Foundations of Physical Law
3 The most primitive concepts

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Foundations of Physics

We hope to justify claim that Foundations of Physics is a subject in its own right and with its own methodology.

Next step almost entirely inductive, but also deeply mathematical.

It is not conventional mathematical physics because that is a product of complexity and emergence, not of fundamental simplicities,

but we will certainly lead to it, and we will show how in the lectures that follow.

Foundations of Physics

We have methodology and mathematical toolkit. Will now apply these to physics at its most fundamental, ‘embryonic’ level.

If our application is correct, then physics can never be looked at in the same way again.

Not an exercise in philosophy, but a completely physical discussion pitched at the most primitive level at which physical concepts can be identified as such.

Questions that cannot be answered in physics at a more developed, emergent level, may find answers when pitched at this foundational level.

Foundations of Physics

Inductive mode needs to be in overdrive. We need entirely new techniques of organizing the information confronting us in every direction.

Need to use methodological principles and pre-prepared mathematics to see past all the conflicting claims and the layers of complication that nature, as well as ourselves, has put in the way of getting directly to the central core of information.

We need an instinctive feel for the ‘big picture’ to sort out the ideas that are truly basic from the accumulation of complexity that is beginning to interfere with them.

Foundations of Physics

Big simple ideas, like why time flows only one way, need big simple answers – not ones based on the emergent consequences – and we have to develop an aptitude for recognising the recurring patterns.

Inductive thinking very different from pure speculation. No constraints on the ideas we generate, but very definite and very strong constraints on the ones we *accept*, because the methodology allows us immediately to work out the consequences that a *foundational* idea will generate.

Foundations of Physics

The very general application means that foundational ideas that are wrong will immediately generate a mass of unacceptable consequences.

Ones that are broadly correct produce smaller issues, which successive iterations are likely to resolve.

Working in a purely abstract mode and avoiding model-dependent ideas mean that faults in the structure can't be hidden in the details, as they can with a model.

To a large extent, it either works or it doesn't, and the main problems arise with deciding which ideas are more basic than which others.

The most primitive concepts

The most 'primitive' are those that survive as we change our scale of operation.

Include space, time, something representing matter in its point-like state and something representing energy or the connections between the points of matter.

It is clear that the last two, in some sense, represent the sources of the four known physical interactions, which, apart from space and time, seem to be all that could be truly fundamental in physics.

Charge

The concept related to the point-like nature of matter can be identified as charge.

This is not as simple as we might think. Apart from electric charge, the weak and strong interactions require a similar concept, and the parallel is certainly assumed in the concept of 'charge conjugation'.

In view of the already successful partial unification involved in the electroweak theory, and the potential for further unification at higher energy scales, it seems more meaningful to define a single parameter with three components than to imagine that there are three totally separate concepts.

Charge

Something seems to break the symmetry between the three charge components that we might otherwise expect in such a picture, and that has been assumed to exist at some particular energy regime in Grand Unification, and that is suggested by the common $U(1)$ component that each of the forces has.

Need a bold conjecture, based perhaps on the idea that we may have seen the pattern before.

Charge

Our mathematical discussion how ‘packaging’ of two 3-D structures can lead to a broken symmetry.

An ‘embryo’ theory must be expected to show differences from a fully-fledged emergent one, so we can make a provisional assumption that this is what is happening here.

If it is correct assumption, then the exact mathematical structure for symmetry-breaking which we have established in the previous lecture should be reflected in the physics that emerges.

If they aren’t, it will soon become obvious.

Mass

The other concept that might be primitive is the source of the gravitational interaction, and it has to be differentiated from the other three.

Energy, mass-energy or mass.

We call it ‘mass’, rather than energy. One reason is to emphasize its role as a source for gravity; we will show later how the concept of ‘energy’ in the quantum sense and its relation to mass seen as the gravitational source are emergent properties from the packaging of the more primitive structure

Mass

There is no physical system in which the rest mass can actually be separated from some dynamic component. All mass is, in fact, in some sense dynamic, even the so-called rest mass can be seen as a product of the subtle quantum dynamical process known as *zitterbewegung*.

Mass

The concepts identified here could almost have been derived by the old method of dimensional analysis and they feature at a fundamental level in such a deeply significant result as the *CPT* theorem.

Purely abstract – none, as we will show, is any more ‘real’ than any other. They can all be seen almost as manifestations of pure algebra.

We don’t assume they are the most fundamental imaginable concepts, but the most fundamental imaginable that can be described as purely *physical*. In the last lecture we will seek to derive them.

Measurement

The most primitive abstract concepts must be more fundamental than either the laws of physics or the structure of matter – neither of these can be imagined without them.

Significantly, of all the four concepts we think may be primitive, only one, space, is actually susceptible to direct measurement

though measurement is the only means we have of investigating nature.

We have created many ingenious devices for measuring and recording data, but however sophisticated the system, they can always be reduced to the equivalent of moving a pointer across a scale.

Measurement

But we have found it necessary to channel our measurements of space through three other conceptual structures as well, as though they had an independent existence.

The entertainment industry uses attributes of space, such as shape, colour and sound vibration, to simulate things that are not meant to be spatial.

The make-believe of films, sound recordings and holograms, for example, depends entirely on the idea that variations in space can induce in the viewer or listener a sense of the passage of time or the presence of matter.

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Measurement

Space measurement is also universal – any object or collection of objects will automatically create a measurement standard for space, and this happens at all times and in all places throughout the universe.

Though we can sense time passing through the laws of thermodynamics, and can detect whether the simulation of a sequence of events is running in the wrong direction, we can never actually measure it.

When we claim to be ‘measuring’ time, we are really only measuring space. We have to find some kind of device in which something repeatedly traverses the same section of space, so that we can construct a time interval from the frequency of repetition.

Measurement

Traditional devices – pendulum clocks, watches governed by a balance spring, use *isochronous* SHM, regular in its repeating cycle.

The alternative was to use an astronomical measure, like the rotation of the Earth or the orbit of the Earth round the Sun.

Relatively modern devices like atomic and digital clocks work on the same principle, with internal oscillations which are counted automatically.

But in all the devices the space is observed not the time.

Measurement

We need special conditions to ‘measure’ time, and these would be impossible without acceleration and force. Even sending a light signal over a known path requires a reflection.

Acceleration and force are second order in time and so are yet further removed from anything like a direct time measurement.

The same is true of ‘measurements’ of mass and charge, again only possible in the presence of force, and again require the equivalent of a pointer moving over a scale.

Measurement

The special nature of space long of interest to philosophers and scientists, and several have tried to reduce the whole of nature to this one quantity.

Descartes. Einstein. Time became a fourth dimension of the new combined concept of ‘space-time’ in SR, while mass-energy expressed as space-time curvature in GR. Kaluza and Klein added a fifth dimension for electromagnetism.

No one has come close to incorporating the rest of the Standard Model into a space-like structure even by incorporating yet more dimensions, though this appears to be the ultimate aim of string theory.

Measurement

The mass of solid objects is determined by observing the force of gravity on them at the Earth’s surface, and this involves both a spatial measurement and the use of a clock and its spatial repetitions.

The same is true for astronomical objects, where we observe the dynamics due to gravity on a large scale.

There are various ways of measuring the mass-energy of a particle, but they all involve using a force or a heating effect, with consequent reliance on spatial observation.

Charge, of course, is not even detectable without force.

Measurement

Problems even with such ‘unifications’ as we have already achieved.

A unification using a single multi-dimensional spacelike structure does not fulfil criterion of leaving nothing arbitrary in a fundamental theory.

Space itself, and dimensionality, remain unexplained, and no explanation of why the dimensions had such distinct manifestations.

If a multidimensional ‘space-time’ really is the foundation for physics, there is nothing more foundational to explain its components.

Measurement

In any case, quantum mechanics seems to be telling us that space and time are fundamentally different in ways that suggest a theory relying on a *complete* union, while suggesting some interesting consequences, will fail at some significant point.

One of the most significant differences in quantum mechanics is that space is an observable, while time is not, exactly in line with our more general analysis.

The indications are that the union between space and time in relativity, is an emergent one, at the first stage of complexity, not a foundational one that can’t be broken down further.

Duality and identity

We can't reduce all physical concepts to space, but there are *relationships* between space and the other fundamental concepts.

An alternative to establishing *identity* between concepts is to find *symmetries* between them.

We can guess that at this level it will be the most basic one, duality, represented by the C_2 group.

Perfect duality, some characteristics in which two concepts are identical, some in which they are absolutely opposite. We might not immediately know that dual concepts are not identical.

Duality and identity

Perhaps this applies to space and time, or even space and all other quantities.

Possibly, dual, rather than identical.

Duality offers a better route to explanation than identity, because if we only have space, then there is no route to explaining space itself.

But dualities might only be explicable if we can explain the concepts themselves.

We will explore three possible cases of fundamental dualities.

Conserved	*Nonconserved*
Mass	Space
Charge	Time

Conservation and nonconservation

Some of the most fundamental laws are about conservation.

Descriptions of physical systems seem to involve a statement, direct or indirect, that some quantity is conserved – for example, mass-energy, momentum, angular momentum, charge – while others – for example, space and time – are not.

Conservation laws are also very specific. Mass and charge are not just globally, but *locally* conserved. Each elementary charge and each element of mass has an *identity*, or unique label, which it carries with it throughout any changes brought about by its interactions.

Conservation and nonconservation

Surprisingly, there is a property that can be called 'nonconservation' and it is just as definite a property as conservation, and an exact dual to it.

This is the property of variable quantities, in particular, space and time, quantities which have *no identity*.

We can't single out a unit of space and time, like we can those of mass and charge, and there are three major symmetries which say exactly that.

the translation symmetry of time

the translation symmetry of space

the rotation symmetry of space

Translation and rotation asymmetry

Conserved quantities, by contrast, are translation and rotation asymmetric. Each unit is unique. One cannot be replaced by another. Mass and charge have *identities*. So, we have

the translation asymmetry of mass
the translation asymmetry of charge
the rotation asymmetry of charge

The last is especially important. The three types of charge do not rotate into each other. The 3 types of charge are separately conserved (baryon and lepton conservation).

Gauge invariance

Another key property of nonconserved quantities is *gauge invariance*. Field terms remain unchanged if we arbitrarily change potentials, due to translations (or rotations) in the space and time coordinates.

In effect, we can arbitrary changes in *the coordinates* which don't produce changes in the values of conserved quantities such as charge, energy, momentum and angular momentum.

Gauge invariance

Significantly, gauge invariance in the Standard Model is *local*, exactly like the conservation laws. Local conservation has an exact dual in local nonconservation.

Gauge invariance and translation and rotation symmetry are not merely passive constraints.

They force us to construct physical equations in such a way that nonconserved quantities have properties exactly opposite to those of the conserved ones, and that it is *explicitly shown* that this is the case.

Differential equations

In general, physics structures itself in terms of *differential equations* which ensure that the conserved quantities – mass and charge, and others derived from them, such as energy, momentum and angular momentum – remain unchanged while the nonconserved or variable quantities vary absolutely.

This means that the nonconserved or variable quantities are expressed in physics equations as *differentials*, dx, dt, directly expressing this variation.

Quantum mechanics

The intrinsic variability or nonconservation of space and time can be seen as the ultimate origin of the path-integral approach to quantum mechanics, where we must sum over all possible paths. None can be privileged.

The idea that 'God plays dice' in the quantum state should no longer trouble us if we accept the logic of defining space and time as nonconserved quantities. This means that they are not fixed and should be subject to absolute variation.

It is only the fact that conservation principles should hold at the same time, that restricts the range of variation when systems interact with each other. When the interactions are on a massive scale, we can even make a classical 'measurement'!

Quantum mechanics

But it isn't the *measurement* that makes the situation become classical.

The degree of variability, in fact, becomes restricted by the application of external potentials, requiring new conservation conditions, as the isolated system interacts with its external environment (the 'rest of the universe').

The so-called 'collapse of the wavefunction' is nothing more significant than the extension of an isolated quantum system to incorporate some part of its environment, so introducing a degree of decoherence.

Noether's theorem

According to a well-known mathematical theorem, to every variational property there is a conserved quantity. So

translation symmetry of time ≡ conservation of energy
translation symmetry of space ≡ conservation of momentum
rotation symmetry of space ≡ conservation of angular momentum

Noether's theorem is a natural consequence of defining conservation and nonconservation properties symmetrically.

So nonconservation of time ≡ conservation of mass (energy) …

Noether's theorem

We can extend Noether's theorem, purely by symmetry:

conservation of	**conservation of**	**symmetry of**
energy	mass	time translation
momentum	magnitude of charge	space translation
angular momentum	type of charge	space rotation

The first, we can see, might be related to the gauge invariance we have discussed. But what about the second?

Conserved	*Nonconserved*
Mass	Space
Charge	Time
Real	***Imaginary***
Mass	Time
Space	Charge

Space and time

Relativity combines space and time in a *4-vector*, with 3 real parts (space) and one imaginary part (time).

Pythagoras' theorem in 4-D

$$r^2 = x^2 + y^2 + z^2 - c^2t^2 = x^2 + y^2 + z^2 + i^2c^2t^2$$

4-vector:

$$r = \mathbf{i}x + \mathbf{j}y + \mathbf{k}z + ict$$

Space and time

And from the Einstein energy-momentum relation (with $c = 1$)

$$m = E^2 - p_x^{\,2} - p_y^{\,2} - p_z^{\,2} = i^2E^2 - p_x^{\,2} - p_y^{\,2} - p_z^{\,2}$$

$$p_x^{\,2} + p_y^{\,2} + p_z^{\,2} - E^2 = p_x^{\,2} + p_y^{\,2} + p_z^{\,2} - i^2E^2$$

we can extract the 4-vector

$$\mathbf{i}p_x + \mathbf{j}p_y + \mathbf{k}p_z + iE,$$

which, with the c terms included, becomes

$$\mathbf{i}p_xc + \mathbf{j}p_yc + \mathbf{k}p_zc + iE.$$

Space and time

Why is the time or energy component imaginary compared to the space or momentum? Why do they have different norms.

A *physical* argument based on relativity would say that the light signal is retarded. But we have to remember that, at the foundational level, there is no light and there is no relativity.

Sometimes, the 3 + 1 real-imaginary representation of space and time is described as a mathematical ‘trick’.

Vectors in our representation require an imaginary fourth component (a pseudoscalar) because they derive from complexified quaternions.

Space and time

There is also a physical explanation.

Quantities containing time to the first power, such as uniform velocity, have no real significance or physical meaning. This only comes when they incorporate time squared, as we find with acceleration and force.

Time ‘measurement’ requires these quantities, even for time-measuring devices that use light itself.

This is totally consistent with what we might expect for an imaginary quantity, but there is yet another physical reason.

Space and time

Imaginary quantities are intrinsically dual. They have + and – solutions which can’t be distinguished.

If we use imaginary numbers, then we automatically accept the duality that is built into them.

Time flows only one way – detect from increased entropy.

Physical equations, however, have two directions of *time symmetry*.

Space and time

Even if we can't reverse time, we can extract physical meaning from reversing the sign of the time parameter, as with *CP* violation in particle physics. This constitutes the famous reversibility paradox.

However, there is no paradox at all if time really is imaginary, and the one-way flow of time comes from an entirely different aspect of the parameter (as we will see).

A parallel case can be seen in relativistic quantum mechanics where two signs of the energy parameter derive from time via its representation as $\partial / \partial t$, though there is only one sign of physical energy.

Mass and charge

If our methodology is correct, then we might expect to find a real-imaginary distinction occurring also with mass and charge.

We have the problem that our picture of 'charge' is complicated by the broken symmetry involved, with significant distinctions between all three interactions for which we suppose it is the source.

Let's suppose that a real unbroken symmetry is there which is exact in principle, and that the breaking of the symmetry is an effect of emergence or complexity, as our analysis of the Clifford algebra would suggest.

Mass and charge

This is also consistent with current thinking, since it is widely believed that there is some energy regime at which the weak, strong and electric interactions would lose their distinguishing features and become alike.

They are already alike in at least one aspect. They have a ‘Coulomb’ or inverse square force term, representing the $U(1)$ symmetry of a scalar phase, as also does Newtonian gravity (an aspect which is imported even into GR as the Newtonian potential).

Mass and charge

The weak and strong interactions differ from the electric interaction in having additional terms in their force laws which give them additional properties.

It is possible then that, at grand unification, these extra components could be seen to shrink, leaving all three interactions as purely Coulomb in form.

So, we will investigate this term first.

Mass and charge

The inverse square force or Coulomb force has a relatively simple explanation. It is the exact result we would expect for a charged point source in a 3-dimensional space with spherical symmetry.

In all known interactions, it relates to the coupling constant. This is not strictly the charge (in effect, a pure existence condition), but one can define the *magnitude* of the charge (electric, weak or strong) in terms of the electromagnetic, weak or strong coupling constants.

In quantum terms, the coupling constant squared is the probability of absorbing or emitting the boson that carries the interaction, and this will be zero if the particular charge is not present.

Mass and charge

Why do identical masses attract; identical charges repel?

$$F = -constant \times \frac{m_1 m_2}{r^2}$$

$$F = constant \times \frac{e_1 e_2}{r^2}$$

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Masses are real; charges imaginary.

Mass and charge

But there are three charges: electric, strong and weak.

Are they alike in being mutually repelling when identical?

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Yes! We can use a quaternion with components $\boldsymbol{i}s$, $\boldsymbol{j}e$, $\boldsymbol{k}w$, where s, e and w are the strong, electric and weak charges.

Mass and charge

A quaternion needs a real fourth term, or scalar, just as our multivariate vectors needed a pseudoscalar, and mass is available to play this part.

We can then propose that the units of charge and mass could act as the three imaginary plus one real parts of a quaternion, just as the units of space and time act as the three real and one imaginary parts of a multivariate 4-vector.

space	time	charge	mass
$\mathbf{i}x\ \mathbf{j}y\ \mathbf{k}z$	it	$\boldsymbol{i}s\ \boldsymbol{j}e\ \boldsymbol{k}w$	$1m$

Mass and charge

Perhaps this is the final vindication of Hamilton, giving quaternions a direct role in nature, as well as the indirect one of being the progenitor of the 4-vectors linking space and time.

In fact the quaternion representation is logically ‘prior’, not only in the mathematical sense, but also in the physical sense,

the character of space-time being predetermined by the necessity of symmetry with charge-mass, whose structure is completely determined by the quaternionic form.

Mass and charge

Such a symmetry would constrain the vector character of space to the extended form required from a complexified quaternion or Clifford algebra, and not the restricted form of the Gibbs-Heaviside algebra,

meaning that spin would be automatically factored in to the structure of space, and not be an unexplained additional extra brought in with quantum mechanics.

Mass and charge

This depends on choosing charge to be the imaginary quantity, rather than mass. So, could we have chosen mass to be imaginary instead of charge?

Here, we return to the fundamental property of imaginary numbers (including quaternions) that they can only exist as a dual pair, with both + and – signs.

Mass, as we know it, has only one sign, whether we call it positive or negative, there is only one version. Mass is 'unipolar'. With charge, we always have both + and – versions, whether the charges are electric, strong or weak.

Mass and charge

This is the explanation of ‘antimatter’. For every particle with a charge structure of *any* kind, there has to be a particle with charges of the opposite sign.

We even call the particle and antiparticle switching *charge conjugation*, and we are fully aware that it isn’t just about particles with *electric* charge.

Neutrons, with no electric charge, have antiparticles, because the neutron still has strong and weak charges, and the antiparticle requires these to take the opposite signs. Masses are the same.

Mass and charge

There is also another reason for imaginary time / charge and real space / mass.

We can observe space directly by observation or measurement, or through its squared value, in Pythagoras' theorem or vector addition,

whereas time can only be apprehended through its squared value in force or acceleration

Mass and charge

Similarly, we can apprehend mass physically in two different ways, either directly, through inertia or force = mass × acceleration, or through its squared value, through gravitation.

Inertia allows us to apprehend a mass even if no other mass is present, though we need at least two masses for gravitation.

For charge, there is only apprehension through the squared quantity, via Coulomb's law.

Ultimately, for both time and charge, the imaginary status is not a mathematical convention; it represents a real physical property.

Conserved	*Nonconserved*
Mass	Space
Charge	Time
Real	*Imaginary*
Mass	Charge
Space	Time
Anticommutative	*Commutative*
Space	Mass
Charge	Time

Commutative and anticommutative

Already our methodology is forcing certain constraints on the way we view the fundamental parameters. They are connected by at least two dualities, and there is, in fact, a third.

Mass and time are, respectively, scalar and pseudoscalar, and so commutative, while space, as a vector, must be anticommutative.

A quaternionic quantity (charge) must be anticommutative as well. Anticommutativity requires a quantity to be both dimensional and specifically 3-D, and the reverse argument is also true.

Commutative quantities, like time and mass, must be non-D, or, as it is sometimes termed, one-D.

Commutative and anticommutative

In addition to dimensionality, another very significant consequence may be the discreteness or discontinuity introduced with the fact that the three components of an anticommutative system are very much like the components of a closed discrete set.

Can we now postulate that anticommutative or dimensional quantities are necessarily also discrete or divisible, while commutative or nondimensional are correspondingly always continuous or indivisible?

If 3-D quantities are the only discrete ones, this would be yet another remarkable consequence of Hamilton's original discovery.

Commutative and anticommutative

Continuous quantities clearly not dimensional, because a dimensional system needs an origin, a zero or crossover point, and this is incompatible with continuity.

Also, a one-dimensional quantity can’t be measured, because scaling requires crossover points into another dimension.

Often claimed that a point in space has zero D, a line 1-D and an area 2-D, but a single dimension cannot generate structure. A line can only be seen as a 1-D structure, within a 2-D world, which itself can only exist in a 3-D one, because a 2-D mathematical structure always necessarily generates a third.

Commutative and anticommutative

Is there a counter-argument that space, as a 3-D quantity, is necessarily also discrete?

The answer has to be yes. If space weren't discrete we couldn't observe it. The whole of our measuring process is based on the fact that space is fundamentally discrete.

Previous confusion due to a fundamental misunderstanding about the nature of real numbers, and the lack of a methodology which could separate the different aspects of the fundamental parameters into primitive properties.

Commutative and anticommutative

Certainly, *charge*, which may be 3-D, is discrete, because it comes in fixed point-like units or ‘singularities’, which are easily countable.

Space is nothing like this. Often represented by a real number line, which gives the appearance of being continuous.

However, space, unlike charge, is a nonconserved quantity, and its units can’t be fixed. Its discreteness is one that is endlessly reconstructed. The real number line is not absolutely continuous, it is *infinitely divisible*.

Commutative and anticommutative

Space presents exactly the characteristics required by non-standard arithmetic and non-Archimedean geometry.

It is made up of real numbers constructed by an algorithmic process, and so is necessarily countable.

If it were not, then measurement, and dimensionality, would be impossible.

Commutative and anticommutative

There is, as far as we know, a very deep distinction between space and time, beyond any consideration of their mathematical nature as real and imaginary quantities.

Time cannot be split into dimensions in the same way as space, which means that it cannot be discrete and must be continuous.

This allows us to complete the explanation of the reversibility paradox.

Commutative and anticommutative

As a ‘nondimensional’ and continuous quantity, time is necessarily irreversible. To reverse time, we would have to create a discontinuity or zero-point. In addition, as a nondiscrete quantity, time can never be observed, which is exactly what quantum mechanics tells us.

Observability always requires discreteness.

This is why we treat time as the *independent variable*, by comparison with space. We write dx / dt, in fundamental equations, not dt / dx, because time varies independently of our measurements, represented by dx, which respond in turn to the unmeasurable variation in time.

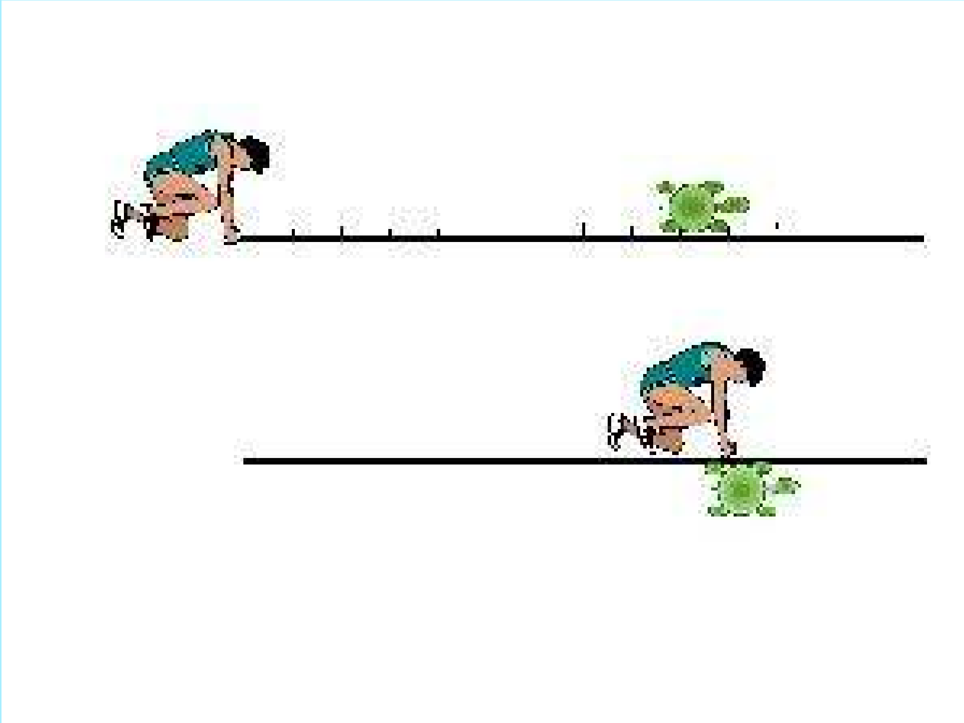
Commutative and anticommutative

While space, which is infinitely divisible, time is *absolutely continuous*, that is not divisible at all.

The two conditions are mathematically opposite, about as different as any physical or mathematical properties could conceivably be, though a fundamental duality in nature allows one to be substituted for the other.

Clocks do not measure time, but a space with which it has an indirect relation; the divisions that we measure are those of the space which is repeatedly traversed, and they require the fact that space, as a dimensional quantity can be reversed because we can define it to have an origin. They also, very often, use the fact that space has more than one dimension.

Zeno's paradox



Achilles never catches the tortoise, however fast he is, if he gives it a start.

Zeno's paradox

'One can, therefore, conclude that the idea of the infinite divisibility of time must be rejected, or ... one must recognize that it is ... a logical fiction.' Motion is 'impossible if time (and, correlatively, space) is divisible ad infinitum'.

G. J. Whitrow

'Either one can seek to deny the notion of 'becoming', in which case time assumes essentially space-like properties; or one must reject the assumption that time, like space, is infinitely divisible into ever smaller portions.'

Peter Coveney and Roger Highfield

Two methods of calculus

One of the usual strategies for tackling the problem has been to invoke calculus, and in particular to define it in terms of the ‘limit’ of a function as approaching a particular value.

As we have seen, however, there are two valid methods of calculus, only one of which involves limits.

We can now see that they are, in fact, based on differentiation with respect to two different quantities with different physical properties, namely, space and time.

Two methods of calculus

Calculus, in principle, has nothing to do with the distinction between continuity and discontinuity, but is concerned with whether a quantity is variable or conserved.

Variable quantities can be either continuous or discontinuities, and this leads to two ways of approaching the differential.

If we differentiate with a discrete quantity like space, we end up with infinitesimals and non-standard analysis (cf lecture 2). If we differentiate with respect to time, we generate standard analysis and the theory of limits.

Two methods of calculus

One requires an imagined line that is absolutely continuous, the other one that is infinitely divisible.

It is a classic example of the ‘unreasonable effectiveness of physics in mathematics’ parallelling the ‘unreasonable effectiveness of mathematics in physics’.

Remarkably, only the method of limits can be used to ‘solve’ Zeno’s problem, and this is because it is really concerned with differentiation with respect to time.

Space-time

The same duality also applies in physics, though this time at the first level of complexity.

The mathematical connection between space and time does not automatically require the kind of *physical* connection supposed by Minkowski.

In fact the connection is, as we will show, a result of ‘packaging’. This will become clearer in the next lecture.

Space-time

The physical identity is denied by QM, which proclaims that time, unlike space, is not an observable. The reason will emerge only when we make quantum mechanics relativistic.

In principle, the combination of space and time in a 4-vector format, while possible mathematically, cannot be done in a physical way. We are obliged to go to the nearest physical equivalent by either making time spacelike or space timelike.

Wave-particle duality

We will argue that this is the origin of wave-particle duality, for the first solution makes everything discrete, or particle-like, while the second makes everything continuous, or wavelike.

The mathematical connection diverges into two physical connections, neither of which is completely valid.

Wave-particle duality would not exist if space-time was a truly physical quantity.

Wave-particle duality

The duality, which runs through both classical and quantum physics, extends even to the existence of two forms of nonrelativistic quantum mechanics.

Heisenberg gives us the particle-like solution, while Schrödinger gives us a version based on waves. Neither is more valid than the other, but the ideas of the theories cannot be mixed. In all cases, classical as well as quantum, the duality is absolute.

Nature gives us duality between discrete and continuous processes where space and time occur at the same level, and this is because of the fundamental difference between them due to their respective properties of discreteness and continuity.

Is mass continuous or discrete?

We have just one more issue to be resolved in relation to discreteness and continuity, but it is a very important one.

Where does mass stand with regard to this question?

There can only be one answer, either from symmetry or its intrinsic nondimensionality. Such a parameter can only be completely continuous in the same way as time.

Is mass continuous or discrete?

There is no mass which is fully discrete, even though we are accustomed to defining a rest mass or invariant mass for fundamental particles. No particle, in fact, has such a mass, for all are in dynamic motion with the relativistic energy which this involves. Even the ‘rest’ mass arises dynamically from the subtle QM motion known as *zitterbewegung*.

But mass-energy, in any case, is a continuum which is present at all points in space. Several different forms – the Higgs field or vacuum = 246 GeV of energy at every point in space, without which the rest masses could not be generated.

Is mass continuous or discrete?

It is possible, in fact, that the discrete and continuous options could be responsible for the respective ideas of the local and global, the transition from global to local gauge invariance being the point in the Higgs mechanism at which the continuous field leads to the generation of a discrete invariant mass.

Besides the Higgs field, manifestations of continuous mass include the zero-point energy and even ordinary fields, which cannot be localised at points. It is the continuity of mass which is the reason for its 'unipolarity' or single sign, and for the absence of a zero or crossover point, which would indicate dimensionality.

Three exact dualities

We have now identified three dualities which appear to be astonishingly exact, and over a long period of testing I have yet to find an exception.

If they represent a truly primitive level in physics, this is exactly what we would expect to find.

We would expect nature to be neither totally conserved nor totally nonconserved, neither totally real nor totally imaginary, neither totally continuous nor totally discrete.

Three exact dualities

In the case of the last distinction, it is impossible to imagine defining discreteness without also describing continuity. We can only know what something is if we also know what it isn't.

The thing that seems to be completely excluded at this level is *extended* discreteness, except insofar as it applies to space.

Continuity is often described as an 'illusion', but, at the fundamental level, where abstractions are dominant, 'illusions' or ideas are an intrinsic part of reality – we couldn't even have an idea unless it was somehow part of the abstractions which nature makes available. At this level, we would not expect to find the 'best fit' compromises that might appear at a more complex level.

Three exact dualities

While it has often been claimed that physics would fit ‘reality’ better if we made it totally discrete, like measurement, continuity always seems to force its way in, as in the second law of thermodynamics.

The four fundamental parameters can also be interpreted in terms of system and measurement, ontology and epistemology, neither being dominant over the other.

Three exact dualities

We can make the system (or theoretical superstructure) discrete, as Heisenberg did, but then continuity will appear in the measurement, as in the Heisenberg uncertainty.

Alternatively, we can make the system continuous, as Schrödinger did, and find that discreteness appears in the measurement, in this case the ‘collapse of the wavefunction’.

As with the divisions between conserved and nonconserved, and real and imaginary parameters, this one seems to be an exact symmetry of absolute opposites.

The End