energy and, as we have indicated, *zitterbewegung*, which connect with both spin and the uncertainty principle. Interpreting the *zitterbewegung* as a dipolar switching between fermion and vacuum antifermion states, we can describe this in terms of a weak dipole moment $(\hbar c / 2)^{3/2} / M_Z c^2$, of magnitude $8.965 \times 10^{-18}$ $e$ m ($1.44 \times 10^{-36}$ Cm). Because of the specific appearance of the ½ $\hbar c$ term for spin ($s$) in $\mu = gqs / 2m$, an identical expression can additionally be used to define a weak magnetic moment, of order $4.64 \times 10^{-5} \times$ the magnetic moment of the electron. The existence of such a dipole moment would make the spin ½ term an expression of the dipolarity of the weak vacuum, and a physical representation of the weak interaction as a link between the fermion and vacuum, or between real space and vacuum space. The possible appearance of an imaginary factor $i$ in $A$ is interesting in relation to the requirement of a complex potential or vacuum for $CP$ violation in the pure weak interaction.

The value of $A$ we have assumed seems to be the one required to link a number of different physical manifestations of the weak interaction as related to fermionic structure. But, in any case, the solution we have derived indicates that an additional potential of the form $Cr^n$, where $n$ is an integer greater than 1 or less than –1, has the effect of creating a harmonic oscillator solution for the nilpotent operator, irrespective of the value of $n$. In fact, we can show that any polynomial sum of potentials of this form will produce the same result, and consequently virtually any function of $r$ other than the pure Coulomb of inverse linear, and the combination of inverse and direct linear. Potentials of this kind will emerge from any system in which there is complexity, aggregation, or a multiplicity of sources, even if the individual sources have Coulomb or linear potentials. In the case of dipolar weak sources, the minimum additional term will be of the form $Cr^{-3}$, and so will provide the correct characteristics for the weak interaction from the kind of potential that weak sources must necessarily produce. In addition, because this solution is exclusive for distance related potentials of the form $Cr^n$, except where $r = 1$ or –1, we have also, in effect, shown that a fermion interaction specified in relation to a spherically symmetric point source has only three physical manifestations, and that these are the ones associated with the electric (or other pure Coulomb), strong and weak interactions.

There is just one further aspect of the weak interaction that we need to resolve, certainly when it extends beyond the fermion-vacuum interaction to one between different fermion states. The Coulomb interaction, as we have seen, manifestly obeys a $U(1)$ symmetry, and the $SU(3)$ symmetry for the strong interaction can be seen from the nilpotent structures we have identified for baryons. Can we establish that the symmetry group of the weak interaction as we have identified it is $SU(2)$? The dual state $\pm i\mathbf{k}E$ which produces the dipolar switching is, of course, manifestly an $SU(2)$ symmetry, and this can be related to the fact that the spherical symmetry of the point source proceeds from its independence from the handedness of the rotation, which, in terms of Noether's theorem, becomes the conservation of the handedness of angular momentum.

There are, however, *two different* $SU(2)$ symmetries involved, though, as we have previously hinted, they are related. The $SU(2)$ of spin describes two helicity states, left- and right-handed. However, *another* $SU(2)$ symmetry, weak isospin, describes a fermionic weak interaction as being independent of whether or not an electric charge is present and generating its own contribution to mass. The relation to the sign of $ikE$ is based on the fact that the weakly interacting part with the positive sign of energy is purely left-handed, and the right-handed component has the 'wrong' sign of $\mathbf{p}$ relative to $ikE$. Weak isospin, in effect, tells us that the weak interaction, though often occurring simultaneously and in combination with the electric interaction, has an independent origin. In this symmetry, the $SU(2)$ is a switching between different *ratios* of left- and right-handedness, and so of mass, determined by the presence or absence of the electric charge in one of the two states. So a superposition of, say, $\alpha_1(ikE_1 + \mathbf{i}\mathbf{p}_1 + jm_1) + \alpha_2(ikE_1 - \mathbf{i}\mathbf{p}_1 + jm_1)$ might become $\beta_1(ikE_2 + \mathbf{i}\mathbf{p}_2 + jm_2) + \beta_2(ikE_2 - \mathbf{i}\mathbf{p}_2 + jm_2)$, with both spin 1 and spin 0 vertices, the additional spin 0 vertex (the one which changes the mass) being equivalent to a fermion $(\pm ikE \pm \mathbf{i}\mathbf{p} + jm)$ acting on an 'electric' vacuum of the form $-j\ (\pm ikE \pm \mathbf{i}\mathbf{p} + jm)\ j$. In effect, a combination of the electric vacuum operator ($j$) and the weak one ($k$) produces a partial transition in the sign of $\mathbf{p}$, giving a basic indication of the way in which these forces are connected. (Gluons produce a direct transition in the strong interaction, and could be represented by terms like $(ikE + i\ \mathbf{i}p_x)\ i\ (ikE + i\ \mathbf{i}p_x)\ i$ reflecting the equivalence of the action on a nilpotent operator of a 'strong vacuum'.) It is significant that, while all the forces contribute to the production of the scalar mass term, the electric force is the only which contributes to nothing else.

The coupling of a massless fermion, say $(ikE_1 + \mathbf{i}\mathbf{p}_1)$, to a Higgs boson, say $(ikE + \mathbf{i}\mathbf{p} + jm)\ (-ikE - \mathbf{i}\mathbf{p} + jm)$, to produce a massive fermion, say $(ikE_2 + \mathbf{i}\mathbf{p}_2 + jm_2)$, can be imagined as occurring at a vertex between the created fermion $(ikE_2 + \mathbf{i}\mathbf{p}_2 + jm_2)$ and the antistate $(ikE_1 - \mathbf{i}\mathbf{p}_1)$, to the annihilated massless fermion, with subsequent equalization of energy and momentum states. If we imagine a vertex involving a fermion superposing $(ikE + \mathbf{i}\mathbf{p} + jm)$ and $(ikE - \mathbf{i}\mathbf{p} + jm)$ with an antifermion superposing $(-ikE + \mathbf{i}\mathbf{p} + jm)$ and $(-ikE - \mathbf{i}\mathbf{p} + jm)$, then there will be a minimum of two spin 1 combinations and two spin 0 combinations, meaning that the vertex will be massive (with Higgs coupling) and carry a non-weak (i.e. electric) charge. So, a process such as a weak isospin transition, which, to use a very basic model, converts something like $(ikE_1 + \mathbf{i}\mathbf{p}_1 + jm_1)$ (representing isospin up) to something like $\alpha_1\ (ikE_2 + \mathbf{i}\mathbf{p}_2 + jm_2) + \alpha_2\ (ikE_2 - \mathbf{i}\mathbf{p}_2 + jm_2)$ (representing isospin down), requires an additional Higgs boson vertex (spin 0) to accommodate the right-handed part of the isospin down state, when the left-handed part interacts weakly. This is, of course, what we mean when we say that the $W$ and $Z$ bosons have mass. The mass balance is done through separate vertices involving the Higgs boson.

We have now seen that the nilpotent structure, with its pseudoscalar, vector and scalar components, *already incorporates the fundamental interactions*. A nilpotent fermion defined by this mathematical formalism is *necessarily* acting according to some or all of these interactions. They arise solely from its internal structure. Coulomb terms are simply the result of spherical symmetry of point sources. Since the Coulomb interaction is purely an expression of the magnitude of a scalar phase, all the terms in the nilpotent contribute, but only one, the passive (scalar) mass term, contributes to nothing else, and that one term in the nilpotent operator has no structure other than magnitude ensures that it must be possible to have an interaction with no symmetry other than $U(1)$. An interaction with this precise property may therefore be defined, and it is the one we define as the *electric* interaction. At the same time, the strong interaction, with its characteristic linear potential, can be represented as we have seen, by the vector properties of the **p** term. It may be significant that the linear potential of the strong interaction is the only one that is optional to the fermion state, the nilpotency not being dependent directly on the vector nature of **p**.

However, yet another interaction is required by the *spinor* structure of the nilpotent operator, and the associated phenomenon of *zitterbewegung*. While the co-existence of two spin states is, in some sense, real, and is accounted for by the presence of mass, the co-existence of two energy states is only meaningful in the context of the simultaneous existence of fermion and vacuum. While the transitions between the two energy states may be virtual in this sense, the *zitterbewegung* would seem to require the production of an intermediate bosonic state at a vertex where one fermionic state is annihilated and another is created to replace by it. This behaviour is, of course, characteristic of the weak interaction, and, in this sense, we can say that the weak interaction, like the electric and strong interactions, is built into the structure of the nilpotent operator, and its nature is determined by that of the pseudoscalar $iE$ operator, whose sign uniquely determines the helicity of a weakly interacting particle, or more specifically its weakly interacting component.

Ultimately, the broken symmetry between the interactions, which is manifested in these structures, emerges when the 8 base units for time, space, mass and charge are compactified into 5 composite generators of the group of order 64 which they construct. Both time, space and mass, and simultaneously charge, are modified in the process. The modification of charge shows the nonlocal or vacuum side of the compactification process, while the compactification to energy, momentum and rest mass shows the local. The algebraic characteristics acquired are manifested nonlocally through the vacua associated with the energy, momentum, and rest mass components of the nilpotent wavefunction. The different algebraic characteristics of the three components then ultimately determine the nature of the local interactions which result from the local symmetries. We have shown that it is possible to go all the way from the nonlocal vacua to the recognizable physical effects which are characteristic of the different forces.

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