Foundations of Physical Law
10 Return to symmetries

Peter Rowlands

THE UNIVERSITY
of LIVERPOOL

A universal rewrite system

The seemingly unbroken consistency of the symmetries described in lectures 3 and 4 gives a strong indication that they are close to a true foundational level in physics, and the apparent applicability of foundational methods in resolving problems in relatively complex parts of physics gives support to this.

If this is a valid inference, there is still one major question to be answered: does Foundations of Physics have a foundation? And, if so, what kind?

Clearly, it would have also to act in some way as a foundation for mathematics, and possibly for a more general approach to information processing.

Rewriting and rewrite systems

The twentieth century success of digital computing made a number of people think that nature might be structured on an information system, similar to the workings of a computer. So, can this be true here?

Modern computers are based on the Turing machine, digital logic, and the rewrite or production system, which is essentially the basis of programming and algorithmic processes of any kind. So, we have a pretty clear idea of how they work. A conventional rewrite system has 4 fixed components:

alphabet
rewrite rules (productions)
a start 'axiom' or symbol
stopping criteria

Example: Fibonacci numbers

As an example we can devise a process for generating Fibonacci numbers. There are two rules: p1: A → B; p2: B → AB. We begin with generation 0, and a single symbol A. Rule p1 then tells us to replace A with B. In the next generation, B becomes AB, and the process then repeats indefinitely:

N=0	A	length 1
N=1	→B	1
N=2	→ AB	2
N=3	→ BAB	3
N=4	→ ABBAB	5
N=5	→ BABABBAB	8
N=6	→ ABBABBABABBAB	13
N=7	→ BABABBAB...	

Example: Fibonacci numbers

It may be significant that the rules A → B and B → AB reproduce the structure of 3-D (quaternion) algebra

$$\boldsymbol{i} \rightarrow \boldsymbol{j} \qquad \boldsymbol{j} \rightarrow \boldsymbol{i}\,\boldsymbol{j} = \boldsymbol{k}$$

and that a string like BABABBABABBABBABABBAB appears to be creating a fractal-like structure in 3-D space, but situated in the AB or ***ij*** plane, as in holography.

The logarithmic spiral then becomes a way of expressing 3-dimensionality in the plane, with the increasing length of the intervals substituting for penetration into the third dimension.

Universal system / alphabet

If we had a system in which all four elements – alphabet, start object, rules, and stopping criteria – could be varied, the rewrite structure and alphabet would become universal, and perhaps self-generating.

The algebraic structure which we believe lies at the foundations of physics, has the appearance of being generated by a process, possibly an information process.

Can we use the concept of totality zero to find the process or the algorithm? On the basis of the foundational principles we have declared in the first lecture, should we expect it to be universal?

Universal system / alphabet

No universal rewrite system being known, Bernard Diaz and I set out to see if nature actually supplied one, and if it looked like what was driving physics and mathematics as set out in the Klein-4 parameter group and the nilpotent package and algebraic structure used for fermionic physics.

In this search it is important not to assume any prior knowledge of mathematics, not even the natural numbers.

This is very difficult to do, as counting has become a basis for virtually everything we do, and, even if we avoid actual counting, is difficult to avoid the use of it in our language.

Universal system / alphabet

We assume that the universe, or any alphabet which it contains, is always a zero totality state, with no unique description, and so infinitely degenerate.

This means that we have to continually regenerate the alphabet, in such a way that it is always new, but there is no limit or stopping criteria, as the new state created is always another nonunique zero totality.

It is a kind of *zero attractor*.

A non-zero deviation from 0 (say R) will always incorporate an automatic mechanism for recovering the zero (say the ‘conjugate’ R^*), but the zero totality which results, say (R, R^*), will not be unique, and will necessarily lead to a new structure.

Universal system / alphabet

The characteristics of the process which emerge from this include self-similarity, scale-independence, duality, bifurcation, and holism.

The process differs from other rewrite processes in having no fixed starting or ending point, and an alphabet and production rules that are endlessly reconstructed during the process. So, it fulfils our conditions for universality.

The self-similarity is a necessary consequence of the lack of a fixed starting point. It suggests that, if there are physical applications at one level, then there are likely to be applications also at others. As we scale up from small to larger systems, we can imagine that some principle such as the renormalization group takes effect to maintain the form of the structures generated by the rewrite process.

Universal system / alphabet

We ensure that a structure or alphabet is new by defining the position of all previous structures within it as subalphabets. The process will then continue indefinitely.

Effectively, the process involves defining a series of *cardinalities*. Successive alphabets absorb the previous ones in the sequence, so creating a new cardinality.

The cardinalities are like Cantor’s cardinalities of infinity, but are cardinalities of zero instead.

Universal system / alphabet

From the point of view of the observer, i.e. someone ‘inside’ the system (‘universe’, ‘nature’, ‘reality’), we have to start from (R, R^*), which is the minimum description of a zero totality alphabet (or of a zero totality universe in physical terms).

The successive stages are all zeros, as we go from one zero cardinality or totality to the next, and we ensure that they are cardinalities by always including the previous cardinality or alphabet. So (R, R^*, A, A^*), for example, includes (R, R^*).

We may start at any arbitrary zero-totality alphabet but there is no natural beginning or end to the process. Because all the stages are cardinalities or zero totality alphabets, the process is always holistic. We have to include everything.

Universal system / alphabet

A convenient though not unique way of representing the process is by a ‘concatenation’ or placing together, with no algebraic significance, of any given alphabet with respect to either its components or subalphabets or itself.

Since an alphabet is defined to be a cardinality, then anything other than itself must necessarily be a ‘subalphabet’ and the concatenation will produce nothing new. Only concatenation of the entire alphabet with itself will produce a new cardinality or zero totality alphabet.

Universal system / alphabet

It is convenient to represent these two aspects of the process by the symbols

$\Rightarrow$, for *create*, in which every alphabet produces a new one which incorporates itself as a component,

and $\rightarrow$, for *conserve*, which means that nothing new is created by concatenating with a subalphabet.

conserve: (subalphabet) (alphabet) $\rightarrow$ (alphabet)

there is nothing new

create: (alphabet) (alphabet) $\Rightarrow$ (new alphabet)

a zero totality is not unique

Universal system / alphabet

The process is simultaneously recursive, creating everything E (all symbols) at once, and iterative, creating a single symbol only.

It is also fractal and can begin or end at any stage.

Since we intend that it should describe or create both time and 3D space, we can think of it as prior to both.

The create and conserve aspects must also be simultaneous; we only know which new alphabet will emerge when we have ensured that all possible concatenations with subalphabets yield only the alphabet itself.

Universal system / alphabet

Suppose, we then start with a zero totality alphabet with the form (*R*, *R**). We have to assume that this is not necessarily the beginning, though it is the point where we as observers start from.

So, this already 'bifurcated' state will have started from a previous alphabet, which we assume we can't access directly, because we have no structure for it.

If we describe this as *R*, then the * or *R** character creates the 'doubling' process. (Although we now see this in terms of the number 2, we have to imagine that we have not yet invented this method of labelling, and the same applies to the term 'duality'.)

Universal system / alphabet

Before we create (*R*, *R**), we have to assume that (*R*) is a zero totality alphabet, but it is a zero to which we have no access.

In effect, we are trying to posit an ontology that exists before the epistemology or observation, begins with (*R*, *R**). So, we assume that it must happen without being able to observe it.

Universal system / alphabet

Applying the conserve process $\rightarrow$ to concatenate (R, R^*) with its subalphabets should produce nothing new. No concept of 'ordering' is needed in this process, but each term must be distinct. So

$$(R)\ (R, R^*) \rightarrow (R, R^*)$$
$$(R^*)\ (R, R^*) \rightarrow (R^*, R) \rightarrow (R, R^*)$$

It follows immediately that these concatenations lead to rules of the form:

$$(R)\ (R) \rightarrow (R)\ ;\ (R^*)\ (R) \rightarrow (R^*)\ ;$$
$$(R)\ (R^*) \rightarrow (R^*)\ ;\ \ (R^*)\ (R^*) \rightarrow (R)$$

Universal system / alphabet

The next stage is to show that the zero-totality alphabet (R, R^*) is not unique, and that a concatenation with itself will produce a new zero-totality alphabet.

Our first suggestion might be something like (A, A^*), but, with the terms undefined, this is indistinguishable from (R, R^*),

and so the only way to ensure that the new alphabet is distinguishable from the old is by incorporating the old one, and we need to do this in such a way that we ensure that the subalphabets yield nothing new. So we try

$$(R, R^*)\,(R, R^*) \Rightarrow (R, R^*, A, A^*) \qquad (1)$$

Universal system / alphabet

Having used the 'create' mechanism, we now apply the conserve operation ($\rightarrow$) to this new alphabet, and concatenate with the subalphabets. So

$$(R)\ (R, R^*, A, A^*) \rightarrow (R, R^*, A, A^*) \rightarrow (R, R^*, A, A^*)$$
$$(R^*)\ (R, R^*, A, A^*) \rightarrow (R^*, R, A^*, A) \rightarrow (R, R^*, A, A^*)$$
$$(A)\ (R, R^*, A, A^*) \rightarrow (A, A^*, R^*, R) \rightarrow (R, R^*, A, A^*)$$
$$(A^*)\ (R, R^*, A, A^*) \rightarrow (A^*, A, R, R^*) \rightarrow (R, R^*, A, A^*)$$

As before the order of the terms is different for each operation, as we require, but the total is the same, and we soon quickly realise that (R, R^*) and (A, A^*) can only be different if

$$A\,A \rightarrow R^*, \text{ etc., while } R\,R \rightarrow R.$$

Universal system / alphabet

Duality is intrinsic to the process. The operation () () $\Rightarrow$ (,) describes how we go from one zero totality alphabet – or description of the universe – to the next one up.

The (,) is a kind of 'doubling' or 'bifurcation'.

So we could write the result of (1) in the form (R, R*, A, A*), and represent (R, R*) (R, R*) as a kind of doubling, to create a new cardinality (R, R*, A, A*), almost like transforming the second (R, R*) into (A, A*).

Universal system / alphabet

The next stage presents a new problem, for

$$(R, R^*, A, A^*)\ (R, R^*, A, A^*) \Rightarrow (R, R^*, A, A^*, B, B^*)$$

would fail the application of the conserve mechanism ($\rightarrow$) by introducing new concatenated *terms* like AB, AB^*, which lie outside the alphabet. This means that we must include these in advance, as in

$$(R, R^*, A, A^*)\ (R, R^*, A, A^*) \Rightarrow (R, R^*, A, A^*, B, B^*, AB, AB^*).$$

A universal rewrite system

The question that remains is: does this new alphabet satisfy all our requirements, when we concatenate separately with (R), (R^*), (A), (A^*), (B), (B^*), (AB), (AB^*)? The process is straightforward for the first six concatenations:

$(R)\ (R, R^*, A, A^*, B, B^*, AB, AB^*) \rightarrow (R, R^*, A, A^*, B, B^*, AB, AB^*)$
$(R^*)\ (R, R^*, A, A^*, B, B^*, AB, AB^*) \rightarrow (R^*, R, A^*, A, B^*, B, AB^*, AB)$
$(A)\ (R, R^*, A, A^*, B, B^*, AB, AB^*) \rightarrow (A, A^*, R^*, R, AB, AB^*, B, B^*)$
$(A^*)\ (R, R^*, A, A^*, B, B^*, AB, AB^*) \rightarrow (A^*, A, R, R^*, AB^*, AB, B^*, B)$
$(B)\ (R, R^*, A, A^*, B, B^*, AB, AB^*) \rightarrow (B, B^*, AB, AB^*, R^*, R, A, A^*)$
$(B^*)\ (R, R^*, A, A^*, B, B^*, AB, AB^*) \rightarrow (B^*, B, AB^*, AB, R, R^*, A^*, A)$

A universal rewrite system

But concatenations of the *concatenated terms*, (*AB*) and (*AB**), on themselves and on each other appear to leave us with two options, which we can describe as ‘commutative’ and ‘anticommutative’:

$$(AB)\,(AB) \rightarrow (R) \qquad (\textit{commutative})$$
$$(AB)\,(AB) \rightarrow (R^*) \qquad (\textit{anticommutative})$$

In fact, however, there is no choice, for *only the anticommutative option* produces something new.

Labelling is arbitrary in the rewrite structure, and so the labels *A* and *B* alone cannot distinguish these terms from each other – this can only be done if they produce distinguishable outcomes.

A universal rewrite system

The commutative option leaves *A* and *B* indistinguishable except by labelling, and so does not extend the alphabet. We are obliged to default on the anticommutative option, which means that the last two concatenations become:

$(AB)\ (R, R^*, A, A^*, B, B^*, AB, AB^*) \rightarrow (AB, AB^*, B, B^*, A, A^*, R^*, R)$
$(AB^*)\ (R, R^*, A, A^*, B, B^*, AB, AB^*) \rightarrow (AB^*, AB, B^*, B, A^*, A, R, R^*)$

This solution for *A* and *B* cannot be repeated to include new terms, such as (*C*), (*D*),..., when we extend alphabet to higher stages because an inconsistency will always reveal itself at some point in the analysis.

A universal rewrite system

Anticommutativity produces a closed ‘cycle’ with components (*A*, *B*, *AB*) and their conjugates (*A**, *B**, *AB**), and prevents further terms, such *C*, *D*, etc., from anticommuting with them in a consistent manner.

However, successive cycles of the form (*A*, *B*, *AB*), (*C*, *D*, *CD*), etc., can be introduced into the structure, if they commute with each other, and this can be continued indefinitely.

All of the terms then have a unique identity *because they each have a unique partner*, and the successive alphabets can be seen as a regular series of identically structured closed anticommutative cycles, each of which commutes with all the others.

A universal rewrite system

This structure is familiar to us in the form of the infinite series of finite (binary) integers of conventional mathematics, each alphabet representing a new integer.

We can regard the closed cycles as an infinite ordinal sequence, and so establishing for the first time in this process both the number 1 and the binary symbol 1 of classical Boolean logic as a conjugation state of 0, with the alphabets structuring themselves as an infinite series of binary digits.

Mathematics and digital logic become emergent properties of a rewrite process which has no specific defined starting point, and can be reconstructed endlessly in a fractal manner, with self-similarity at all stages and a zero attractor.

A universal rewrite system

The universal rewrite system is a pure description of process with many representations.

It can, for example, be presented as a series of ‘doublings’ or ‘bifurcations’, analogous to the initial creation of (R, R^*),

even when they represent ‘complexification’ (the introduction of a new anticommutative cycle) or ‘dimensionalization’ (the closing of the cycle) rather than conjugation (the zeroing process used in the first alphabet).

A universal rewrite system

So, one way of writing

$$(R, R^*, A, A^*)\ (R, R^*, A, A^*) \Rightarrow (R, R^*, A, A^*, B, B^*, AB, AB^*)$$

would be as

$$(R, R^*, A, A^*)\ (R, R^*, A, A^*) \Rightarrow (R, R^*, A, A^*, \quad B, B^*, AB, AB^*)$$

in which we retain the old alphabet (R, R^*, A, A^*) and a dual (B, B^*, AB, AB^*) formed by some process, as we did with R and R^*.

In practical terms, we introduce a new character (B).

Mathematical representation

A significant representation is the mathematical one, for the effective creation of a discrete integer system means that, by applying this to the original terms (*R*, *A*, *B*, etc.), we can also generate an entire arithmetic and an algebra.

Once we have generated integers, the rest of the constructible number system will follow automatically, along with arithmetical operations.

At the same time, application of the constructed number systems to the undefined state with which the process began suggests that this state, which is not intrinsically discrete, can be interpreted in terms of a continuity of real numbers in the Cantorian sense.

Mathematical representation

In principle, discreteness appears in the construction only when we introduce anticommutativity or ‘dimensionality’, and specifically 3-dimensionality.

Physically, 3-dimensionality, or anticommutativity, becomes the ultimate source of discreteness in a zero totality universe, and we observe, in all cases, that 3-dimensionality requires discreteness, and discreteness requires 3-dimensionality.

The iterative process in symbolic form

	0	$\mathbf{\Delta_a}$	$\mathbf{\Delta_b}$	$\mathbf{\Delta_c}$	**…**	$\mathbf{\Delta_n}$
0	00	$0\Delta_a$	$0\Delta_b$	$0\Delta_c$		$0\Delta_n$
$\mathbf{\Delta_a}$	$\Delta_a 0$	$\Delta_a\Delta_a$	$\Delta_a\Delta_b$	$\Delta_a\Delta_c$		$\Delta_a\Delta_n$
$\mathbf{\Delta_b}$	$\Delta_b 0$	$\Delta_b\Delta_a$	$\Delta_b\Delta_b$	$\Delta_b\Delta_c$		$\Delta_b\Delta_n$
$\mathbf{\Delta_c}$	$\Delta_c 0$	$\Delta_c\Delta_a$	$\Delta_c\Delta_b$	$\Delta_c\Delta_c$		$\Delta_c\Delta_n$
…						
$\mathbf{\Delta_n}$	$\Delta_n 0$	$\Delta_n\Delta_a$	$\Delta_n\Delta_b$	$\Delta_n\Delta_c$		$\Delta_n\Delta_n$

The iterative process in symbolic form

The symbols represent the alphabets:

Δ_a (R)

Δ_b (R, R^*)

Δ_c (R, R^*, A, A^*)

Δ_d $(R, R^*, A, A^*, B, B^*, AB, AB^*)$

Δ_e $(R, R^*, A, A^*, B, B^*, AB, AB^*, C, C^*,$
$AC, AC^*, BC, BC^*, ABC, ABC^*)$

...

An algebraic structure

The process is not restricted to any specific mathematical interpretation, but incorporates digital logic and binary integers, while a convenient consequence is the algebraic series:

$(1, -1)$

$(1, -1) \quad \times (1, \boldsymbol{i}_1)$

$(1, -1) \quad \times (1, \boldsymbol{i}_1) \times (1, \boldsymbol{j}_1)$

$(1, -1) \quad \times (1, \boldsymbol{i}_1) \times (1, \boldsymbol{j}_1) \quad \times (1, \boldsymbol{i}_2)$

$(1, -1) \quad \times (1, \boldsymbol{i}_1) \times (1, \boldsymbol{j}_1) \quad \times (1, \boldsymbol{i}_2) \times (1, \boldsymbol{j}_2)$

$(1, -1) \quad \times (1, \boldsymbol{i}_1) \times (1, \boldsymbol{j}_1) \quad \times (1, \boldsymbol{i}_2) \times (1, \boldsymbol{j}_2) \quad \times (1, \boldsymbol{i}_3) \ldots$

At the point where the cycle repeats, we have what can be recognised as a Clifford algebra – the algebra of 3-D space, where the vectors **i**, **j**, **k** are constructed from $\boldsymbol{i}_2\, \boldsymbol{i}_3$, $\boldsymbol{j}_2\, \boldsymbol{i}_3$, $\boldsymbol{i}_2\, \boldsymbol{j}_2\, \boldsymbol{i}_3$, and $\boldsymbol{i}_1$, $\boldsymbol{j}_1$, $\boldsymbol{i}_1 \boldsymbol{j}_1 = \boldsymbol{k}_1$ and $\boldsymbol{i}_2$, $\boldsymbol{j}_2$, $\boldsymbol{i}_2 \boldsymbol{j}_2 = \boldsymbol{k}_2$ are (mutually commutative) quaternion algebras of the form $\boldsymbol{i}$, $\boldsymbol{j}$, $\boldsymbol{k}$

An algebraic structure

The anticommutative pairs *A*, *B*; *C*, *D*; *E* … now become successive quaternion units, $\boldsymbol{i}_1$, $\boldsymbol{j}_1$; $\boldsymbol{i}_2$, $\boldsymbol{j}_2$; $\boldsymbol{i}_3$ …, each of which is commutative to all the others.

By the fourth stage, we have repetition, which then continues indefinitely.

An incomplete set of quaternion units (for example, $\boldsymbol{i}_3$ in the sixth alphabet) becomes equivalent to the algebra of complex numbers.

Mathematically, we can see the process of the creation of the zero totality alphabets as one of conjugation, followed by repeated cycles of complexification and dimensionalization.

.

An algebraic structure

$(1, -1)$
$(1, -1) \times (1, \boldsymbol{i})$
$(1, -1) \times (1, \boldsymbol{i}) \times (1, \boldsymbol{j})$
$(1, -1) \times (1, \boldsymbol{i}) \times (1, \boldsymbol{j}) \times (1, \mathbf{i})$
$(1, -1) \times (1, \boldsymbol{i}) \times (1, \boldsymbol{j}) \times (1, \mathbf{i}) \times (1, \mathbf{j})$
$(1, -1) \times (1, \boldsymbol{i}) \times (1, \boldsymbol{j}) \times (1, \mathbf{i}) \times (1, \mathbf{j}) \times (1, i)$ ….

Clifford vector algebra notably produces these subalgebras in the reverse order to the universal rewrite system, which generates, in its first four alphabets, scalars, pseudoscalars, quaternions and vectors, along with the scalar subalgebras of pseudoscalars and quaternions.

An algebraic structure

These, as we have seen, generate an algebra which is isomorphic to that of the gamma matrices of the Dirac equation, as used in relativistic quantum mechanics.

Mathematically, it represents the commutative combination of the first complete Clifford algebra (that of 3D space) with all its subalgebras.

So, in many respects, the sixth alphabet represents a particularly significant stage in the rewrite process, the first at which the repetitive nature of the sequence becomes fully established, and this is, therefore, in effect the rewrite system order code.

An algebraic structure

Significantly, if we take all these algebras as *independently true*, and hence commutative, as the rewrite structure seems to suggest we should, since each is a complete description of zero totality, then we require an algebra that is a commutative combination of vectors, bivectors, trivectors and scalars, or vectors, quaternions, pseudoscalars and scalars. This turns out to be equivalent to the algebra of the sixth alphabet, a group structure of order 64 with elements:

$\pm\, \mathbf{i}$	$\pm\, \mathbf{j}$	$\pm\, \mathbf{k}$	$\pm\, i\mathbf{i}$	$\pm\, i\mathbf{j}$	$\pm\, i\mathbf{k}$	$\pm\, i$	$\pm\, 1$
$\pm\, \boldsymbol{i}$	$\pm\, \boldsymbol{j}$	$\pm\, \boldsymbol{k}$	$\pm\, i\boldsymbol{i}$	$\pm\, i\boldsymbol{j}$	$\pm\, i\boldsymbol{k}$		
$\pm\, \mathbf{i}\boldsymbol{i}$	$\pm\, \mathbf{i}\boldsymbol{j}$	$\pm\, \mathbf{i}\boldsymbol{k}$	$\pm\, i\mathbf{i}\boldsymbol{i}$	$\pm\, i\mathbf{i}\boldsymbol{j}$	$\pm\, i\mathbf{i}\boldsymbol{k}$		
$\pm\, \mathbf{j}\boldsymbol{i}$	$\pm\, \mathbf{j}\boldsymbol{j}$	$\pm\, \mathbf{j}\boldsymbol{k}$	$\pm\, i\mathbf{j}\boldsymbol{i}$	$\pm\, i\mathbf{j}\boldsymbol{j}$	$\pm\, i\mathbf{j}\boldsymbol{k}$		
$\pm\, \mathbf{k}\boldsymbol{i}$	$\pm\, \mathbf{k}\boldsymbol{j}$	$\pm\, \mathbf{k}\boldsymbol{k}$	$\pm\, i\mathbf{k}\boldsymbol{i}$	$\pm\, i\mathbf{k}\boldsymbol{j}$	$\pm\, i\mathbf{k}\boldsymbol{k}$		

Physical application

In physical terms, the first four alphabets suggest the successive emergence of descriptions of the universe in terms of the fundamental parameters mass, time, charge and space, which are described by the algebras created with these alphabets.

$(1, -1)$	real	mass
$(1, -1) \times (1, \boldsymbol{i}_1)$	complex	time
$(1, -1) \times (1, \boldsymbol{i}_1) \times (1, \boldsymbol{j}_1)$	quaternion	charge
$(1, -1) \times (1, \boldsymbol{i}_1) \times (1, \boldsymbol{j}_1) \times (1, \boldsymbol{i}_2)$	vector	space

Physical application

The simultaneous existence of these independent descriptions of the universe can only be accomplished at the level of the sixth alphabet.

This is also the first level at which the processes of conjugation, complexification and dimensionalization, which are successively introduced with the first three alphabets, can be finally incorporated into a repetitive sequence.

It is like commutativity and anticommutativity, where the first continues generating possibilities to infinity, while the second closes after a limited finite sequence of three terms.

Physical application

Since such terms as *R*, *A*, *B*, *C*, etc., are generated independently of each other by a unique process, no relative numerical ‘values’ are automatically attached to them, and it is possible, when we use one of the mathematical representations, to choose values for the terms of this alphabet in such a way that its self-product becomes zero and all subsequent alphabets are automatically zeroed as well.

This is the nilpotent condition. It can be achieved in an infinite number of ways, each of which is unique, and it appears to apply both to quantum physics and to systems at much higher levels.

Physical application

The nilpotent condition defines its conjugate as a kind of mirror reflection of itself rather than an intrinsically new state, or defined new symbol – physically, we call it ‘vacuum’.

Significantly, the ‘conjugate’ state to the creation of the initial symbol 1 in binary arithmetic is not defined by a new symbol either, but by a kind of number ‘vacuum’.

In binary, 1 = 1, but –1 = ... 1111111111111111111.

Physical application

The rewrite process is a fractal one in which alphabets at one level can become the units at another. We can therefore imagine an interpretation of the rewrite structure in which every new symbol, say *A*, *B*, *C*, ... represents the creation of a new nilpotent alphabet.

Since the units are now already defined as unique, the connections between them require only the generation of commutative algebraic coefficients, which can then continue to infinity.

This suggests the creation of something like a Hilbert space of nilpotent alphabets, defined by a commutative Grassmann algebra. The coefficients increase with every new nilpotent alphabet, but they have no intrinsic significance, serving only to distinguish one nilpotent alphabet from another.

Entropy and information

This section, based on a recent paper with Peter Marcer, presents entropy as a concept that occurs as a result of the universal rewrite structure independently of any specific application to physics.

A significant fact about the universal rewrite system is that, because every alphabet always includes the previous one, it is intrinsically irreversible. Every new alphabet is always necessarily an extension of the last. And it provides the simplest of all definitions of entropy.

Entropy and information

The constant bifurcation at every new creation produces 2^n components at the nth stage.

If we take the standard definition of entropy, where W is the number of available microstates, $S = k \ln W$, where k is only there because we have historically separated the measurement of temperature from that of energy on the basis of the properties of water.

Really, temperature T has no meaning except as kT. We can't use T in any physics without k. We can't use T in any physics without k. So the true measure of standard entropy is $\ln W$, which is just a number.

Entropy and information

Now, we can simply redefine entropy, say S', as the number n, the order of the alphabet from any given beginning. Then, the previous measure of entropy S becomes $k\, n \ln 2$, just a choice of numerical factor: $S' = S / k \ln 2$.

The number of equally probable microstates at the nth stage is 2^n, and this gives a measure of the increasing complexification / disorder. This is true whatever the rewrite process is, and at whatever level – quantum physics, chemistry, biology, the brain, macrosystems of every kind.

Entropy and information

Because entropy always increases in our experience, we can use this as evidence that the rewrite system is always bifurcating, and the rate of entropy increase will be a measure of the bifurcation that actually occurs, and at the same time a standard clock.

We can actually see how rewrite operates in any given system, and we can establish that it is a universal process.

Entropy and information

Since digital logic and information in the computational sense may be seen as a specialised result of the universal rewrite procedure, this definition of entropy would accord with Shannon's view of 1948 that at least $kT \ln 2$ must be dissipated per transmitted bit of information, based on the assumption that communication in linear systems is done by waves, and that part of the energy of which the message consists must be dissipated, and so lost from the message.

Entropy and information

If the energy in the message is indeed dissipated, this may be a minimum dissipation, and also an information measure.

Self-organization allows us to take this loss as in fact a measure, not simply of the loss, but of the information which enables the self-organization to proceed accordingly, and to treat the self-organization as if it is a complete binary tree of ordinal measure and has a natural canonical power series structure, which the universal rewrite construction shows must indeed be the case.

At some scale, the measure of information transfer and entropy increase is determined directly by the level of the alphabet reached in the universal rewrite process.

Entropy and information

Temperature is, of course, a global parameter, which shows up, for example, in the radiation formula as kT because it concerns the summation over each linearly independent wave phenomenon phase θ across the entire universe, i.e. from zero to infinity, which is how k, $\hbar$ and c become fundamental global scaling constants of the universal rewrite system.

Of course, the information in nilpotent quantum mechanics is not lost, but rather accumulated to make the global process irreversible. It passes from nonlocal / indistinguishable to local / distinguishable.

Entropy and information

Translations from nonlocal to local effects cause time asymmetry because the local requires asymmetric, timelike and consequently irreversible solutions, whereas the nonlocal does not.

All nonlocal processes also have local manifestations, and this appears to be how the time component of the vacuum manifests itself in local effects.

Observability is an indication of an event in the present, for the future remains as part of the unobservable or nonlocal vacuum.

Entropy and information

This is especially true of biological systems, which have evolved to be highly organized and composed of many interconnecting parts.

Referring to the bifurcation process which is the manifestation of the universal rewrite system, we can say that the rate at which it happens must be proportional to the (free) energy involved.

The higher the energy, the higher the rate of bifurcation events.

Near chaotic systems, involving nonlinearity and high connectivity of the components, transfer energy at near maximum efficiency, and so bifurcate rapidly, generating a correspondingly large measure of entropy.

Entropy and information

Rapid information transfer and states of high entropy become strongly correlated.

The process in general acts as a ‘clock’, with the time interval determined by the rate at which the available options are doubled.

The fact that all natural systems are entropic, and irreversible in time, is evidence that all act in terms of the universal rewrite process.

Entropy and information

The growth of a chaotic system, like an event in quantum mechanics, provides a perfect parallel with the universal rewrite system.

It has been observed that in a typical situation leading to chaos, say the growth of an animal population, there comes a point at which, when the growth rate increases above a certain value, the equation produces a bifurcation between two possible outcomes.

Further increases in the growth rate produce a series of further bifurcations of each bifurcation, at a frequency determined by a single scale factor for preserving self-similarity, the universal Feigenbaum number 4.669 … . This can be seen as a characteristic extension of an alphabet by the creation of a new one, exactly as in the 'create' process involved in universal rewrite.

Entropy and information

The rewrite system describes the evolution of a *process* rather than a physically-defined system, though the process might itself require a bifurcation in the system.

In effect, a near-chaotic system becomes subject to particularly rapid overall change because of its high degree of nonlinearity and interconnectivity, and the bifurcation occurs at the level of the whole system or the process applied to the whole system, rather than in only a part of it.

When the process applies collectively, rather than to just part of the system, the expansion of the rewrite structure leads to a complete bifurcation or doubling of the options, and we would expect this to happen repeatedly.

Entropy and information

A system operating near chaos, with a high degree of nonlinearity and connectedness of its parts, will have a high efficiency in transferring free energy, and be subject to rapid development.

The existence of a universal scale factor in the outcome may be taken as a consequence of the relative holism of a system or process on the edge of chaos.

Complexity and maximum entropy

The process also relates to the growth of complexity in natural systems. In principle, nature creates systems and objects whose most required state is self-annihilation with the rest of the universe, or the universal vacuum.

Everything in nature constantly strives towards this end, resulting in local combinations of systems with the nearest available manifestation of a process tending towards this, e.g. fermions and antifermions.

Since complete zeroing isn’t possible locally, the result is complexity and combinations where the symmetry is imperfect or broken, and where the parts continually strive to make further connections.

Complexity and maximum entropy

The nearest to ‘stability’ occurs when an object combines with an ‘environment’ that fulfils a maximal approximation to the desired vacuum connection (e.g. a nucleus in an atom, a bulk molecule in condensed matter, a cell in an organism, an aerobic bacterium being absorbed within an archaeum).

To create and maintain this pseudo-vacuum environment requires a maximal number of connections and interactions to be made and maintained, and hence a maximal generation of entropy.

Since the most ‘desired’ state is the combination (and consequent annihilation) of any system with the universal background, then the tendency of the evolution of the universe will always be in the direction of maximum entropy.

Duality and the factor 2

Both physics and mathematics encompass a fundamental principle of duality at their very bases.

Essentially, this is how we create ‘something from nothing’.

If the ultimate thing that we wish to describe is really ‘nothing’, then we can only create ‘something’ as part of a dual pair, in which each thing is opposed by another thing which negates it.

We can describe this mathematically in terms of the simplest known symmetry group (C_2), which is essentially equivalent to an object and its mirror image (or ‘dual’), whose components are the positive and negative versions of a quantity which may be left undefined.

Duality and the factor 2

This has a surprisingly simple manifestation everywhere as the factor 2 or ½, which sometimes becomes equivalent to squaring or square-rooting.

Of course, duality does not always imply equal status, and may incorporate *chirality*, as in the different status of + and – units in binary numbering.

Duality, in addition, is not a single operation, and the process requires indefinite extension, in the form $C_2 \times C_2 \times C_2 \times \ldots$.

If we begin with a unit, there will be an infinite series of 'duals' to this unit, via a process which must be carried out with respect to all previous duals (that is, that the entire set of characters generated becomes the new 'unit') and the total result must be zero at every stage.

Duality and the factor 2

There are essentially 3 dualities in the parameter group:

nonconserved / conserved
imaginary / real
commutative / anticommutative

Physical dualities always emerge from one or more of these, and they are often interchangeable.

Examples of the first include action + reaction, absorption + emission, radiation + reaction, potential v. kinetic energy, relativistic v. rest mass, uniform v. uniformly accelerated motion, and even rectangles v. triangles. It manifests itself in the use of pairs of *conjugate variables* to define a system, in both classical and in quantum physics.

Duality and the factor 2

Examples of the second include bosons v. fermions, electric and magnetic fields in Maxwell's equations, and space-like v. time-like systems. It allows transformations to be made, for example, between space and time representations. It is the one which occurs in relativistic contexts. A more subtle form of it occurs in the creation of massive particle states at the expense of components of charge.

Examples of the third include fermion + 'environment' (Aharonov-Bohm, Berry phase, Jahn-Teller, etc.), space-like v. time-like systems, particles v. waves, Heisenberg v. Schrödinger / the harmonic oscillator, quantum mechanics v stochastic electrodynamics / zero point energy; 4π v. 2π rotation, and all cases in which physical dimensionality or noncommutativity is involved.

Duality and the factor 2

So the factor 2 may be seen, for example, as a result of action and reaction (A); commutation relations (C); absorption and emission (E); object and environment (O); relativity (R); the virial relation (V); or continuity and discontinuity (X).

The colour coding comes from the fundamental duality from which it emerges. Many of these explanations overlap in the case of individual phenomena, suggesting that they are really all part of some more general overall process:

Duality and the factor 2

Kinematics			V				X
Gases	A		V				
Orbits	A		V				X
Radiation pressure	A	E	V				
Gravitational light deflection			V		R		
Fermion / boson spin			V	O	R	C	
Zero-point energy	A		V			C	X
Radiation reaction	A	E	V		R		
SR paradoxes	A	E					

action and reaction (A); absorption and emission (E); the virial relation (V); object and environment (O); relativity (R); commutation relations (C); continuity and discontinuity (X).

Duality and the factor 2

The factor 2 seems to work mainly in one direction. So, the constant terms produce effects which are 2 × the changing terms, the real produce ones which are 2 × the imaginary, and the discrete produces ones which are 2 × the continuous: the multiplication occurs in the direction which doubles the options.

The first combines + and – cases where it remains constant; the second involves squaring imaginary parameters to produce real ones; and the third combines dimensionality and noncommutativity with discreteness, and so doubles the elements.

Duality and the factor 2

However, doubling of options in one direction may be balanced by halving the options in another.

The factor appears when we look at a process from a one-sided point of view, and the complete description of a system tends to lead to the overall elimination of the factor.

The use of the factor 2 is a two-way process, and the system can only be described in complete terms by taking both the halving and doubling into account.

Physical phenomena involving the factor tend to incorporate, in some form, the opposing sets of characteristics.

Duality and the factor 2

As an entertainment, we can look at the famous equation $e^{i\pi} = -1$ and see that it is a remarkable case of a combination of all three dualities!

If we take mass as being conserved, real and nondimensional, and with positive real unit 1, we will see that

e	is defined by differentiation (nonconserved),
i	is imaginary, and
π	is defined by 3-dimensionality,

and that these 3 act together to produce a collective dual to positive unit 1. (The combination has parallels to *CPT* in physics, which, of course, combines the properties of the 3 quantities which are dual to mass: charge, space and time.)

Anticommutativity and the factor 3

3-dimensionality, one of the most profound and fundamental concepts in physics, has its origin in ideas of anticommutativity, which may be antecedent to the concept of number.

It seems to be responsible for all discreteness in physical systems, and in particular for quantization, as well as for symmetry breaking between the forces, for many significant aspects of particle structure, and for most of the manifestations of the number 3 that are considered fundamental in physics. The Dirac equation is specially structured to accommodate it.

We have 3 dimensions of space, 3 nongravitational interactions, 3 fundamental symmetries (*C*, *P* and *T*), 3 conserved dynamical quantities (momentum, angular momentum and energy), 3 quarks in a baryon, 3 generations of fermions (which can be related to *C*, *P* and *T*).

Anticommutativity and the factor 3

No other dimensionality, not even that of ‘4-dimensional’ space-time, has any fundamental physical significance.

The connection between space and time is basically 3-D (***k***, ***i***, ***j***), and not privileged with respect to mass and charge. Time is *not* part of space, but of *another* 3-dimensionality, though the differences can be masked when we take the scalar product.

An ordinary connection between space and time, not mediated by this second 3-D, leads to wave-particle duality, where one parameter has to take in the other’s physical aspects. Even within the higher dimensionalities of the Dirac algebra, the nilpotent structure shows its fundamental 3-dimensionality, and it is this inherent 3-dimensionality which allows us to develop a fully renormalizable formal theory of quantum gravitational inertia.

Anticommutativity and the factor 3

Just as there are two 3-D structures that are fundamental in nature, so there are also two manifestations of 3-dimensionality, the nonconserved and the conserved.

The nonconserved has an unbroken symmetry, with axes not separately identifiable; in the conserved case, the symmetry is broken or chiral, with axes separately identifiable.

The symmetry-breaking always has the same structure: one term is complexified; one term is associated with an unbroken 3-dimensionality; and the remaining term is purely scalar. The whole structure is invariably nilpotent, reflecting the conserved nature, and either the scalar or the complexified (pseudoscalar) term becomes redundant, except as a ‘book-keeper’.

Anticommutativity and the factor 3

Our quantized, i.e. 3-D, picture denies us the opportunity of representing time as a fourth dimension, denying it status as a physical observable.

In a *3-D* theory, time occupies the place of the 'book-keeper', as energy does in the Dirac state, the quantity which preserves conservation or conjugation, but adds only the information of + or –.

We only know the direction of the sequence that preserves causality, not a *measure* of time in the same sense as we measure space, in the same way as energy only tells us whether the system is a fermion or antifermion.

Anticommutativity and the factor 3

This fact is well known as a stumbling block to proponents of a quantum theory of gravity, which automatically incorporates time as a physical fourth dimension.

The fact that the number system we use in mathematics may have a 3-dimensional origin is of profound significance.

It means that we can’t arbitrarily choose the number of dimensions we apply to quantities like space and time without contradicting the principles on which these concepts, and related ones, such as quantization and conservation, were founded.

Anticommutativity and the factor 3

The separate roles for the three axes in a 3-dimensional system with identifiable components has a remarkable similarity with the processes involved in creating the infinite algebra of the rewrite system.

The role of ***j*** is essentially that of complexification, the beginning of a new and as yet incomplete new quaternion system.
The role of ***i*** is to introduce dimensionalization, while ***k*** is restricted to the 'book-keeping' role of conjugation or conservation.

These also run parallel to the roles of scalar, vector and pseudoscalar quantities (which an extra *i* factor has transformed from the sequence pseudoscalar, quaternion, scalar). It is this parallelism which makes it possible to create a closed parameter system with zero totality and in-built repetition.

Anticommutativity and the factor 3

pseudoscalar	quaternion	scalar	(1)
scalar	vector	pseudoscalar	(2)
mass	space	time	
m	$\mathbf{p}$	E	
t	$\mathbf{r}$	t	
e	s	w	
C	P	T	
$\boldsymbol{j}$	$\boldsymbol{i}$	$\boldsymbol{k}$	
magnitude	direction	orientation	
complexification	dimensionalization	conjugation	
complexification	dimensionalization	conservation	

The ‘dimensional’ term here is in the second column and the ‘book-keeping’ term in the third. (1) = (2) × i and (2) = (1) × i.

Symmetry and self-organization

Physics uses many groups, geometries, algebras, and displays many symmetries, but the only pure symmetries that matter are all based on 2 and 3, and the most fundamental broken symmetries are based on 5.

The table of numbers related to particles in lecture 9 includes practically all the integers that are fundamentally important in physics and in many other scientific areas, including biology, and they are all based on 2, 3 and 5, and these, in turn, all emerge from the universal rewrite system.

It may be that this system is a general description of process which applies to mathematics, computation, physics, chemistry, biology, and any self-organizing system. Investigations in a considerable number of areas appear to show signs of the same structures and symmetries, in exactly the way we said that nature made it possible for us to understand things way outside our experience.

Symmetry and self-organization

The nilpotent relation between the defined system and the rest of the universe, which emerges in the nilpotent form of QM could possibly be an indicator of a process of self-organization which occurs at increasing levels of complexity. The particular characteristics of this type of self-organizing include double 3-D; a 5-fold broken symmetry; geometric phase; uniqueness of the objects and unique birthordering; irreversibility; dissipation; chirality, a harmonic oscillator mechanism, *zitterbewegung*; fractality of dimension 2; the holographic principle and quantum holography.

Research is ongoing in this area, with special (but not unique) reference to biology, where it is clear that some sort of information process is a bigger driver than the specific chemical processes involved.

Symmetry and self-organization

I hope that this lecture series has shown that the Foundations of Physical Law is a very different subject from anything we might previously have imagined. It requires its own methodology and philosophy, and even to a certain extent its own mathematics. It requires, in addition, a great deal of inductive thinking.

Yet it is also rigorous and leads to mathematically and experimentally testable results. Besides creating its own results, it also adds significantly to physics worked out at a greater level of complexity. Its methods and symmetries can suggest answers where none are available by any other means, and resolve anomalies where this had previously seemed impossible. I would like to think that anyone who has followed this series of lectures will see that the possibilities opened up are almost limitless with many opportunities for significant research.

Acknowledgements

The lectures concern a long-term research project. The most significant publication is a book entitled *Zero to Infinity: The Foundations of Physics*, published by World Scientific, 2007.

Before 1997 the project was a solitary enterprise and much of it still is. But there have been a number of collaborations since then with John Cullerne, Bernard Diaz, Peter Marcer, Brian Koberlein and Vanessa Hill.

I would also like to mention the continuing support from a number of members of Liverpool's Physics Department, and in particular from Mike Houlden and John Dainton.

The End