Investigating the Foundations of Physical Law
10 Return to symmetries

A universal rewrite system

The seemingly unbroken consistency of the symmetries described in lectures 3 and 4 gives a strong indication that they are close to a true foundational level in physics, and the apparent applicability of foundational methods in resolving problems in relatively complex parts of physics gives support to this. If this is a valid inference, there is still one major question to be answered: does Foundations of Physics have a foundation? And, if so, what kind? Clearly, it would have also to act in some way as a foundation for mathematics, and possibly for a more general approach to information processing.

The twentieth century success of digital computing made a number of people think that nature might be structured on an information system, similar to the workings of a computer. So, can this be true here? Modern computers are based on the Turing machine, digital logic, and the rewrite or production system, which is essentially the basis of programming and algorithmic processes of any kind. So, we have a pretty clear idea of how they work. A conventional rewrite system has 4 fixed components:

alphabet
rewrite rules (productions)
a start 'axiom' or symbol
stopping criteria

As an example (suggested by my computer scientist colleague, Bernard Diaz) we can devise a process for generating Fibonacci numbers. There are two rules: p1: A $\rightarrow$ B; p2: B $\rightarrow$ AB. We begin with generation 0, and a single symbol A. Rule p1 then tells us to replace A with B. In the next generation, B becomes AB, and the process then repeats indefinitely:

N=0	A	length of string	1	
N=1	$\rightarrow$B		1	
N=2	$\rightarrow$ AB		2	
N=3	$\rightarrow$ BAB		3	
N=4	$\rightarrow$ ABBAB		5	
N=5	$\rightarrow$ BABABBAB		8	
N=6	$\rightarrow$ ABBABBABABBAB		13	
N=7	$\rightarrow$ BABABBABABBABBABABBAB		21	...

It may be significant that the rules A $\rightarrow$ B and B $\rightarrow$ AB seem to be suggesting the structure of 3-dimensional (quaternion) algebra

$$\boldsymbol{i} \rightarrow \boldsymbol{j} \qquad \boldsymbol{j} \rightarrow \boldsymbol{i}\,\boldsymbol{j} = \boldsymbol{k}$$

and that a string like BABABBABABBABBABABBAB appears to be creating a fractal-like structure in 3-dimensional space, but situated in the AB or ***ij*** plane, as in holography. The logarithmic spiral then becomes a way of expressing 3-dimensionality in the plane, with the increasing length of the intervals substituting for penetration into the third dimension.

Now, if we had a system in which all four elements – alphabet, start object, rules, and stopping criteria – could be varied, the rewrite structure and alphabet would become universal, and perhaps self-generating. The algebraic structure which we believe lies at the foundations of physics, has the appearance of being generated by a process, possibly an information process. Can we use the concept of totality zero to find the process or the algorithm? On the basis of the foundational principles we have declared in the first lecture, should we expect it to be universal?

No universal rewrite system being known, Bernard Diaz and I set out to see if nature actually supplied one, and if it looked like what was driving physics and mathematics as set out in the Klein-4 parameter group and the nilpotent package and algebraic structure used for fermionic physics. In this search it is important not to assume any prior knowledge of mathematics, not even the natural numbers. This is very difficult to do, as counting has become a basis for virtually everything we do, and, even if we avoid actual counting, is difficult to avoid the use of it in our language.

This is what we came up with. We assume that the universe, or any alphabet which it contains, is always a zero totality state, with no unique description, and so infinitely degenerate. This means that we have to continually regenerate the alphabet, in such a way that it is always new, but there is no limit or stopping criteria, as the new state created is always another nonunique zero totality. It is a kind of *zero attractor*. A non-zero deviation from 0 (say *R*) will always incorporate an automatic mechanism for recovering the zero (say the 'conjugate' *R**), but the zero totality which results, say (*R*, *R**), will not be unique, and will necessarily lead to a new structure.

The characteristics of the process which emerge from this include self-similarity, scale-independence, duality, bifurcation, and holism. The process differs from other rewrite processes in having no fixed starting or ending point, and an alphabet and production rules that are endlessly reconstructed during the process. So, it fulfils our conditions for universality. The self-similarity is a necessary consequence of the lack of a fixed starting point. It suggests that, if there are physical applications at one level, then there are likely to be applications also at others. As we scale up from small to larger systems, we can imagine that some principle such as the renormalization group takes effect to maintain the form of the structures generated by the rewrite process.

We ensure that a structure or alphabet is new by defining the position of all previous structures within it as subalphabets. The process will then continue indefinitely. Effectively, the process involves defining a series of *cardinalities*. Successive alphabets absorb the previous ones in the sequence, so creating a new cardinality. The cardinalities are like Cantor's cardinalities of infinity, but are cardinalities of zero instead. From the point of view of the observer, i.e. someone 'inside' the system ('universe', 'nature', 'reality'), we have to start from (R, R*), which is the minimum description of a zero totality alphabet (or of a zero totality universe in physical terms). The successive stages are all zeros, as we go from one zero cardinality or totality to the next, and we ensure that they are cardinalities by always including the previous cardinality or alphabet. So (R, R*, A, A*), for example, includes (R, R*). We may start at any arbitrary zero-totality alphabet but there is no natural beginning or end to the process. Because all the stages are cardinalities or zero totality alphabets, the process is always holistic. We have to include everything.

A convenient though not unique way of representing the process is by a 'concatenation' or placing together, with no algebraic significance, of any given alphabet with respect to either its components or subalphabets or itself. Since an alphabet is defined to be a cardinality, then anything other than itself must necessarily be a 'subalphabet' and the concatenation will produce nothing new. Only concatenation of the entire alphabet with itself will produce a new cardinality or zero totality alphabet. It is convenient to represent these two aspects of the process by the symbols $\Rightarrow$, for *create*, in which every alphabet produces a new one which incorporates itself as a component, and $\rightarrow$, for *conserve*, which means that nothing new is created by concatenating with a subalphabet.

conserve:	(subalphabet) (alphabet) $\rightarrow$ (alphabet)	*there is nothing new*
create:	(alphabet) (alphabet) $\Rightarrow$ (new alphabet)	*a zero totality is not unique*

The process is simultaneously recursive, creating everything E (all symbols) at once, and iterative, creating a single symbol only. It is also fractal and can begin or end at any stage. Since we intend that it should describe or create both time and 3D space, we can think of it as prior to both. The create and conserve aspects must also be simultaneous; we only know which new alphabet will emerge when we have ensured that all possible concatenations with subalphabets yield only the alphabet itself.

Suppose, we then start with a zero totality alphabet with the form (R, R*). Of course, we have to assume that this is not necessarily the beginning, though it is the point where we as observers start from. So, this already 'bifurcated' state will have started from a previous alphabet, which we assume we can't access directly, because we have no structure for it. If we describe this as R, then the * or R* character creates the 'doubling' process. (Although we now see this in terms of the number 2, we have to imagine that we have not yet invented this method of labelling, and the same applies to the term 'duality'.) Before we create (R, R^*), we have to assume that (R) is a zero totality alphabet, but it is a zero to which we have no access. In effect, we are trying to posit an ontology that exists before the epistemology or observation, begins with (R, R^*). So, we assume that it must happen without being able to observe it.

Applying the conserve process $(\rightarrow)$ to concatenate (R, R^*) with its subalphabets should produce nothing new. No concept of 'ordering' is needed in this process, but each term must be distinct. So

$$(R)\ (R, R^*) \rightarrow (R, R^*)$$
$$(R^*)\ (R, R^*) \rightarrow (R^*, R) \rightarrow (R, R^*)$$

It follows immediately that these concatenations lead to rules of the form:

$$(R)\ (R) \rightarrow (R)\ ;\ (R^*)\ (R) \rightarrow (R^*)\ ;$$
$$(R)\ (R^*) \rightarrow (R^*)\ ;\ \ (R^*)\ (R^*) \rightarrow (R)$$

The next stage is to show that the zero-totality alphabet (R, R^*) is not unique, and that a concatenation with itself will produce a new zero-totality alphabet. Our first suggestion might be something like (A, A^*), but, with the terms undefined, this is indistinguishable from (R, R^*), and so the only way to ensure that the new alphabet is distinguishable from the old is by incorporating the old one, and we need to do this in such a way that we ensure that the subalphabets yield nothing new. So we try

$$(R, R^*)\ (R, R^*) \Rightarrow (R, R^*, A, A^*) \qquad (1)$$

Having used the 'create' mechanism, we now apply the conserve operation $(\rightarrow)$ to this new alphabet, and concatenate with the subalphabets. So

$$(R)\ (R, R^*, A, A^*) \rightarrow (R, R^*, A, A^*) \rightarrow (R, R^*, A, A^*)$$
$$(R^*)\ (R, R^*, A, A^*)\ \ \rightarrow (R^*, R, A^*, A) \rightarrow (R, R^*, A, A^*)$$
$$(A)\ (R, R^*, A, A^*) \rightarrow\ \ (A, A^*, R^*, R) \rightarrow (R, R^*, A, A^*)$$
$$(A^*)\ (R, R^*, A, A^*) \rightarrow (A^*, A, R, R^*) \rightarrow (R, R^*, A, A^*)$$

As before the order of the terms is different for each operation, as we require, but the total is the same, and we soon quickly realise that (R, R^*) and (A, A^*) can only be different if

$$A\ A \rightarrow R^*, \text{ etc., while } R\ R \rightarrow R.$$

Duality is intrinsic to the process. The operation () () $\Rightarrow$ (,) describes how we go from one zero totality alphabet – or description of the universe – to the next one up. The (,) is a kind of 'doubling' or 'bifurcation'. So we could write the result of (1) in the form $(R, R^*,\ \ A, A^*)$, and represent $(R, R^*)\ (R, R^*)$ as a kind of doubling, to create a new cardinality (R, R^*, A, A^*), almost like transforming the second (R, R^*) into (A, A^*).

The next stage presents a new problem, for

$$(R, R^*, A, A^*)\ (R, R^*, A, A^*) \Rightarrow (R, R^*, A, A^*, B, B^*)$$

would fail the application of the conserve mechanism ($\rightarrow$) by introducing new concatenated *terms* like AB, AB^*, which lie outside the alphabet. This means that we must include these in advance, as in

$$(R, R^*, A, A^*)\ (R, R^*, A, A^*) \Rightarrow (R, R^*, A, A^*, B, B^*, AB, AB^*).$$

The question that remains is: does this new alphabet satisfy all our requirements, when we concatenate separately with (R), (R^*), (A), (A^*), (B), (B^*), (AB), (AB^*)? The process is straightforward for the first six concatenations:

$$(R)\ (R, R^*, A, A^*, B, B^*, AB, AB^*) \rightarrow (R, R^*, A, A^*, B, B^*, AB, AB^*)$$
$$(R^*)\ (R, R^*, A, A^*, B, B^*, AB, AB^*) \rightarrow (R^*, R, A^*, A, B^*, B, AB^*, AB)$$
$$(A)\ (R, R^*, A, A^*, B, B^*, AB, AB^*) \rightarrow (A, A^*, R^*, R, AB, AB^*, B, B^*)$$
$$(A^*)\ (R, R^*, A, A^*, B, B^*, AB, AB^*) \rightarrow (A^*, A, R, R^*, AB^*, AB, B^*, B)$$
$$(B)\ (R, R^*, A, A^*, B, B^*, AB, AB^*) \rightarrow (B, B^*, AB, AB^*, R^*, R, A, A^*)$$
$$(B^*)\ (R, R^*, A, A^*, B, B^*, AB, AB^*) \rightarrow (B^*, B, AB^*, AB, R, R^*, A^*, A)$$

But concatenations of the *concatenated terms*, (AB) and (AB^*), on themselves and on each other appear to leave us with two options, which we can describe as 'commutative' and 'anticommutative':

$$(AB)\ (AB) \rightarrow (R) \qquad (\textit{commutative})$$
$$(AB)\ (AB) \rightarrow (R^*) \qquad (\textit{anticommutative})$$

In fact, however, there is no choice, for *only the anticommutative option* produces something new. Labelling is arbitrary in the rewrite structure, and so the labels A and B alone cannot distinguish these terms from each other – this can only be done if they produce distinguishable outcomes. The commutative option leaves A and B indistinguishable except by labelling, and so does not extend the alphabet. We are obliged to default on the anticommutative option, which means that the last two concatenations become:

$$(AB)\ (R, R^*, A, A^*, B, B^*, AB, AB^*) \rightarrow (AB, AB^*, B, B^*, A, A^*, R^*, R)$$
$$(AB^*)\ (R, R^*, A, A^*, B, B^*, AB, AB^*) \rightarrow (AB^*, AB, B^*, B, A^*, A, R, R^*)$$

This solution for A and B cannot be repeated to include new terms, such as (C), (D),..., when we extend alphabet to higher stages because an inconsistency will always reveal itself at some point in the analysis. Anticommutativity produces a closed 'cycle' with components (A, B, AB) and their conjugates (A^*, B^*, AB^*), and prevents further terms, such C, D, etc., from anticommuting with them in a consistent manner. However, successive cycles of the form (A, B, AB), (C, D, CD), etc., can be introduced into the structure, if they commute with each other, and this can be continued indefinitely. All of the terms then have a unique identity *because they each have a unique partner*, and the successive alphabets can be seen as a regular series of identically structured closed anticommutative cycles, each of which commutes with all the others.

This structure is familiar to us in the form of the infinite series of finite (binary) integers of conventional mathematics, each alphabet representing a new integer. We can regard the closed cycles as an infinite ordinal sequence, and so establishing for the first time in this process both the number 1 and the binary symbol 1 of classical Boolean logic as a conjugation state of 0, with the alphabets structuring themselves as an infinite series of binary digits. Mathematics and digital logic become emergent properties of a rewrite process which has no specific defined starting point, and can be reconstructed endlessly in a fractal manner, with self-similarity at all stages and a zero attractor.

The universal rewrite system is a pure description of process with many representations. It can, for example, be presented as a series of 'doublings' or 'bifurcations', analogous to the initial creation of (R, R^*), even when they represent 'complexification' (the introduction of a new anticommutative cycle) or 'dimensionalization' (the closing of the cycle) rather than conjugation (the zeroing process used in the first alphabet). So, one way of writing

$$(R, R^*, A, A^*)\ (R, R^*, A, A^*) \Rightarrow (R, R^*, A, A^*, B, B^*, AB, AB^*)$$

would be as

$$(R, R^*, A, A^*)\ (R, R^*, A, A^*) \Rightarrow (R, R^*, A, A^*, \quad B, B^*, AB, AB^*)$$

in which we retain the old alphabet (R, R^*, A, A^*) and a dual (B, B^*, AB, AB^*) formed by some process, as we did with R and R^*. In practical terms, we introduce a new character (B).

Mathematical representation

A significant representation is the mathematical one, for the effective creation of a discrete integer system means that, by applying this to the original terms (R, A, B, etc.), we can also generate an entire arithmetic and an algebra. Once we have generated integers, the rest of the constructible number system will follow automatically, along with arithmetical operations. At the same time, application of the constructed number systems to the undefined state with which the process began suggests that this state, which is not intrinsically discrete, can be interpreted in terms of a continuity of real numbers in the Cantorian sense.

In principle, discreteness appears in the construction only when we introduce anticommutativity or 'dimensionality', and specifically 3-dimensionality. Physically, 3-dimensionality, or anticommutativity, becomes the ultimate source of discreteness in a zero totality universe, and we observe, in all cases, that 3-dimensionality requires discreteness, and discreteness requires 3-dimensionality.

The rewrite process can be represented in symbolic form in the table:

	0	$\boldsymbol{\Delta_a}$	$\boldsymbol{\Delta_b}$	$\boldsymbol{\Delta_c}$	**…**	$\boldsymbol{\Delta_n}$
0	00	$0\Delta_a$	$0\Delta_b$	$0\Delta_c$		$0\Delta_n$
$\boldsymbol{\Delta_a}$	$\Delta_a 0$	$\Delta_a\Delta_a$	$\Delta_a\Delta_b$	$\Delta_a\Delta_c$		$\Delta_a\Delta_n$
$\boldsymbol{\Delta_b}$	$\Delta_b 0$	$\Delta_b\Delta_a$	$\Delta_b\Delta_b$	$\Delta_b\Delta_c$		$\Delta_b\Delta_n$
$\boldsymbol{\Delta_c}$	$\Delta_c 0$	$\Delta_c\Delta_a$	$\Delta_c\Delta_b$	$\Delta_c\Delta_c$		$\Delta_c\Delta_n$
:						
$\boldsymbol{\Delta_n}$	$\Delta_n 0$	$\Delta_n\Delta_a$	$\Delta_n\Delta_b$	$\Delta_n\Delta_c$		$\Delta_n\Delta_n$

Here, the Δ symbols represent the alphabets:

Δ_a (R)
Δ_b (R, R^*)
Δ_c (R, R^*, A, A^*)
Δ_d $(R, R^*, A, A^*, B, B^*, AB, AB^*)$
Δ_e $(R, R^*, A, A^*, B, B^*, AB, AB^*, C, C^*, AC, AC^*, BC, BC^*, ABC, ABC^*)$ ...

The process is not restricted to any specific mathematical interpretation, but incorporates digital logic and binary integers, while a convenient consequence is the algebraic series:

$$(1, -1)$$
$$(1, -1) \times (1, \boldsymbol{i}_1)$$
$$(1, -1) \times (1, \boldsymbol{i}_1) \times (1, \boldsymbol{j}_1)$$
$$(1, -1) \times (1, \boldsymbol{i}_1) \times (1, \boldsymbol{j}_1) \times (1, \boldsymbol{i}_2)$$
$$(1, -1) \times (1, \boldsymbol{i}_1) \times (1, \boldsymbol{j}_1) \times (1, \boldsymbol{i}_2) \times (1, \boldsymbol{j}_2)$$
$$(1, -1) \times (1, \boldsymbol{i}_1) \times (1, \boldsymbol{j}_1) \times (1, \boldsymbol{i}_2) \times (1, \boldsymbol{j}_2) \times (1, \boldsymbol{i}_3) \ldots$$

The anticommutative pairs *A*, *B*; *C*, *D*; *E* … now become successive quaternion units, $\boldsymbol{i}_1, \boldsymbol{j}_1$; $\boldsymbol{i}_2, \boldsymbol{j}_2$); $\boldsymbol{i}_3$ …, each of which is commutative to all the others. By the fourth stage, we have repetition, which then continues indefinitely. An incomplete set of quaternion units (for example, $\boldsymbol{i}_3$ in the sixth alphabet) becomes equivalent to the algebra of complex numbers. Mathematically, we can see the process of the creation of the zero totality alphabets as one of conjugation, followed by repeated cycles of complexification and dimensionalization.

At the point where the cycle repeats, we have what can be recognised as a Clifford algebra – the algebra of 3-D space, where the vectors **i**, **j**, **k** are constructed from $i_2\, i_3$, $j_2\, i_3$, $i_2\, j_2\, i_3$, and i_1, j_1, $i_1\, j_1 = k_1$ and i_2, j_2, $i_2\, j_2 = k_2$ are (mutually commutative) quaternion algebras of the form ***i***, ***j***, ***k***.

$$(1, -1)$$
$$(1, -1) \times (1, i)$$
$$(1, -1) \times (1, i) \times (1, j)$$
$$(1, -1) \times (1, i) \times (1, j) \times (1, \mathbf{i})$$
$$(1, -1) \times (1, i) \times (1, j) \times (1, \mathbf{i}) \times (1, \mathbf{j})$$
$$(1, -1) \times (1, i) \times (1, j) \times (1, \mathbf{i}) \times (1, \mathbf{j}) \times (1, i) \ldots$$

Standard Clifford vector algebra notably produces these subalgebras in the reverse order to the universal rewrite system, which generates, in its first four alphabets, scalars, pseudoscalars, quaternions and vectors, along with the scalar subalgebras of pseudoscalars and quaternions.

Significantly, if we take all these algebras as *independently true*, and hence commutative, as the rewrite structure seems to suggest we should, since each is a complete description of zero totality, then we require an algebra that is a commutative combination of vectors, bivectors, trivectors and scalars, or vectors, quaternions, pseudoscalars and scalars. This turns out to be equivalent to the algebra of the sixth alphabet, a group structure of order 64 with elements:

± **i**	± **j**	± **k**	± *i***i**	± *i***j**	± *i***k**	± *i*	± 1
± ***i***	± ***j***	± ***k***	± *i****i***	± *i****j***	± *i****k***		
± **i*i***	± **i*j***	± **i*k***	± *i***i*i***	± *i***i*j***	± *i***i*k***		
± **j*i***	± **j*j***	± **j*k***	± *i***j*i***	± *i***j*j***	± *i***j*k***		
± **k*i***	± **k*j***	± **k*k***	± *i***k*i***	± *i***k*j***	± *i***k*k***		

These, as we have seen, generate an algebra which is isomorphic to that of the gamma matrices of the Dirac equation, as used in relativistic quantum mechanics. Mathematically, it represents the commutative combination of the first complete Clifford algebra (that of 3D space) with all its subalgebras. So, in many respects, the sixth alphabet represents a particularly significant stage in the rewrite process, the first at which the repetitive nature of the sequence becomes fully established, and this is, therefore, in effect the rewrite system order code.

Physical application

In physical terms, the first four alphabets suggest the successive emergence of descriptions of the universe in terms of the fundamental parameters mass, time, charge and space, which are described by the algebras created with these alphabets.

(1, –1)	real	mass
(1, –1) × (1, $\boldsymbol{i}_1$)	complex	time
(1, –1) × (1, $\boldsymbol{i}_1$) × (1, $\boldsymbol{j}_1$)	quaternion	charge
(1, –1) × (1, $\boldsymbol{i}_1$) × (1, $\boldsymbol{j}_1$) × (1, $\boldsymbol{i}_2$)	vector	space

The simultaneous existence of these independent descriptions of the universe can only be accomplished at the level of the sixth alphabet. This is also the first level at which the processes of conjugation, complexification and dimensionalization, which are successively introduced with the first three alphabets, can be finally incorporated into a repetitive sequence. It is like commutativity and anticommutativity, where the first continues generating possibilities to infinity, while the second closes after a limited finite sequence of three terms.

Since such terms as *R*, *A*, *B*, *C*, etc., are generated independently of each other by a unique process, no relative numerical 'values' are automatically attached to them, and it is possible, when we use one of the mathematical representations, to choose values for the terms of this alphabet in such a way that its self-product becomes zero and all subsequent alphabets are automatically zeroed as well. This is the nilpotent condition. It can be achieved in an infinite number of ways, each of which is unique, and it appears to apply both to quantum physics and to systems at much higher levels. The nilpotent condition defines its conjugate as a kind of mirror reflection of itself rather than an intrinsically new state, or defined new symbol – physically, we call it 'vacuum'. Significantly, the 'conjugate' state to the creation of the initial symbol 1 in binary arithmetic is not defined by a new symbol either, but by a kind of number 'vacuum'. In binary, 1 = 1, but –1 = ... 1111111111111111111.

Now, the rewrite process is a fractal one in which alphabets at one level can become the units at another. We can therefore imagine an interpretation of the rewrite structure in which every new symbol, say *A*, *B*, *C*, ... represents the creation of a new nilpotent alphabet. Since the units are now already defined as unique, the connections between them require only the generation of commutative algebraic coefficients, which can then continue to infinity. This suggests the creation of something like a Hilbert space of nilpotent alphabets, defined by a commutative Grassmann algebra. The coefficients increase with every new nilpotent alphabet, but they have no intrinsic significance, serving only to distinguish one nilpotent alphabet from another.

Entropy and information

This section, based on a recent paper with Peter Marcer (*IJCAS*, 2013), presents entropy as a concept that occurs as a result of the universal rewrite structure independently of any specific application to physics. A significant fact about the universal rewrite system is that, because every alphabet always includes the previous one, it is intrinsically irreversible. Every new alphabet is always necessarily an extension of the last. And it provides the simplest of all definitions of entropy. The constant bifurcation at every new creation produces 2^n components at the n^{th} stage. If we take the standard definition of entropy, where W is the number of available microstates, $S = k \ln W$, where k is only there because we have historically separated the measurement of temperature from that of energy on the basis of the properties of water. Really, temperature T has no meaning except as kT. We can't use T in any physics without k. So the true measure of standard entropy is $\ln W$, which is just a number. Now, we can simply redefine entropy, say S', as the number n, the order of the alphabet from any given beginning. Then, the previous measure of entropy S becomes $k\, n \ln 2$, just a choice of numerical factor: $S' = S / k \ln 2$. The number of equally probable microstates at the n^{th} stage is 2^n, and this gives a measure of the increasing complexification / disorder. This is true whatever the rewrite process is, and at whatever level – quantum physics, chemistry, biology, the brain, macrosystems of every kind. Because entropy always increases in our experience, we can use this as evidence that the rewrite system is always bifurcating, and the rate of entropy increase will be a measure of the bifurcation that actually occurs, and at the same time a standard clock. We can actually see how rewrite operates in any given system, and we can establish that it is a universal process.

Since digital logic and information in the computational sense may be seen as a specialised result of the universal rewrite procedure, this definition of entropy would accord with Shannon's view of 1948 that at least $kT \ln 2$ must be dissipated per transmitted bit of information, based on the assumption that communication in linear systems is done by waves, and that part of the energy of which the message consists must be dissipated, and so lost from the message. A paper by Rolf Landauer on 'Energy requirement in communication' (1987) begins with this then-accepted view. If the energy in the message is indeed dissipated, this may be a minimum dissipation, and also an information measure. Self-organization allows us to take this loss as in fact a measure, not simply of the loss, but of the information which enables the self-organization to proceed accordingly, and to treat the self-organization as if it is a complete binary tree of ordinal measure and has a natural canonical power series structure, which the universal rewrite construction shows must indeed be the case. At some scale, the measure of information transfer and entropy increase is determined directly by the level of the alphabet reached in the universal rewrite process.

Temperature is, of course, a global parameter, which shows up, for example, in the radiation formula as kT because it concerns the summation over each linearly independent wave phenomenon phase θ across the entire universe, i.e. from zero to infinity, which is how k, $\hbar$ and c become fundamental global scaling constants of the universal rewrite system. Of course, the information in nilpotent quantum mechanics is not lost, but rather accumulated to make the global process irreversible. It passes from nonlocal / indistinguishable to local / distinguishable. Translations from nonlocal to local effects cause time asymmetry because the local requires asymmetric, timelike and consequently irreversible solutions, whereas the nonlocal does not. All nonlocal processes also have local manifestations, and this appears to be how the time component of the vacuum manifests itself in local effects. Observability is an indication of an event in the present, for the future remains as part of the unobservable or nonlocal vacuum.

Referring to the bifurcation process which is the manifestation of the universal rewrite system, we can say that the rate at which it happens must be proportional to the (free) energy involved. The higher the energy, the higher the rate of bifurcation events. Near chaotic systems, involving nonlinearity and high connectivity of the components, transfer energy at near maximum efficiency, and so bifurcate rapidly, generating a correspondingly large measure of entropy. This is especially true of biological systems, which have evolved to be highly organized and composed of many interconnecting parts. Rapid information transfer and states of high entropy become strongly correlated. The process in general acts as a 'clock', with the time interval determined by the rate at which the available options are doubled. The fact that all natural systems are entropic, and irreversible in time, is evidence that all act in terms of the universal rewrite process.

The growth of a chaotic system, like an event in quantum mechanics, provides a perfect parallel with the universal rewrite system. It has been observed that in a typical situation leading to chaos, say the growth of an animal population, there comes a point at which, when the growth rate increases above a certain value, the equation produces a bifurcation between two possible outcomes. Further increases in the growth rate produce a series of further bifurcations of each bifurcation, at a frequency determined by a single scale factor for preserving self-similarity, the universal Feigenbaum number 4.669 … . This can be seen as a characteristic extension of an alphabet by the creation of a new one, exactly as in the 'create' process involved in universal rewrite.

The rewrite system describes the evolution of a *process* rather than a physically-defined system, though the process might itself require a bifurcation in the system. In effect, a near-chaotic system becomes subject to particularly rapid overall change because of its high degree of nonlinearity and interconnectivity, and the bifurcation occurs at the level of the whole system or the process applied to the whole system, rather than in only a part of it. When the process applies collectively, rather than to just part of the system, the expansion of the rewrite structure leads to a complete bifurcation or doubling of the options, and we would expect this to happen repeatedly. A system operating near chaos, with a high degree of nonlinearity and connectedness of its parts, will have a high efficiency in transferring free energy, and be subject to rapid development. The existence of a universal scale factor in the outcome may be taken as a consequence of the relative holism of a system or process on the edge of chaos.

The process also relates to the growth of complexity in natural systems. In principle, nature creates systems and objects whose most required state is self-annihilation with the rest of the universe, or the universal vacuum. Everything in nature constantly strives towards this end, resulting in local combinations of systems with the nearest available manifestation of a process tending towards this, e.g. fermions and antifermions. Since complete zeroing isn't possible locally, the result is complexity and combinations where the symmetry is imperfect or broken, and where the parts continually strive to make further connections. The nearest to 'stability' occurs when an object combines with an 'environment' that fulfils a maximal approximation to the desired vacuum connection (e.g. a nucleus in an atom, a bulk molecule in condensed matter, a cell in an organism, an aerobic bacterium being absorbed within an archaeum). To create and maintain this pseudo-vacuum environment requires a maximal number of connections and interactions to be made and maintained, and hence a maximal generation of entropy. Since the most 'desired' state is the combination (and consequent annihilation) of any system with the universal background, then the tendency of the evolution of the universe will always be in the direction of maximum entropy.

Duality and the factor 2

Both physics and mathematics encompass a fundamental principle of duality at their very bases. Essentially, this is how we create 'something from nothing'. If the ultimate thing that we wish to describe is really 'nothing', then we can only create 'something' as part of a dual pair, in which each thing is opposed by another thing which negates it. We can describe this mathematically in terms of the simplest known symmetry group (C_2), which is essentially equivalent to an object and its mirror image (or 'dual'), whose components are the positive and negative versions of a quantity which may be left undefined.

This has a surprisingly simple manifestation everywhere as the factor 2 or ½, which sometimes becomes equivalent to squaring or square-rooting. Of course, duality does not always imply equal status, and may incorporate *chirality*, as in the different status of + and – units in binary numbering. Duality, in addition, is not a single operation, and the process requires indefinite extension, in the form $C_2 \times C_2 \times C_2 \times \ldots$. If we begin with a unit, there will be an infinite series of 'duals' to this unit, via a process which must be carried out with respect to all previous duals (that is, that the entire set of characters generated becomes the new 'unit') and the total result must be zero at every stage.

There are essentially 3 dualities in the parameter group:

nonconserved / conserved   imaginary / real   commutative / anticommutative

Physical dualities always emerge from one or more of these, and they are often interchangeable.

Examples of the first include action + reaction, absorption + emission, radiation + reaction, potential v. kinetic energy, relativistic v. rest mass, uniform v. uniformly accelerated motion, and even rectangles v. triangles. It manifests itself in the use of pairs of *conjugate variables* to define a system, in both classical and in quantum physics.

Examples of the second include bosons v. fermions, electric and magnetic fields in Maxwell's equations, and space-like v. time-like systems. It allows transformations to be made, for example, between space and time representations. It is the one which occurs in relativistic contexts. A more subtle form of it occurs in the creation of massive particle states at the expense of components of charge.

Examples of the third include fermion + 'environment' (Aharonov-Bohm, Berry phase, Jahn-Teller, etc.), space-like v. time-like systems, particles v. waves, Heisenberg v. Schrödinger / the harmonic oscillator, quantum mechanics v stochastic electrodynamics / zero point energy; 4π v. 2π rotation, and all cases in which physical dimensionality or noncommutativity is involved.

So the factor 2 may be seen, for example, as a result of action and reaction (A); commutation relations (C); absorption and emission (E); object and environment (O); relativity (R); the virial relation (V); or continuity and discontinuity (X). The colour coding comes from the fundamental duality from which it emerges. Many of these explanations overlap in the case of individual phenomena, suggesting that they are really all part of some more general overall process:

Kinematics						V	X
Gases	A					V	
Orbits	A					V	X
Radiation pressure	A		E			V	
Gravitational light deflection					R	V	
Fermion / boson spin		C		O	R	V	
Zero-point energy	A	C				V	X
Radiation reaction	A		E		R	V	
SR paradoxes	A		E				

The factor 2 seems to work mainly in one direction. So, the constant terms produce effects which are 2 × the changing terms, the real produce ones which are 2 × the imaginary, and the discrete produces ones which are 2 × the continuous: the multiplication occurs in the direction which doubles the options. The first combines + and – cases where it remains constant; the second involves squaring imaginary parameters to produce real ones; and the third combines dimensionality and noncommutativity with discreteness, and so doubles the elements. However, doubling of options in one direction may be balanced by halving the options in another. The factor appears when we look at a process from a one-sided point of view, and the complete description of a system tends to lead to the overall elimination of the factor. The use of the factor 2 is a two-way process, and the system can only be described in complete terms by taking both the halving and doubling into account. Physical phenomena involving the factor tend to incorporate, in some form, the opposing sets of characteristics.

It will be interesting to look at some of the many cases, especially where there is obvious crossover. One is the Argand diagram, where 2 dimensions of space become a different duality between real and imaginary. In pure geometry, the area of a triangle, ½ × length of base × perpendicular height, translates into the graphical representation of kinematics as motion under uniform acceleration (a) as a straight-line v-t graph. Area under the graph is distance travelled, ½ vt. Under *uniform v*, the distance would have been given by the area of a rectangle, vt. The factor 2 distinguishes steady conditions and steadily *changing* conditions. Under uniform acceleration, we have the 'mean speed theorem', $s = ½ (u + v) t$ and $v^2 = u^2 + 2as$. A more general variation gives us ½ mv^2 for kinetic energy and $p^2 / 2m$. Even more generally, we have the virial theorem, $V = 2T$. Kinetic energy gives the action side of Newton's third law, potential energy concerns both action and reaction. Newton derived mv^2 / r for centripetal force, or mv^2 for orbital potential energy, by having the satellite object being 'reflected' off the circle of the orbit, in a polygon with an increasing number of sides, which, in the limiting case, becomes a circle. The imagined physical reflection, by doubling the momentum through action and reaction, then produces the potential, rather than kinetic, energy formula.

A real reflection of ideal gas molecules off the walls of a container produces a momentum doubling, which indicates steady-state conditions, though it is immediately removed by the fact that we have to calculate the average time between collisions ($t = 2a / v$) as the time taken to travel *twice* the length of the container (a). The average force then becomes the momentum change / time = $2\ mv / t = mv^2 / a$, and the pressure due to one molecule in a cubical container of side a becomes mv^2 / a^3, or mv^2 / V (volume), leading, for n molecules, to the direct pressure-density relationship, which we call Boyle's law ($P = \rho \bar{c}^2 / 3$, where $\bar{c}$ is the root mean square velocity). The kinetic behaviour of the ideal gas molecules is actually irrelevant to the derivation since the system describes a steady-state dynamics with positions of molecules constant on a time-average. Taking into account the three dimensions between which the velocity is distributed, the ratio of pressure and density (P / ρ) is derived from the *potential energy* term mv^2 for each molecule and is equal to one third of the average of the squared velocity, or $\bar{c}^2 / 3$.

Photons, which, unlike material particles, are relativistic objects, surprisingly behave in exactly the same way in a 'photon gas', producing a radiation pressure of the form $P = \rho c^2 / 3$, with the relativistic energy $E = mc^2$ behaving exactly like a classical potential energy term, and with no mysterious 'relativistic factor' at work. We can consider the photons as being reflected off the walls of the container in exactly the same way as the molecules of materials although the real process obviously also involves absorption and re-emission. In addition, even though free photons have no kinematics, it is also perfectly possible to treat photons acting under the constraint of certain forces as though they have. This is why it is possible to use the standard Newtonian escape velocity equation to derive the Schwarzschild limit for a black hole, with no transition to a 'relativistic' value.

As we have seen in lecture 9, we can derive the full double gravitational bending of light, using the kinetic equation, for orbit creation, rather than the potential energy equation used for steady-state conditions. Of course, we *can* use both special and general relativity to derive the effect, but the cause of the effect is independent of the particular version of physics we use to calculate it. In every case where a 'relativistic' correction (either special or general) seems to 'cause' the doubling of a physical effect, the relativistic aspect, like classical kinetic energy, is providing a way of incorporating the effect of *changing conditions* if we begin with the potential, rather than the kinetic, energy equation. Authors have had conflicting views about the doubling, but if we use the potential energy equation where the kinetic energy equation is appropriate, or *vice versa*, then we can find correct physical reasons for almost *any* additional term which doubles the effect predicted.

The same applies to the anomalous magnetic moment or, equivalently, the gyromagnetic ratio, of a Bohr electron acquiring energy in a magnetic field, and so ultimately spin ½. Here, using an equation for steady state conditions, we find only half the measured value, but a relativistic effect (the Thomas precession) doubles the value. But, if we use a kinetic energy equation for changing conditions, for example, at the instant we 'switch on' the field, then we get the correct value immediately. The equation, in fact, $\frac{1}{2} m(\omega^2 - \omega_0^2) = e\omega_0 rB$, is only a disguised version of the kinematic equation $v^2 - u^2 = 2as$. Spin is neither relativistic nor quantum in origin, though it can be derived using relativistic or quantum theories – we have already seen that spin ½ can be derived from the anticommutativity of **p** as much as from the addition of the Thomas precession. When it is derived from the Schrödinger equation, it is simultaneously derived from the classical kinetic energy term, and, at the same time, produced by the anticommuting nature of the momentum operator.

Applying the Schrödinger equation to the quantum harmonic oscillator requires a *varying* potential energy term, ½ $m\omega^2x^2$, taken directly from the classical kinetic energy term ½mv^2. The ½ in this expression leads by direct derivation to the ½ in the expression for the ground state or 'zero-point' energy of the system. The same zpe relates to $\hbar$ / 2 in the Heisenberg Uncertainty Principle, though the factor ½ there also generated by anticommutativity in the same way as for electron spin.

The fact that the factor 2 in spin states, which establishes a distinction between bosons and fermions, can be shown to originate ultimately in the virial relation between kinetic and potential energies, has fundamental significance with respect to the fermion / vacuum duality. Kinetic energy is always associated with rest mass m_0, undergoing a continuous change, potential energy is associated with 'relativistic' mass because this term is actually *defined* through a potential energy-type expression ($E = mc^2$), and this implies an equilibrium state with an 'environment', requiring a discrete transition for any change.

The particle and its 'environment' are two 'halves' of a more complete whole. For a material particle, when we expand its relativistic mass-energy term (mc^2) to find its kinetic energy (½ m_0v^2), we either take the relativistic energy conservation equation as a 'relativistic' mass or potential energy equation, incorporating the particle and its interaction with its environment, and then quantize to a Klein-Gordon equation, with integral spin; or, we separate out the kinetic energy term using the rest mass m_0, by taking the square root of $E^2 = \gamma^2 m_0^2c^4$ to obtain $E = m_0c^2 + m_0v^2$ / 2 ..., and, if we choose, quantize to the Schrödinger equation, and spin ½. The ½ occurs in the act of square-rooting, or the splitting of 0 into two nilpotents in the Dirac equation; the ½ in the nonrelativistic Schrödinger approximation is a manifestation of this which we can trace through the ½ in the relativistic binomial approximation. If we go directly to the Dirac equation to obtain the spin ½ term, we see that the same result emerges from the behaviour of the anticommuting terms; the anticommuting property is a direct result of taking the quaternion state vector as a nilpotent. So the anticommuting and binomial factors have precisely the same origin.

The connection between spin and statistics becomes obvious: square-rooting the scalar (and, so, commutative) operator, associated with an integral spin state, to produce two ½-spin states requires the introduction of quaternion operators which are necessarily anticommutative. So particles with integral spins (bosons) follow the Bose-Einstein statistics associated with commutative wavefunctions, while particles with ½-integral spins (fermions) follow the Fermi-Dirac statistics associated with anticommutative ones.

The factor 2 also links the continuous with the discontinuous. Expressions involving half units of $\hbar$, representing an average or integrated increase from 0 to $\hbar$, are characteristic of continuous aspects of physics, while those involving integral ones are characteristic of discontinuous aspects. The Schrödinger theory is an example of a continuous option, while the Heisenberg theory is discontinuous. Stochastic electrodynamics (SED), which is based on the existence of zero-point energy of value $\hbar\omega$ / 2, is another completely continuous theory, which has developed as a rival to the purely discrete theory of the quantum with energy $\hbar\omega$.

Again, the Klein-Gordon equation, based on potential energy, is effectively a 'discrete' one, in the space-time sense, whereas the Dirac equation, like the Schrödinger equation, based on kinetic energy, is effectively 'continuous'. 'Discrete' (or steady state) energy equations employ terms which are twice the size of those describing 'continuous' (or changing) conditions; and the distinction is transferred into quantum mechanics with the quantum energy equations which are based on classical ones. The choice between the factors 1 and ½ for spin, and other related quantities, seems to be made at the same time as that between timelike and spacelike equations, and between discrete and continuous physics. In fact, the ½ ratio between the spins of fermions and bosons provides a classic instance in which there are alternative explanations using *any one* of the three fundamental dualities.

duality	**method**
conserved / nonconserved	**potential energy /** kinetic energy
real / complex	**nonrelativistic /** relativistic
nondimensional / dimensional	**commutative /** anticommutative

There is, of course, only one system, whatever the description, and both options have to incorporate the alternative in some way. Though a single duality separates alternative theories, such as Heisenberg and Schrödinger, or quantum mechanics and stochastic electrodynamics, it is invariably open to more than one interpretation because each pair of parameters is always separated by two distinct dualities, and the separate interpretations ultimately act together when we consider a phenomenon in relation to its place in the overall 'environment' of the physical universe.

The Schrödinger approach is a continuous one based on ½$\hbar$, but incorporates discreteness (based on $\hbar$) in the process of measurement – the 'collapse of the wavefunction'. The Heisenberg approach, assumes a discrete system based on $\hbar$, but incorporates continuity (and ½$\hbar$) in the process of measurement, via the uncertainty principle and zero-point energy. There is always a route by which ½$\hbar\omega$ in one context can become $\hbar\omega$ in another. So, in black-body radiation, the spontaneous emission of energy of value $\hbar\omega$ combines the effects of ½$\hbar\omega$ units of energy provided by both oscillators and zero-point field. A fermionic object on its own shows changing behaviour, requiring an integration which generates a factor ½ in the kinetic energy term, and a sign change when it rotates through 2π, while a conservative 'system' of object plus environment shows unchanging behaviour, requiring a potential energy term which is twice the kinetic energy.

The ½ $\hbar\omega \rightarrow \hbar\omega$ transition for black body radiation can also be explained in terms of radiation reaction, which is connected again with the distinction between the relativistic and rest masses of an object. Rest mass effectively defines an isolated object, with *kinetic* energy. *Relativistic* mass, on the other hand, already incorporates the effects of the environment. For a photon, which has no rest mass, and only a relativistic mass, the energy mc^2 behaves exactly like a classical potential energy term, as when a photon gas produces the radiation pressure $\rho c^2 / 3$. We take into account both action and reaction because the doubling of the value of the energy term comes from doubling the momentum when the photons rebound from the walls of the container, or, alternatively, are absorbed and re-emitted. Exactly, the same thing happens with radiation reaction, thus explaining an otherwise 'mysterious' doubling of energy from ½$h\nu$ to $h\nu$. In a more classical context, Feynman and Wheeler find a doubling of the contribution of the retarded wave in electromagnetic theory, at the expense of the advanced wave, by assuming that the vacuum behaves as a perfect absorber and reradiator of radiation. This is effectively the same as privileging matter over vacuum, or one direction of time.

Radiation reaction is, basically just another version of Newton's third law. For the anomalous magnetic moment of the electron, many of the same results are also explained by special relativity, but the correct magnetic moment for the electron is obtained, without relativity, by treating the transmission of light as a two-step process involving absorption and emission (C. K. Whitney), which is, again, action and reaction. Two-step processes also remove those SR paradoxes which involve apparent reciprocity (Whitney); in effect, SR, by including only one side of the calculation, effectively removes reciprocity, and so leads to such things as asymmetric ageing in the twin paradox.

The 'environment' can apply to either a material or vacuum contribution. The duality here connects with the boson / fermion distinction and the spin 1 / ½ division, supersymmetry, vacuum polarization, pair production, renormalization, *zitterbewegung*, etc. The halving of energy in 'isolating' the fermion from its vacuum or material 'environment' is the same process as mathematically square-rooting the quantum operator via the Dirac equation. Energy principles determine that all fermions, in whatever circumstances, may be regarded either as isolated spin ½ objects or as spin 1 objects in conjunction with some particular material or vacuum environment, or, indeed, the 'rest of the universe'; and fermions with spin ½ automatically become spin 1 particles when taken in conjunction with their environment, whatever that may be.

In the Berry phase examples (Jahn-Teller effect, Aharonov-Bohm effect, etc.), the spin-½, ½-wavelength-inducing nature of the fermionic state is a product of discreteness in both the fermion (and its charge) and the space in which it acts. The very act of creating a discrete particle requires a splitting of the continuum vacuum into *two* discrete halves, as with a rectangle into two triangles or (relating the concept of discreteness to that of dimensionality) two square roots of 0. That the doubling mechanism also applies in purely mathematical, as well as in physical, contexts is evident from the topological explanation of the Aharonov-Bohm effect, though the physical and mathematical applications must ultimately have the same origin.

The very concept of duality also implies that the actual processes of counting and generating numbers are created at the same time as the concepts of discreteness, nonconservation, and orderability are separated from those of continuity, conservation, and nonorderability. The mathematical processes of addition and squaring are, in effect, 'created' at the same time as the physical quantities to which they apply, while all the other fundamental mathematical concepts and processes (e.g. the Dedekind cut) are, in some way, defined by dualling. The factor 2 thus expresses dualities which are fundamental to the creation of both mathematics and physics, and duality provides a philosophy on which both can be based.

Square-rooting and halving in mathematics have an intimate relationship, which is manifested physically in the relation between vector spin terms of bosons and fermions and their respective uses of double or single nilpotent operators, in addition to the halving approximation used to find the kinetic energy term in the binomial expansion for relativistic mass. This relationship is determined entirely by the fact that 3-D Pythagorean addition is a dualistic process, with a numerical doubling arising from noncommutativity, and this applies to both the vector operators used for space and momentum, and the quaternion operators used in the Dirac nilpotent.

A few cases of the general idea of duality are of particular interest. At 0.25, the idealised electroweak mixing parameter $\sin^2\theta_W$ calculated from $Tr\ (t_3^2)\ /\ Tr\ (Q^2)$ becomes equivalent to making the weak charge value twice that of the electric charge at the energy at which the mixing takes place. The ultimate reason for this is the fact that the quantum number for weak charge (t_3) is half that for electric charge (Q), because of the $SU(2)$ nature or dipolarity introduced by the complexifying factor i, while the compensating factors of weak isospin and single-handedness contrive to halve the respective numbers of electric and weak states simultaneously – which is, of course, no coincidence, because both are aspects of weak dipolarity. Ultimately, then, the weak charge has, ideally, twice the magnitude of the electric charge at the energy at which they are mixed because the weak charge is dipolar, and the weak charge is dipolar because it is complexified. Could the magnitude of the *strong* charge, at the same energy of interaction, be *twice that of the weak charge*, through a further doubling effect due to dimensionalization? It is certainly of this order at the energy of the electroweak scale. Among other less obvious examples of the factor 2 dual process are the magnetic flux quantum term, $\hbar / 2e$, in the time-dependent voltage of the Josephson effect, $U(t) = (\hbar / 2e)\ \partial\phi / \partial t$, produced by a *changing* phase difference, ϕ, and the pair production producing only one observable fermion in Hawking radiation.

As an entertainment, we can look at the famous equation $e^{i\pi} = -1$ and see that it is a remarkable case of a combination of all three dualities! If we take mass as being conserved, real and nondimensional, and with positive real unit 1, we will see that e is defined by differentiation (nonconserved), i is imaginary, and π is defined by 3-dimensionality, and that these 3 act together to produce a collective dual to positive unit 1. (The combination has parallels to *CPT* in physics, which, of course, combines the properties of the 3 quantities which are dual to mass: charge, space and time.)

In terms of the mathematical structure generated by the universal rewrite structure, it would be possible to classify the physical phenomena involving the factor 2 as resulting from the three distinct mathematical processes, which can be identified as *conjugation*, *complexification* and *dimensionalization*, and which are manifested, respectively, through opposite signs (or equivalent), the distinction between real and imaginary components, and the introduction of cyclic dimensionality.

action and reaction	conjugation
commutation relations	dimensionalization
absorption and emission	conjugation
object and environment	conjugation
relativity	complexification
electroweak mixing	complexification
the virial relation	conjugation
continuity and discontinuity	conjugation / dimensionalization

In t may be that, in larger scale systems, which particular duality is invoked depends on the point at which the first zero-totality alphabet is defined.

Overall, the factor 2 appears as either the link between the continuous and discrete physical domains, or between the changing and the fixed, or the real and imaginary (orderable and nonorderable), the three dualities of the Klein-4 parameter group, and, in every physical instance, between more than one of these. Duality seems to be the necessary result of any attempt to create singularity. The infinite imaging of the fermion state in the vacuum and the nonlocal connection between all the state vectors in the entire universe, or infinite entanglement of all nilpotent fermion states, described mathematically in terms of Hilbert space, provide an extension of the dualling process to infinity. Ultimately, it would seem, duality is not merely a 'component' of physics but an expression of the fundamental nature of physics itself.

Anticommutativity: the factor 3

3-dimensionality, one of the most profound and fundamental concepts in physics, has its origin in ideas of anticommutativity, which may be antecedent to the concept of number. It seems to be responsible for all discreteness in physical systems, and in particular for quantization, as well as for symmetry breaking between the forces, for many significant aspects of particle structure, and for most of the manifestations of the number 3 that are considered fundamental in physics. The Dirac equation is specially structured to accommodate it. We have 3 dimensions of space, 3 nongravitational interactions, 3 fundamental symmetries (*C*, *P* and *T*), 3 conserved dynamical quantities (momentum, angular momentum and energy), 3 quarks in a baryon, 3 generations of fermions (which can be related to *C*, *P* and *T*).

No other dimensionality, not even that of '4-dimensional' space-time, has any fundamental physical significance. The connection between space and time is basically 3-dimensional (***k***, ***i***, ***j***), and not privileged with respect to mass and charge. Time is *not* part of space, but of *another* 3-dimensionality, though the differences can be masked when we take the scalar product. An ordinary connection between space and time, not mediated by this second 3-dimensionality, leads to wave-particle duality, where one parameter has to take in the other's physical aspects. Even within the higher dimensionalities of the Dirac algebra, the nilpotent structure shows its fundamental 3-dimensionality, and it is this inherent 3-dimensionality which allows us to develop a fully renormalizable formal theory of quantum gravitational inertia.

Just as there are two 3-dimensional structures that are fundamental in nature, so there are also two manifestations of 3-dimensionality, the nonconserved and the conserved. The nonconserved has an unbroken symmetry, with axes not separately identifiable; in the conserved case, the symmetry is broken or chiral, with axes separately identifiable. The symmetry-breaking always has the same structure: one term is complexified; one term is associated with an unbroken 3-dimensionality; and the remaining term is purely scalar. The whole structure is invariably nilpotent, reflecting the conserved nature, and either the scalar or the complexified (pseudoscalar) term becomes redundant, except as a 'book-keeper'.

Our quantized, i.e. 3-dimensional, picture denies us the opportunity of representing time as a fourth dimension, denying it status as a physical observable. In a *3-dimensional* theory, time occupies the place of the 'book-keeper', as energy does in the Dirac state, the quantity which preserves conservation or conjugation, but adds only the information of + or –. We only know the direction of the sequence that preserves causality, not a *measure* of time in the same sense as we measure space, in the same way as energy only tells us whether the system is a fermion or antifermion. This fact is well known as a stumbling block to proponents of a quantum theory of gravity, which automatically incorporates time as a physical fourth dimension. The fact that the number system we use in mathematics may have a 3-dimensional origin is of profound significance. It means that we can't arbitrarily choose the number of dimensions we apply to quantities like space and time without contradicting the principles on which these concepts, and related ones, such as quantization and conservation, were founded.

The separate roles for the three axes in a 3-dimensional system with identifiable components has a remarkable similarity with the processes involved in creating the infinite algebra of the rewrite system. The role of ***j*** is essentially that of complexification, the beginning of a new and as yet incomplete new quaternion system. The role of ***i*** is to introduce dimensionalization, while ***k*** is restricted to the 'book-keeping' role of conjugation or conservation. These also run parallel to the roles of scalar, vector and pseudoscalar quantities (which an extra *i* factor has transformed from the sequence pseudoscalar, quaternion, scalar). It is this parallelism which makes it possible to create a closed parameter system with zero totality and in-built repetition.

pseudoscalar	quaternion	scalar	(1)
scalar	vector	pseudoscalar	(2)
mass	space	time	
m	$\mathbf{p}$	E	
τ	$\mathbf{r}$	t	
e	s	w	
C	P	T	
j	***i***	***k***	
magnitude	direction	orientation	
complexification	dimensionalization	conjugation	
complexification	dimensionalization	conservation	

The 'dimensional' term here is in the second column and the 'book-keeping' term in the third. (1) = (2) × *i* and (2) = (1) × *i*. It may be that we can also include momentum-angular momentum-energy and space translation-space rotation-time translation. The last row refers to the properties of the parameter group.

Symmetry and self-organization

Physics uses many groups, geometries, algebras, and displays many symmetries, but the only pure symmetries that matter are all based on 2 and 3, and the most fundamental broken symmetries are based on 5. The table of numbers related to particles in lecture 9 includes practically all the integers that are fundamentally important in physics and in many other scientific areas, including biology, and they are all based on 2, 3 and 5, and these, in turn, all emerge from the universal rewrite system. It may be that this system is a general description of process which applies to mathematics, computation, physics, chemistry, biology, and any self-organizing system. Investigations in a considerable number of areas appear to show signs of the same structures and symmetries, in exactly the way we said that nature made it possible for us to understand things way outside our experience.

The nilpotent relation between the defined system and the rest of the universe, which emerges in the nilpotent form of quantum mechanics could possibly be an indicator of a process of self-organization which occurs at increasing levels of complexity. The particular characteristics of this type of self-organizing include double 3-dimensionality; a 5-fold broken symmetry; geometric phase; uniqueness of the objects and unique birthordering; irreversibility; dissipation; chirality, a harmonic oscillator mechanism, *zitterbewegung*; fractality of dimension 2; the holographic principle and quantum holography. Research is ongoing in this area, with special (but not unique) reference to biology, where it is clear that some sort of information process is a bigger driver than the specific chemical processes involved.

I hope that this lecture series has shown that the Foundations of Physics is a very different subject from anything we might previously have imagined. It requires its own methodology and philosophy, and even to a certain extent its own mathematics. It requires, in addition, a great deal of inductive thinking. Yet it is also rigorous and leads to mathematically and experimentally testable results. Besides creating its own results, it also adds significantly to physics worked out at a greater level of complexity. Its methods and symmetries can suggest answers where none are available by any other means, and resolve anomalies where this had previously seemed impossible. I would like to think that anyone who has followed this series of lectures will see that the possibilities opened up are almost limitless with many opportunities for significant research.

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