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Higher Education Funding, Welfare and Inequality in Equilibrium

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Abstract

This paper analyses quantitatively the effect that higher education funding policies have on welfare and inequality. We evaluate five different higher education financing schemes with a heterogeneous agent model in continuous time. When educational costs are small, differences in outcomes amongst systems are negligible. As the cost of education and the share of debtors in society rises, it becomes preferable to fund education with subsidies, instead of student loans, as there are pecuniary externalities that arise with debt. Although subsidies can generate large steady state welfare gains, transition costs can be large enough to justify the status quo.

JEL codes: D52, D58, E24, I22, I23
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1 Introduction

Student debt is now the second largest type of household debt in the United States, recently surpassing 1.7 trillon dollars. As shown in Figure (1), the average student at an American university is graduating with over \$34,000 of debt and the stock of student debt, which continues to grow, recently reached 8% of all personal disposable income. While the United States is usually held as a basket case, the United Kingdom is not fairing any better. According to the Institute for Fiscal Studies and the Sutton Trust, the average UK student graduates with over £44,000 worth of debt - Kirby (2016). The rising costs of higher education, and issues related to student indebtedness have exacerbated calls in favour of either student loan debt forgiveness and/or free tuition at public universities. Those opposing such policies argue that they are regressive. Since the benefits of higher education largely accrue to the individual pursuing a college degree, while the costs are shared amongst tax payers, many of whom who do not enjoy such benefit, these policies might actually make matters worse (for instance, by reinforcing inequality).

Income contingent student loans have been proposed as an efficient solution for financing tertiary education. They increase access to higher education for low income households by reducing the capital market imperfection in educational investments and lessening income uncertainty with protections against bad shocks. The leading proponents for financing higher education with income contingent student loans argue that such a system is the best suited at balancing equity and efficiency trade-offs, is the 'most efficient' and that 'tax funding (of higher education) is unfair' - Barr and Crawford (2000)[2].

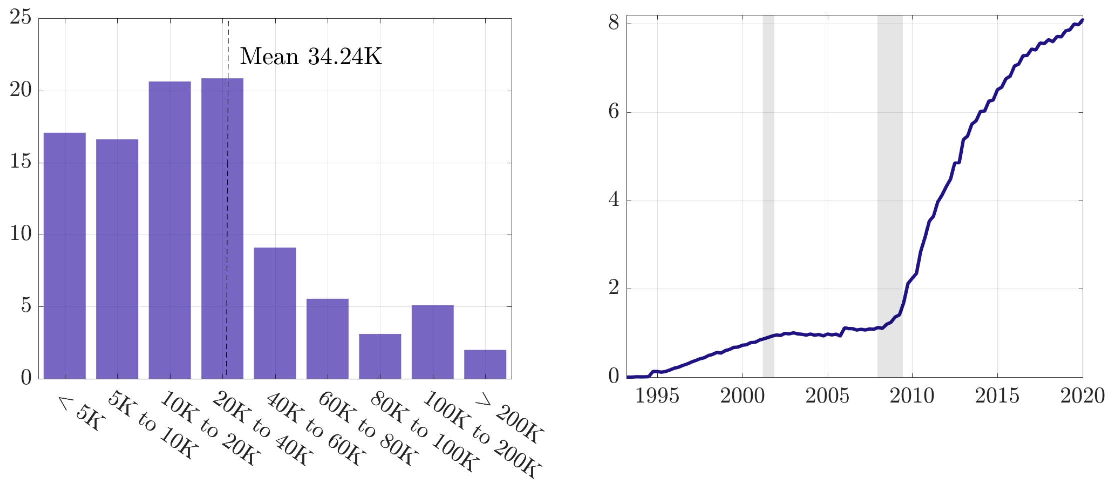

Figure 1: Left: % of borrowers by student loan balances at the 2nd quarter of 2020. Source: U.S. Department of Education. Right: Federal student debt as a percentage of disposable personal income. Source: BEA and Board of Governors

There are considerable reasons to ask if this should be the preferred way to finance tertiary education. First, while there seems to be a consensus, undisputed in some policy circles, on financing higher education with income contingent loans, there is no unique and preferred policy for financing higher education in the OECD[3]. In fact there is plenty of variability, as depicted in Figure (2). Second, contrary to popular perceptions of generous tax financed tertiary education,

[2] *'(income contingent student debt) is efficient, in that it addresses the major capital market imperfection... It is fair, because people with low earnings make low repayments and people with low lifetime earnings do not repay their loan in full... tax funding (of higher education) is unfair'.*

[3] The same can be said of economists working on this field of research. There is no consensus on which is the best way to finance higher education. Even in the small subset of the literature cited further below we may find that either graduate taxes, tuition subsidies, merit grants or income contingent student loans appear to be the preferred policy recommendation.

larger public spending in higher education, relative to GDP, is associated with lower income inequality in the OECD (see Figure (31) in the appendix). Finally, the main emphasis of this article, is that a set of papers in heterogeneous agent macroeconomics have shown that agents' savings behaviour may generate pecuniary externalities that can steer the economy away from efficiency - Aiyagari (1994), Obiols-Homs (2011), Dávila et al. (2012), Nuño and Moll (2018) and Angelopoulos et al. (2017). It is not clear a priori if a system of higher education relying on student loans, tuition subsidies or on graduate taxes may exacerbate the aforementioned externalities by pushing society to under/over accumulate human and physical capital.

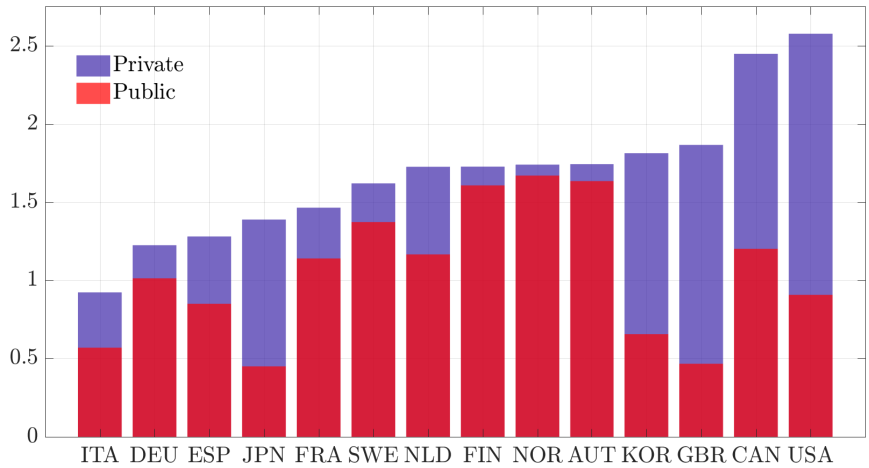

Figure 2: Public and private expenditure on tertiary education relative to GDP in 2015. Source: OECD

In this paper we evaluate the welfare and distributional outcomes of five different higher education financing schemes, using a heterogeneous agent model in continuous time, following Nuño and Moll (2018), extended to allow endogenous educational choices and intergenerational transmission of educational skills. This allows us to evaluate the impact that the most salient features of various educational system (American, Continental European and English and Welsh) have on welfare and inequality. We calibrate the baseline to the United States is, where the government provides a student loan facility and partially subsidises the cost of education. The loans do not have income contingency features, i.e. agents must pay back their student loans regardless of their income. We then introduce two variants that offer income contingency protections; highlighting how small tweaks in the design of the income contingent student loan program can generate significantly different outcomes. Finally, in the last two regimes the government provides support exclusively with tuition subsidies that may cover fully or partially the cost of education; the difference is whether they are funded through general taxation or graduate taxes.

The main contribution of this study is to asses the financing of tertiary education under the light of *the price and quantity effects of debt*[4]; thus making the link between borrowing limits and welfare with higher education financing. The main finding of this paper is that the pecuniary externality of debt manifests itself through the student loan system and becomes more patent as the cost of education rises. When education is relatively easy to achieve, the market failures associated with educational investments do not matter enough to warrant government

[4]Obiols-Homs (2011), shows that too lax borrowing constraints may drag down aggregate welfare. When society has a large fraction of net debtors, the beneficial *quantity effect* of large debt limits (because individuals can continue to optimise and smooth consumption with debt), can be overwhelmed by the *price effect* of more debtors putting upward pressure on the interest rate. Similar effects are major forces driving welfare and efficiency in heterogeneous agent models - Dávila et al. (2012) and Nuño and Moll (2018).

intervention. When the costs of education are calibrated to realistic values, income contingent loans and tuition subsidies provide the best alternatives to finance tertiary education, with the latter yielding the largest welfare gains and drops in inequality. However, as the cost of education and the share of debtors in society gets larger, it becomes much more preferable to increase public support in the form of tuition subsidies, especially if they are financed with graduate taxes. This is particularly important, since tuition costs have been rising in many countries. For instance, these costs have grown consistently faster than CPI, housing and healthcare in the United States - see Figure (32) in the appendix.

By using general equilibrium[5] steady state and transition comparisons, aggregate and individual measures of welfare and a large sensitivity analysis we show that results are affected by two forces: 1) *the shape of the endogenous distribution of income and wealth* and 2) *the price effects of debt* described in Obiols-Homs (2011). With regards to the former, we show that subsidies, as opposed to loans, generate wealth distributions with smaller amounts of the population as net debtors. Additionally, the equilibrium interest rate ends up being higher, which rewards a society with relatively more lenders. Moreover, equilibria with higher net debtor shares tend to be associated with larger wealth inequality. Tuition subsidies place relatively larger shares of the population in the graduate labour market. These distributional impacts have an influence on welfare rankings and in the public cost of higher education. For instance, we show that depending on the design of the student loan system, the fiscal burden generated by the loan program may turn out to be higher than that of tuition subsidies. While the price effects of debt are intricately linked to the distributional outcomes of each higher education financing scheme, we disaggregate welfare gains by wealth, income and ability groups. This allows us to identify which groups gain or lose from higher education reforms.

This article emphasises the importance of evaluating the transitional dynamics of policy changes. While we show substantial steady state gains in terms of consumption equivalent variation of different higher education systems, large transition costs from one regime to another may justify the status-quo. Moving from the baseline to a subsidised system financed by graduate taxes (the system yielding the largest transition welfare gains in the baseline calibration) can be costly enough to eat up more than 70 % of the steady state welfare gains. By comparison, a subsidised system financed by general taxation, which delivers the same steady state gains as that of graduate taxes, loses all of its welfare gains when we take into account transition costs. As a consequence, steady state comparisons of different higher education systems may be misleading for policy.

Related literature: There is a large literature at the cross-roads of macroeconomics, education financing and its distributional impact - García-Peñalosa and Wälde (2000), Bénabou (2002), Hanushek et al. (2003), Bovenberg and Jacobs (2005), Dearden et al. (2008), Johnson (2013), Herrington (2015), Cai and Heathcote (2018) and Luo and Mongey (2019). The closest studies to the one presented here are Ionescu and Simpson (2016), Krueger and Ludwig (2016), Abbott et al. (2019), Hanushek et al. (2014) and Heijdra et al. (2017). In the first article the authors arrive at similar findings as in this paper using a life-cycle environment: tax financed grants can have a larger impact in improving welfare than increasing student loan limits, especially if these are too lax. The present study adds to their results in two ways: endogenising the risk free equilibrium interest rate and factoring transition costs. While the model presented here fails to capture important aspects of lifetime earnings by abstracting from age, it allows us to go beyond steady state comparisons and consider transitional dynamics at a relatively lower computational cost. As will be shown, it is not enough to demonstrate that one regime is better

[5]Welfare gains of policy changes in higher education financing can more than double in partial equilibrium settings. A previous version of this paper included a partial/general equilibrium analysis but we omit such results for the sake of brevity. These results can be reproduced upon request.

than another, the costs of transition must also be taken into account as they can be large enough to significantly lessen the desirability of higher education reforms.

The paper by Krueger and Ludwig (2016) considers transitions, amongst concerns of optimal taxation and education finance. We add to their findings in a number of ways, for instance we consider additional higher education financing systems. We abstract from optimal taxation, so that we can see how results go through even with a flat tax and no public externality in education, as a popular concern against tax-based financing of higher education is that it can be regressive and that in turn, it may reinforce inequality. In this paper we show that even with a tax schedule that is not progressive, we still find that public financing of education can be welfare improving for the vast majority of society. Abbott et al. (2019) cast similar questions as in this study with a detail-rich life-cycle environment. They find that merit based grants and the current student loan system in the U.S. provides substantial increases in welfare. As the study focused on aspects of the U.S. student loan program it did not expand on comparing alternative financing schemes that we consider in this study. Hanushek et al. (2014) compare different higher education funding schemes, as in this paper, with an overlapping generations model. Their findings are somewhat similar to those mentioned herein and we contribute to their results by looking at disaggregated measures of welfare and a large sensitivity analysis of the effects of borrowing constraints. Heijdra et al. (2017) also compare various higher education systems and transitions amongst them, focusing on some of the variants considered in this paper. Our findings complement theirs by assessing higher education financing under the light of the price effects of debt and expanding on why graduate taxes and tuition subsidies can outperform income contingent loan programs.

Finally, the model developed herein contributes to the literature on debt limits and welfare, confirming the presence of price and quantity effects in environments with two types of debt and the simultaneous presence of physical and human capital. For instance, this paper expands on Angelopoulos et al. (2017), who study the pecuniary externalities arising from agents' different savings policies, which vary by education and income profiles. Whereas Angelopoulos et al. (2017) fix exogenously the agent types and restricts flows between groups, this paper endogenises such flows through optimal education choice and evaluates how different higher education systems affect the composition of types in society. Additionally, this work complements findings in Caucutt and Lochner (2020), deepening our understanding of how borrowing constraints affect educational investments not only through the dynamic complementarity of early and late life investments in education - but through price and distributional effects as well.

This paper is structured as follows. In the first section we describe the model. In the next section we show steady state comparisons of the different higher education regimes, where we evaluate welfare gains from each regime, from aggregate and disaggregated perspectives. This is followed by a large sensitivity analysis. In the third section we analyse whether it is worth transitioning from one higher education system to another, specifically from a benchmark towards any of the alternatives considered in this paper. The fourth section concludes.

2 Model

The framework developed herein is based on Aiyagari (1994) and Nuño and Moll (2018). Time is continuous and agents live in a perpetual youth environment. There is a continuum of unit measure of agents that are ex-ante identical (except for their innate ability to graduate from university) and ex-post heterogeneous in their income, wealth, education status and employment state. Agents can invest in education if they find it optimal to do so and can afford its cost. Education is risky; it takes four years on average to graduate, there is a college dropout risk that depends on the agent's innate ability and there is no guarantee that agents will find a job once they graduate. A college degree increases the labour earnings potential of agents and places them in a more favourable labour market[6]. Agents with a college degree are at risk of being hit with a skills depreciation shock, which renders their college degree obsolete.

When an agent dies, a new one replaces it, keeping population constant. Agents derive utility from bequeathing their wealth[7] to the new comers and there is uncertainty regarding the offspring's educational ability. The educational ability and the education status of parent (the exiting agent) influences the educational ability of their offspring (the new comer), as in Abbott et al. (2019).

The production side of the economy follows Nuño and Moll (2018); a representative firm hires labour and rents capital to produce output. The labour input is derived from a CES aggregator of college and non-college educated workers and the distribution of worker types is determined endogenously. The government runs a balanced budget (in steady state and not necessarily during transitions), financing the unemployment insurance program and the higher education system.

This framework is used to rank five different higher education (HE) systems. The baseline regime, called 'non-income contingent loans (NICL)', depicts a system where there is a government backed borrowing facility to fund partially the cost of higher education. NICL can be seen as a broad sketch of higher education financing in the United States[8]. Only agents that can cover $P(1-s)$, the cost of a college degree after government subsidies, either with savings and/or student loans a are allowed to go to university. The next two systems make student loans income-contingent. Only those above a certain earnings threshold repay their student loans, and after 30 years the remaining balance of student debt gets cancelled. The ICL variant has two versions; one is closer to NICL (ICL1) while the other relies on the repayment scheme that is in place in England and Wales (ICL2). These two variants will shed light on how the design of loan repayments can affect outcomes. A fourth regime introduces a government subsidy for tuition fees that is funded by general taxation and is called 'TS'. Finally, a fifth system covers the cost of tuition subsidies with graduate taxes (GT), where only college graduates in employment fund the higher education system. TS and GT do not have student loans. The next subsection outlines a brief overview of agents in the economy. Each type of agent will broadly face the same problem regardless of the HE system. Nonetheless, each regime will have peculiarities affecting agents' budget constraints. Thus, this subsection follows with more details of how the objective and constraints of each type of agent is mathematically formalised for each HE system under consideration.

2.1 Agents

Agents differ permanently on their ability to graduate from university. For a given ability type there are three broad groups of agents in the economy: those currently studying, those without a college degree and those with one. The last two groups are subdivided into two categories: employed and unemployed. Figure (3) illustrates how agents within an ability level move between the five possible types: θ_1^m unemployed and no college degree, θ_2^m employed and no college degree, θ_3^m student, θ_4^m unemployed college graduate and θ_5^m employed college graduate. These are denoted by θ_i^m, where $i = 1, 2, 3, 4, 5$ and where $m = H, L$ is the ability type. The usual flows into and out of employment are written with subscripts denoting the origin and destination (λ_{12} is the flow of non college grads from the unemployed to employed state).

[6]College graduates are less likely to fall into unemployment and find employment at a faster rate.

[7]Agents can only be born with non-negative wealth, they cannot inherit debts. Lenders will charge a premium reflecting death risk when agents have debt.

[8]Thus, when calibrating the model we account for partial tuition subsidies that exist in the U.S.

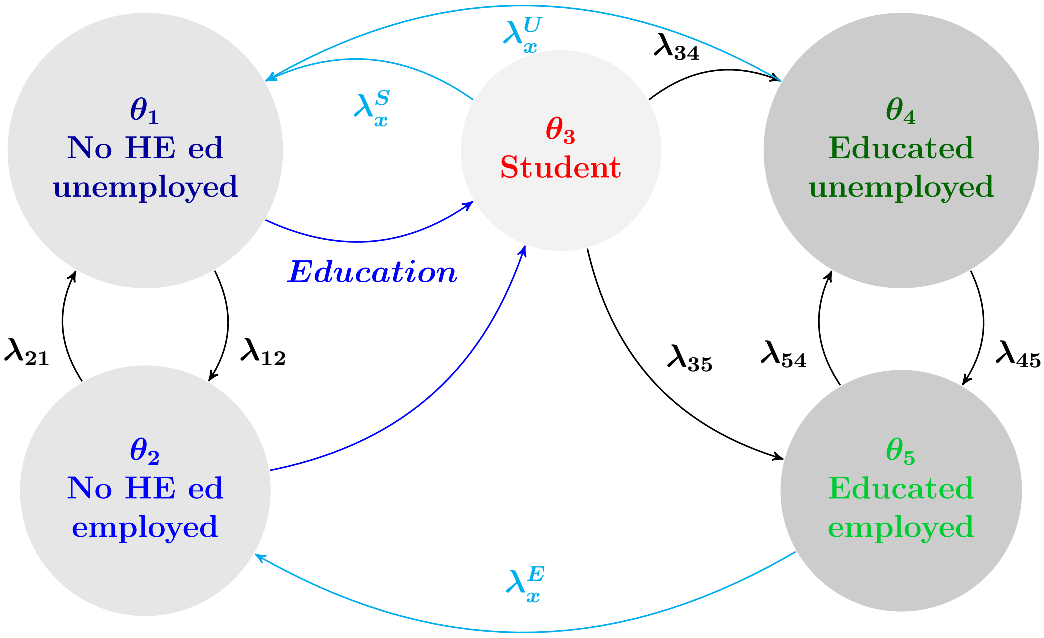

Figure 3: Agent flows

The novelty is the endogenous flow from no HE education to students to HE education. Additionally, a distinctive feature are the flows in the opposite direction λ_x^U and λ_x^E. They represent the rate at which a college degree depreciates. These flows capture how technological advances make redundant some careers that required tertiary education qualifications. This opens the door to study policy in an environment of increasing automation. The approach is not different from Ben-Porath models, where skills can depreciate through time - Ben-Porath (1967) and Manuelli et al. (2012). Furthermore, given that $\lambda_x^U > \lambda_x^E$ we capture how unemployment spells impact skills and labour market outcomes - Arrazola et al. (2005) and Hugonnier et al. (2019). Agents are subject to death risk, with arrival rate κ. When an agent dies the new replacement starts life with wealth bequested by the exiting agent. The level of education and ability of the exiting agent influences the likelihood that the incoming agent has a given level of ability to graduate from university. Finally, agents are subject to college dropout risk, λ_x^S, which itself depends on the permanent ability that the agent receives when it is born.

The transitions between unemployment and employment are such that people with a HE degree tend to face a better job market (higher transition rate into employment and a lower one into unemployment, relative to those without a college degree). All agents face the standard consumption-savings problem with a debt limit on b, the amount of money they have in a checking account. The debt limit $\underline{b}$ is tighter than the natural borrowing limit, i.e. $\underline{b} > -z_1/r$, where z_1 is the lowest possible income - when unemployed without a college degree. Agents pay (receive) interest r if they are net debtors (savers) and r is the risk free rate. When we introduce government guaranteed student loans a, agents will be able to finance the cost of higher education with both a and b. There is also a finite debt limit on student loans $\underline{a}$, not to be confused with the lower limit[9] of the state space in the student loans dimension $\underline{\underline{a}}$.

Common to all agents, preferences are determined by a strictly increasing and strictly concave utility function $u(c)$ and the subjective discount rate ρ. Agents have CRRA preferences in consumption described by $u(c) = \frac{c^{1-\sigma}}{1-\sigma}$. When an agent dies, it also gains from bequeathing

[9]Students can borrow up to $\underline{a}$ to finance education but interest expenses could make student loan balances go beyond $\underline{a}$. We thus set an even lower limit $\underline{\underline{a}}$.

wealth to their offspring $\hat{\phi}\mathbb{E}_m V(z_1, b\mathbb{1}_{\{b\geq 0\}}, 0, m)$, where $\hat{\phi}$ is a parameter capturing how much parents care for the newborn's value function V and where m is the ability type of the newborn. Agents face a mental cost when they become students, captured by $\varepsilon(b)$. In the following subsection I formalise the different type of agents' problem.

2.1.1 Unemployed (θ_1^m) and employed (θ_2^m) and no higher education

As in Achdou et al. (2017) agents maximise utility subject to a flow budget constraint. The only idiosyncratic shock affects income z_i, $i = 1, 2$, which is a two point jump process, where λ_{12} and λ_{21} are the Poisson rates of jumps from unemployed to employed and vice-versa, respectively. Besides choosing consumption, the agent can now choose a time T where, if it has sufficient funds to cover the cost of education, it enrols in university and becomes a student. Since the problem will be solved in the state domain, we will essentially be looking for a free boundary in b (or in b and a in the systems with student loans) that determines when does the agent enrols in university. Let such boundaries in b and a be denoted by a $\dagger$ superscript and let b^* and a^* represent the target points where agents end up at after covering education costs. The general problem of a type θ_1^m or θ_2^m agent in any higher education regime is shown next.

$$
\begin{aligned}
V_i(b,a,m) = \max_{c,T} E_t &\left[\int_t^T e^{-(\rho+\kappa)(s-t)}u(c)\mathrm{d}s + e^{-(\rho+\kappa)(T-t)}(V_3(b^*,a^*,m) - \varepsilon(b^*))\right] \\
& \qquad\qquad\qquad\qquad\qquad\qquad i = 1,2 \; m = H,L \\
\text{s.t. } \mathrm{d}b &= \left(z_i + \left[r + \kappa_{\mathbb{1}\{b<0\}}\right]b - c - \phi_i(b,a)\right)\mathrm{d}t \qquad \text{and} \quad b \geq \underline{b} > -\infty \\
\mathrm{d}a &= [r_A a + \phi_i(b,a) + G_{s,i}(a)]\mathrm{d}t - a\mathrm{d}q_j \qquad \text{and} \quad 0 \geq a \geq \underline{a} > -\infty \\
\mathrm{d}z &= [z_{i+1} - z_i]\mathrm{d}q_{\upsilon_1} - [z_{i+1} - z_i]\mathrm{d}q_{\eta_1} \qquad z_{i+1} > z_i
\end{aligned}
$$

A type θ_1^m agent receives unemployment benefits $z_1 = \mu w_{NC}$, where μ and w_{NC} are the replacement rate and non-college-grad wage, respectively. Type θ_2^m agents receive after tax income $z_2 = (1-\tau)w_{NC}$ and supply labour inelastically. The Poisson process q_ν (q_η) counts when an agent leaves unemployment (loses employment)[10]. The Poisson process q_j counts when the student loan balance is cancelled (the arrival rate of this process is λ_{np}). Agents smooth consumption with b; they pay (receive) interest on b and must pay back student loans a (if they have any). If the agent has student loans, it pays them back according to $\phi_i(b,a)$ and may receive additional government support for its student loan balances[11] with $G_{s,i}(a)$. The functions $\phi_i(b,a)$ and $G_{s,i}(a)$ depend on the peculiarities of each higher education system and they will be described further below. Following Moll (2016) we can show that the solution to this problem satisfies the Hamilton-Jacobi-Bellman (HJB) equation[12]

$$
\begin{aligned}
(\rho+\kappa)V_i = &\max_c u(c) + \kappa\hat{\phi}\mathbb{E}_m V(z_1, b\mathbb{1}_{\{b\geq 0\}}, 0, m) + \frac{\partial V_i}{\partial b}S_b + \frac{\partial V_i}{\partial a}S_a \\
&+ \lambda_{ij}\left[V_j - V_i\right] + \lambda_{np}\left[\tilde{V}_i - V_i\right],
\end{aligned} \tag{1}
$$

where $\tilde{V}$ is the value function where the student loan balance is at zero and where the equation above satisfies the constraint (2)

$$
V_i(b,a,m) > V_3(b,a,m) - \varepsilon(b) \quad i = 1,2, \tag{2}
$$

[10] Arrival rates depend on employment and educational status. See Figure (3) and further below.

[11] The s subscript stands for subsidy. This is a subsidy for student loan repayment, it is not a tuition subsidy.

[12] For notational convenience we will be denoting drifts as S_x. Also, we use notation that omits the dependence of V on b, a and m. We will make these explicit when strictly necessary.

in the region where higher education is not chosen. We can express the problem as a variational inequality

$$\min\{\rho V_i - u(c) - \boldsymbol{A}V_i, V_i - V_3(b^*, a^*) + \varepsilon(b^*)\} = 0, \tag{3}$$

$$\text{where} \quad \boldsymbol{A}V_i = \kappa\left[\hat{\phi}\mathbb{E}_m V(z_1, b\mathbb{1}_{\{b\geq 0\}}, 0, m) - V_i\right] + \frac{\partial V_i}{\partial b}S_b + \frac{\partial V_i}{\partial a}S_a + \lambda_{ij}\left[V_j - V_i\right] + \lambda_{np}\left[\tilde{V}_i - V_i\right].$$

When agents have access to student loans they will cover the cost of education as much as possible with such loans, covering any remaining costs with b[13]. As mentioned earlier, instead of looking for the optimal stopping time T, we will be solving for the threshold values $b_i^\dagger$ and $a_i^\dagger$ where the agent optimally chooses to pay for education (if the agent does not have enough funds to pay, it cannot become a student). In systems such as SF and TS, we encounter single asset problems, e.g. there is no dependence on a, and as a consequence the fourth and sixth terms on the right hand side of equation (1) drop out. Additionally, there is no portfolio problem in the single asset case and as a consequence $V_3(b^*) = V_3(b^\dagger - (1-s)P)$.

The mental cost of becoming a student is defined as[14]

$$\varepsilon(b) = \frac{\varepsilon_0}{1 + b\mathbb{1}_{b\geq 0}}, \tag{4}$$

where $\varepsilon_0 > 0$. In the no schooling region we have the standard first order condition in consumption given by

$$u'(c_i) = \frac{\partial V_i}{\partial b}. \tag{5}$$

Equation (3) is solved as a linear complementarity problem (LCP) - See Moll (2016) and Huang and Pang (1998).

2.1.2 θ_3^m Students

Students work a reduced number of hours and they supply labour inelastically. We scale their labour efficiency accordingly, $z_3 = w_{NC}z_s$. After spending, on average, $\frac{1}{\Delta_{ed}}$ years as a student, the agent may graduate with (without) a job at rate λ_{35} (λ_{34}). There is a risk that the agent will not graduate and become unemployed without a college degree, captured as λ_x^S. Note that $\lambda_{x,m}^S$ varies by innate educational ability m, but we suppress the subscript for ease of notation. Students do not pay income taxes. The HJB equation of students is shown next.

$$\begin{aligned}(\rho+\kappa)V_3 = &\max_c \; u(c) + \kappa\hat{\phi}\mathbb{E}_m V(z_1, b\mathbb{1}_{\{b\geq 0\}}, 0, m) + \frac{\partial V_3}{\partial b}S_b + \frac{\partial V_3}{\partial a}S_a \\ &+ \lambda_{34}V_4 + \lambda_{35}V_5 + \lambda_x^S V_1 - (\lambda_{34} + \lambda_{35} + \lambda_x^S)V_3 + \lambda_{np}\left[\tilde{V}_3 - V_3\right]\end{aligned} \tag{6}$$

The first order condition is analogous to that of (5).

[13] An earlier version of this model allowed for a fully fledged portfolio type problem, where the agent chooses the optimal combination of a^* and b^*. The results are virtually equivalent to those presented here. We keep the less computationally demanding method, which is akin to a so called finance pecking order model - for more on this see section (6.3) in the appendix.

[14] Earlier versions of this paper had a constant mental cost ε_0. The new shape is chosen due to increased computationally stability when computing the value function and the education free boundary.

2.1.3 Unemployed (θ_4^m) and employed (θ_5^m) with higher education

Agents with a college degree earn a college wage premium and thus earn higher income (that is $z_5 = w_C(1-\tau)$ if employed and $z_4 = \mu w_C$, if unemployed). Agents gain (lose) jobs at a higher (lower) rate, when compared to agents without a university education. The two HJB equations for those with a college degree are given by

$$\begin{aligned}(\rho+\kappa)V_i = & \max_c \; u(c) + \kappa\hat{\phi}\mathbb{E}_m V(z_1, b\mathbb{1}_{\{b\geq 0\}}, 0, m) + \frac{\partial V_i}{\partial b}S_b + \frac{\partial V_i}{\partial a}S_a \\ & + \lambda_{ij}\left[V_j - V_i\right] + \lambda_x^k\left[V_{i-3} - V_i\right] + \lambda_{np}\left[\tilde{V}_i - V_i\right] \end{aligned} \quad (7)$$

where $i = 4, 5$, $i \neq j$ and $k = E, U$. $\lambda_x^U > \lambda_x^E$ captures that skills gained by a college degree depreciate faster when the agent is unemployed. The first order condition is analogous to that of (5). Equations (6) and (7) are solved as in Achdou et al. (2017). The next subsection elaborates on the peculiarities of each higher education system and specially on the student loan repayment function $\phi_i(b, a)$ and student loan subsidies $G_{s,i}(a)$.

2.2 Higher education financing and agents' budget constraints

Tuition subsidies (TS) and graduate taxes (GT): The main defining feature of tuition subsidies and graduate taxes is that they are single asset models, i.e. there are no student loans. The cost of education that the agent faces is $P(1-s)$, where P and s are the price and subsidy rate from the state, respectively. When an agent in the TS and GT systems decides to go to university the agent subtracts $P(1-s)$ from its wealth stock and migrates to θ_3^m. As shown further below, the government covers the cost of tuition subsidies by adjusting the income tax rate.

Non income contingent student loans (NICL): Agents are now allowed to pay for higher education with student loans a (or combinations of b and a if the student loan debt limit is binding). The $\phi_i(b, a)$ function describes the student loan repayment scheme when an agent is of type i and has a wealth and student loan balance of b and a.

$$\phi_i(b,a) = \begin{cases} -(r_A+\delta_A)a & \text{for} \quad i = 1,2,4,5 \\ 0 & \text{for} \quad \text{otherwise} \end{cases} \rightarrow \; \mathrm{d}a = \begin{cases} -\delta_A a \mathrm{d}t & \text{for} \quad i = 1,2,4,5 \\ r_A a \mathrm{d}t & \text{for} \quad \text{otherwise} \end{cases}$$

If the agent holds student loans, it pays $(r_A+\delta_A)a$, the interest and amortisation rates on student debt, regardless of its income state. The exception is for students, who accrue debt while at university. Debt forgiveness is not allowed, so the debt cancellation premium $\lambda_{np} = 0$ and thus[15] $r_A = r + \kappa + r_G$. This follows closely federal unsubsidised student loans in the U.S.

Income contingent loan with generous repayment subsidies (ICL1): ICL1 features additional income contingency protections to the previous student loan program. The $\phi_i(b, a)$ function describes the student loan repayment scheme.

$$\phi_i(b,a) = \begin{cases} -(r_A+\delta)a & \text{for} \quad i = 5 \\ 0 & \text{otherwise} \end{cases} \qquad \rightarrow \mathrm{d}a = -\delta_A a\mathrm{d}t - a\mathrm{d}q_j \quad \forall i \quad (8)$$

[15]As NICL mimics the US student loan system, the interest rate on student loans is equal to the sum of the risk free rate plus a spread r_G and κ. This is discussed further below in the calibration section. The government also charges for κ in order to cover unpaid student loan balances arising from death.

Agents pay their student loans only when they reach a high enough income (they reach type θ_5^m, i.e. they become employed and educated). The government covers interest and amortisation payments on student loans otherwise. Furthermore, agents are now allowed to receive debt forgiveness; loans are cancelled, on average, after $1/\lambda_{np}$ years. The Poisson process q_j counts when an agent's student loan balance is cancelled. The government recovers such loses by charging a premium on student loans and thus $r_A = r + \kappa + \lambda_{np}$.

Income contingent loans with repayment subsidies (ICL2): ICL2 builds on NICL as well and adds features of income contingency protections that are different from those of ICL1. Agents are allowed to receive debt forgiveness on the same terms as ICL1. Agents pay their student loans only when they have earnings above the threshold z_T. Any earnings above that threshold are taxed at a rate r_p and this tax contributes towards reducing the student loan balance[16]. Earnings encompass labour and capital income, so any agent with earnings above the threshold will be subject to the tax as long as their student loan balance is not zero. That is, an uneducated agent carrying student loans (say because it suffered a college dropout or skill depreciation shock) that is wealthy in b can still be liable for student loan repayments. Type θ_5^m agents pay an extra interest on their student debt, set to r_A to keep some comparability with ICL1. Students accumulate debt at rate r_A and do not make payments. This system follows closely that of England and Wales[17].

$$\phi_i(b,a) = \begin{cases} r_p \mathbb{1}_{\{a<0\}} \max\{z_i + r\max\{b,0\} - z_T, 0\} & \text{for} \quad i = 1,2,4,5 \\ 0 & \text{for} \quad i = 3 \end{cases}$$

$$\mathrm{d}a = \begin{cases} [r_A a + r_p \mathbb{1}_{\{a<0\}} \max\{z_i + r\max\{b,0\} - z_T, 0\}]\mathrm{d}t - a\mathrm{d}q_j & \text{for} \quad i = 1,2,4,5 \\ r_A a\mathrm{d}t - a\mathrm{d}q_j & \text{for} \quad i = 3 \end{cases}$$

ICL2 differs from ICL1 in two aspects. First ICL1 charges student loan payments depending on type whereas ICL2 does according to earnings. Second, in ICL2 the government does not provide debt repayment subsidies for those receiving income contingency protections. In ICL1 the student loan balance is always decreasing regardless of the income state of the individual; in ICL2 the balance can increase if tax payments on earnings over the threshold z_T are not large enough to cover interest.

In NICL and ICL2 there is one additional subsidy from the state in the student loan program. Any agent with a negative drift at $\underline{a}$, will have interest payments on student debt covered by the government. This is done to prevent mass escaping the state space[18]. These costs are covered through the tax revenue raised from labour income. In the next subsection I describe how agents interact with the other sectors of the economy.

2.3 Firms, government, education and asset market

The rest of the economy is composed of a representative firm, asset market and government. Figure (4) depicts the flows between the different players in the economy. Agents supply labour to a representative firm and receive wages net of taxes in return. Taxes fund unemployment insurance and publicly financed education costs. Agents supply capital to the representative

[16] If these tax payments do not offset interest payments then the student loan balance will keep growing.

[17] As mentioned earlier, in England and Wales student loan interest rates are charged during studies and vary depending on income later in life. The rate charged to students and high income earners tends to be larger than what would be considered a proxy for the risk free rate.

[18] The small amount of mass of agents that reach the lower bound on a (denoted as $\underline{a}$) does not affects results for the calibrations considered in this paper.

firm, through a financial market that is omitted from the figure since it acts as an invisible intermediary. In return, agents receive interest income. The simplified diagram in Figure (4) represents such flows as agents supplying labour and capital and receiving consumption goods and education in return. Figure (5) represents the additional flows in two asset economies (NICL, ICL1 and ICL2), mainly how the government acts as a financial intermediary by supporting the student loan program.

Higher education has a fixed resource cost. It is calibrated as a share of GDP capita so that we can compare different higher education systems with the same relative tuition costs. This is an explicit modelling choice; this assumption is made so that we can evaluate the impact of P in the capital market imperfections of educational investments, and in turn, on the rankings between the different higher education regimes. It is important to highlight the role of P, especially when tuition costs have risen so dramatically in the United States - see Figure (32) - and in many countries.

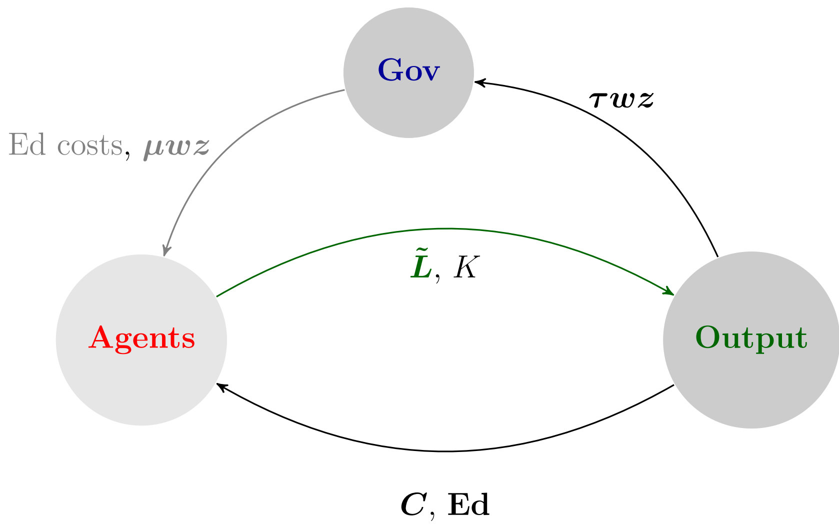

Figure 4: Common flows in all HE systems

2.3.1 Representative firm

As in Aiyagari (1994), there is a representative firm with Cobb-Douglas technology. The firm rents capital, which depreciates at rate δ, from agents and hires labour. The production function is given by

$$Y = AK^{\alpha}\tilde{L}^{1-\alpha}, \tag{9}$$

where A is a positive constant and α is the capital share. The effective labour supply $\tilde{L}$ is given by a CES aggregator of the labour supply of workers with and without a college degree as well as that of students. Remark that students' effective labour supply is scaled by z_s to capture their working hours[19].

$$\tilde{L} = (\chi[\theta_1 + z_s\theta_3]^{\nu} + (1-\chi)\theta_5^{\nu})^{\frac{1}{\nu}}, \tag{10}$$

Factor prices are given by the next three expressions.

[19]Note that $\theta_i = \sum_{m=1}\theta_i^m$.

$$r = \alpha \frac{Y}{K} - \delta, \tag{11}$$

$$w_{NC} = (1-\alpha)\frac{Y}{\tilde{L}^{\nu}}\chi[\theta_1 + z_s\theta_3]^{\nu-1}, \tag{12}$$

$$w_C = (1-\alpha)\frac{Y}{\tilde{L}^{\nu}}(1-\chi)\theta_5^{\nu-1}. \tag{13}$$

Equation (11) gives us capital demand.

2.3.2 Labour income taxes

The government has a balanced budget constraint and raises revenue from labour income with a flat tax applied to workers τ. Additional tax revenue may be raised to cover subsidies to P (in TS), to interest payments or income contingency protections (in ICL1 and ICL2). Hence, the income tax rate τ is shown next.

$$\tau = \overbrace{\frac{\mu[w_L\theta_1 + w_H\theta_4]}{w_L\theta_2 + w_H\theta_5}}^{\tau_{UI}} + \frac{\text{Public Ed spending}}{w_L\theta_2 + w_H\theta_5} \tag{14}$$

The first term, τ_{UI}, is the tax rate needed to cover unemployment benefits. The second term comprises the net expenditures incurred to finance the HE system[20]. The unemployment benefit system is common in all the five regimes being considered. The second term captures the public cost of financing the higher education system. In GT we have two income tax rates.

$$\tau_{NC}^{GT} = \frac{\mu[w_L\theta_1 + w_H\theta_4]}{w_L\theta_2 + w_H\theta_5} \tag{15}$$

$$\tau_{C}^{GT} = \frac{\mu[w_L\theta_1 + w_H\theta_4]}{w_L\theta_2 + w_H\theta_5} + \frac{\text{Public Ed spending}}{w_H\theta_5} \tag{16}$$

The first tax rate, τ_{NC}^{GT}, affects those without a college degree. This group does not participate in public education expenditure, unlike college graduates, which pay τ_C^{GT} as long as they are alive and are not hit with skills depreciation shocks.

2.3.3 Student loan interest rates and asset market

Agents pay their student loan balances according to the debt repayment schemes mentioned above. The government may also raise revenue or cover costs with premiums on student loans. As mentioned above, in all student loan programs, the government charges a premium on loans so as to cover unpaid balances due to death. Additionally, in NICL, the government charges a spread r_G on student loans that generates revenue. Finally, in the ICL regimes, the government raises revenue with additional premiums on student loans, so as to cover debt cancellation.

$$r_A = r + \kappa + \lambda_{np}\mathbb{1}_{\text{ICLs}} + r_G\mathbb{1}_{\text{NICL}} \tag{17}$$

If we denote the total amount of newly issued student loans as A^{new}, the aggregate stock of student debt as A and agents' aggregate net savings as B, we can represent the government's role as an intermediary in student loan programs as follows.

[20]Note that in NICL and ICLs the government incurs expenses to run the student loan program but that it also receives interest income from student loan repayments. So the second component of the tax rate is net expenditure in higher education.

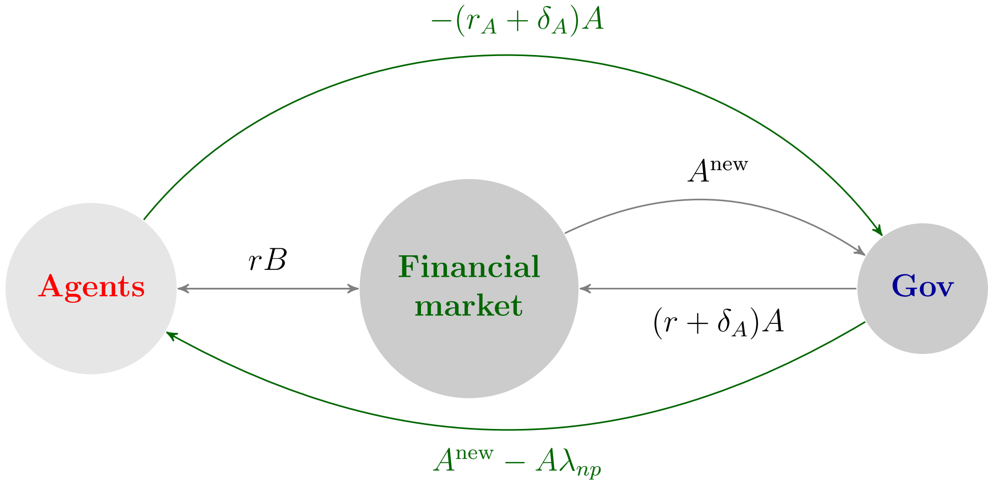

Figure 5: Government intermediation in student loans

The government acts as an intermediary, raising funds in the financial market, issuing student loans to agents and acting as guarantor in case of debt forgiveness. Any loses in the student loan program are covered by the state either though tax revenue and/or student loan premiums. In the next subsection we define what is an equilibrium in the economy and how we rank welfare.

2.4 General equilibrium

The stationary equilibrium in this model is defined by a set of policy functions in consumption and educational investment (given by the HJB equations shown above) for each agent type, a joint income and wealth distribution that is ergodic, a government balanced budget and a risk free rate that clears the asset market. During transitions, the asset market clears at every instant. The income and wealth distribution is governed by the Kolmogorov Forward Equations (KFE)[21]. Market clearing requires $K_S - K_D = 0$, where

$$K_S = \sum_{m=1}^{2} \sum_{i=1}^{5} \int_{\underline{a}}^{0} \int_{\underline{b}}^{\infty} (b + a) g(b, a, i, m) \mathrm{d}b\mathrm{d}a.$$

Capital demand being equal to capital supply implies the national accounting identity[22] $Y = C + I + G + \text{Education costs}$. There is no proof of existence and uniqueness of equilibrium for the model with educational choice. The downward sloping and continuous demand of capital remains the same as in Aiyagari (1994). Nevertheless, capital supply is affected by the different education types - Angelopoulos et al. (2017) - and by the educational choice. Quantitative evaluations for a large parameter space show that it is the case that the aggregate capital supply K_S is monotonically upward sloping, approaching ρ from below, continuous and that there is a single crossing of capital demand and supply. We evaluate aggregate and individual welfare via consumption equivalent gain (CEG), as shown next. Let V_0 and V_c denote the steady state aggregate value in the benchmark and alternative regimes, respectively.

$$\tilde{c} = \left[\left(\frac{V_c}{V_o} \right)^{\frac{1}{1-\sigma}} - 1 \right] * 100 \tag{18}$$

Remark that CEG will be presented in percentage terms. V_c and V_o are computed as follows.

[21] The equations are shown in the appendix.

[22] A heuristic proof is left in the appendix.

$$\sum_{m=1}^{2}\sum_{i=1}^{5}\int_{\underline{a}}^{0}\int_{\underline{b}}^{\infty}V(b,a,i,m)g(b,a,i,m)\mathrm{d}b\mathrm{d}a. \tag{19}$$

The CEG will be computed as an average for the whole economy (as in (19) above) and also for each income and educational ability group. In some cases we will also look at CEG in each point of the state space (using $V(b,a,z,m)$ only), giving us a disaggregated view of which groups in society favour/are against policy changes relative to a common benchmark. Given that each regime yields a different distribution of income and wealth, the disaggregate CEG comparisons will be unweighed comparison of raw value functions. Hence, the aggregate welfare analysis will be complemented by evaluation the gains/losses from HE reforms of different groups and by comparing the income and wealth distributions of HE systems. The benchmark V_o will be set to welfare in NICL. Positive values of $\tilde{c}$ mean that agents in the NICL system would be as well off as in the alternative system if their lifetime consumption is increased by $\tilde{c}$ per cent. Negative values mean that we would have to subtract $\tilde{c}$ per cent of the life time consumption of agents in the NICL regime, in order to make them as worse off as in the alternative higher education system. The numerical method used to solve the model is the finite differences approach presented in Achdou et al. (2017). The agent's decision to become a student is computed with an LCP solver as in Moll (2016) on non-uniform grids.

2.5 Calibration

The baseline calibration of the model is shown in Table (1). The model economy has 36 parameters. These are discussed below in separate categories. The benchmark higher education financing regime is NICL, and we choose parameters by matching selected U.S. data moments and calibrating the rest based on studies focused in the U.S. All parameters are calibrated so that everything is understood in annual terms.

Preferences: Preferences are described by a constant relative risk aversion utility function with risk aversion coefficient σ, set to the standard value of 2, a subjective discount rate ρ, a bequest taste $\hat{\phi}$, the parameter governing the strength of the mental cost of becoming a student ε_0 and the death rate κ. The subjective discount rate is set by matching a capital-output ratio of 2.65, following Krueger and Ludwig (2016). The bequest taste parameter is set by approaching as much as possible a Gini coefficient of wealth of 0.801, following Davies et al. (2011). The mental cost parameter ε_0 is set so as to match the fraction of the population with a college degree. According to the National Center of Education Statistics (NCES) the share of college graduates stood at 35% in 2018. The death rate κ is set to 0.0167, reflecting an average work span of 60 years. The intergenerational transmission of educational ability is taken from Abbott et al. (2019) and we elaborate further on its implementation in the appendix.

Labour market transitions: The labour market transition rates from unemployment to employment, and vice-versa, are taken from Lise et al. (2016). One can see in Table (1) how labour market outcomes are more favourable for graduates as they face a higher probability of being employed and lower probability of falling into unemployment. The transition rates from student to educated is set to $\Delta_{ed} = 0.25$, reflecting that on average it takes four years to complete a bachelor's degree in the U.S.[23]. The flow from student to educated Δ_{ed} is split into transitions to unemployed and educated (λ_{34}), and employed and educated (λ_{35}). According to the NCES, roughly two thirds of students find employment within the first 9 months after graduation - Staklis and Bentz (2016). This figure is roughly constant despite fluctuations over the business cycle. The skills depreciation rate is taken from Manuelli et al. (2012). The magnitude seems to

[23] According to NCES, in the U.S., the most common is to graduate in 4 years. Results with further sensitivity analysis on Δ_{ed} can be reproduced upon request.

	Values	Description	Source
σ	2	CRRA	Nuño and Moll (2018)
λ_{12}	1.884	Transition rate $z_1 \to z_2$	Lise et al. (2016)
λ_{21}	0.132	Transition rate $z_2 \to z_1$	Lise et al. (2016)
λ_{45}	1.608	Transition rate $z_4 \to z_5$	Lise et al. (2016)
λ_{54}	0.072	Transition rate $z_5 \to z_4$	Lise et al. (2016)
λ_{34}	$\Delta_{ed}\frac{1}{3}$	Transition rate $z_3 \to z_4$	Staklis and Bentz (2016)
λ_{35}	$\Delta_{ed}\frac{2}{3}$	Transition rate $z_3 \to z_5$	Staklis and Bentz (2016)
λ_x^E	0.024	Employed obsolescence rate	Manuelli et al. (2012)
λ_x^U	0.048	Unemployed obsolescence	Manuelli et al. (2012) and Arrazola et al. (2005)
$\lambda_{x,1}^S$	0.1940	Dropout rate high ability	Light and Strayer (2000)
$\lambda_{x,2}^S$	0.3064	Dropout rate low ability	Light and Strayer (2000)
s	0.47	Subsidy rate	Athreya et al. (2019)
μ	0.382	Replacement rate	U.S. Department of Labor (2019)
Δ_{ed}	0.25	Inverse years until grad	4 year degree
A	1	Productivity	Normalisation
$\underline{b}$	-0.1454	Exogenous b limit	Athreya et al. (2019)
$\underline{a}$	-0.4636	Exogenous a limit	U.S. Department of Education (2019)
$\underline{\underline{a}}$	-1.0111	Lower bound on a	2.18*$\underline{a}$
$\overline{P}$	1.1090	Education cost †	College Board (2018)
α	1/3	Capital income share	Krueger and Ludwig (2016)
ν	0.6	CES production	Card and Lemieux (2001)
δ	0.0755	Capital depreciation	Krueger and Ludwig (2016)
δ_A	1/30	Amortisation in NICL and ICL1	**
κ	0.0167	Worker death rate	World Bank
λ_{ij}^a	‡	Intergenerational ability trans	Abbott et al. (2019)
r_G	0.0242	NICL student loan spread	B.G. Federal Reserve System (2019)
z_s	0.50625	Student work hours	Carnevale et al. (2015)

Table 1: Exogenously calibrated parameters. † Average cost of going to university.** Maximum maturity for *Standard Repayment* in the US. ‡ The intergenerational transition matrix of ability is shown in the appendix.

be more or less the same among other papers using Ben-Porath type models, for instance Ionescu (2009). The doubling of this rate for those that are unemployed reflects evidence highlighted in Arrazola et al. (2005) and Hugonnier et al. (2019). The replacement rate μ is taken from estimates of the U.S. Department of Labor.

Education, ability and college wage premium: The total cost of education, relative to GDP per capita, rescaled to only take into account workers, and based in a four year education is equal to 1.1090. Scaling educational expenses relative to GDP per capita allows better comparison between different HE systems that may have different steady state outputs. In (35) in the appendix we elaborate on how we compute P. Section (6.6.1) in the appendix also has a sensitivity analysis with larger/smaller costs of education. The χ share in the labour CES aggregator is set so as to match a college wage premium $\psi = w_C/w_{NC}$ of 1.7 in the baseline NICL, following[24] evidence from James (2012) and Valletta (2018). The baseline subsidy rate is set to 0.47, following Athreya et al. (2019). The college dropout rates by educational ability $\lambda_{x,1}^S$ and $\lambda_{x,2}^S$ are taken from Light and Strayer (2000), where we convert in annual rates the probability of not graduating from university and where we combine the two bottom and top AFQT score quartiles into low and high educational ability. We set z_s to 0.50625, based on US' working students average hours worked - Carnevale et al. (2015) and Sonnet (2010).

[24]In a previous version of this paper we imposed a fixed college wage premium on all HE regimes as the last decades have seen a stable, if not rising, college wage premium in both the US and UK despite large increases in the supply of college educated workers. In this paper we let the college wage premium respond to market forces as we change the HE financing policy. Belfield et al. (2018a) and Belfield et al. (2018b) have shown that the premium is driven by substantial heterogeneity. Introducing heterogeneity in ψ is left for an extension.

	Values	Description	Source
λ_{np}	1/30	Premium on student loans	ICL1 and ICL2
r_p	0.09	ICL2 graduate tax	England and Wales (2018)
z_T	0.3246	ICL2 income threshold	England and Wales (2018)

Table 2: Parameters exclusive to ICL1 and ICL2.

Parameter	Value	Description	Target	Model	Data	Source
ρ	0.03875	Discount rate	K/Y	2.65	2.65	Krueger and Ludwig (2016)
ε_0	-0.3237	Mental cost edu	Graduate share	35%	35%	NCES (2018)
χ	0.4232	CES income share	ψ	1.7	1.7	James (2012), Valletta (2018)
$\hat{\phi}$	0.0857	Altruism strength	Wealth Gini	0.6731	0.801	Davies et al. (2011)

Table 3: Endogenously calibrated parameters.

Student loans and debt limits: The amortisation rate in NICL and ICL1 is set to 1/30. This corresponds to the maximum maturity in the *Standard Repayment* schedule for student loans in the United States. The reason we use this rate is twofold. First, higher amortisation rates reduce the state space where consumption can remain positive when indebted and second, a 30 year loan allows some degree of comparability with the other regimes where loans are forgiven after 30 years[25]. The spread r_G that is added to student loans in NICL comes from the Direct unsubsidised student loan interest rate in the academic year 2018-2019 minus the annual average 3 month treasury interest rate[26]. Using the re-scaled GDP per capita method outlined in the appendix in (35), we obtain the values for $\underline{b}$ and $\underline{a}$. In the former we take the value of mean credit limit \$17000 estimated in Athreya et al. (2019), where after converting to 2018 dollars we obtain a debt limit[27] $\underline{b}$ equal to -0.1454. As for the latter, we set the student debt limit (4 year degree cumulative) $\underline{a}$ to -0.4636. This reflects the total aggregate loan limit for independent undergraduate students of \$57500 in 2018, converted following (35). Although agents can borrow up to $\underline{a}$ to go university, interest may push them over this limit. We thus extend the student debt dimension up to $\underline{\underline{a}}$, a parameter that we choose so as to allow the loosest possible limit and such that consumption is always positive for all the experiments considered herein. The result is a value of $\underline{\underline{a}} = 2.18\underline{a}$.

ICL2 parameters: The graduate tax in ICL2 is described by two parameters: z_T and r_p. The threshold z_T is set to mimic England and Wales' student loan system, matching the ratio of the student loan taxable threshold to GDP per capita (rescaled to only account for workers) while the tax rate r_p on earnings above the threshold is set to 9 %. Following the methodology in the calibration for P, we translate z_T to a comparable figure for the US, and we thus set $z_T = 0.3246$. We leave the details of the computation in the appendix.

Production: Three parameters describe the productive technology in the economy. These are the capital elasticity α, total factor productivity A and depreciation δ. The first two are set to commonly used values (0.33 and normalised to 1, respectively). The depreciation rate is taken from Krueger and Ludwig (2016). The elasticity of substitution between labour types is set to 2.5, following Card and Lemieux (2001), which implies $\nu = 0.6$.

[25] For instance, Plan 2 loans in England and Wales and in the ICL systems considered in this paper.

[26] The US student loan program charges interest rates that are well above the government's borrowing rate, and by some accounts, yields net income to the government. The spread captures this source of revenue, lessening the tax burden due to tertiary education.

[27] Aggregate welfare is sensitive to debt limits, as pointed out by Obiols-Homs (2011). Therefore, a large sensitivity analysis on debt limits is carried out to illustrate how they affect the rankings of the higher education funding schemes discussed herein.

3 Steady state results

Table[28] (4) and Figure (6) show welfare gains along with other key aggregates of each system. Welfare gains are relative to the benchmark regime, NICL. Focusing first on Figure (6), we see two striking results. First, while systems with income contingent loans bring substantial gains, these can vary substantially depending on how the student loan program is designed. ICL1, a more generous program delivers larger gains. Second, full or even partial tuition subsidies, yield the largest CEG vis-à-vis NICL. Table (4) shows that welfare rankings move in the same direction with the share of college graduates, lower inequality and lower net debtor shares. Moreover, systems with larger gains tend to have higher equilibrium interest rates *with the lowest shares of the population in net debt*. This last result points to a powerful force driving aggregate welfare: *the price and quantity effects of debt* described in Obiols-Homs (2011). Higher income tax rates do not seem to be useful in predicting welfare losses.

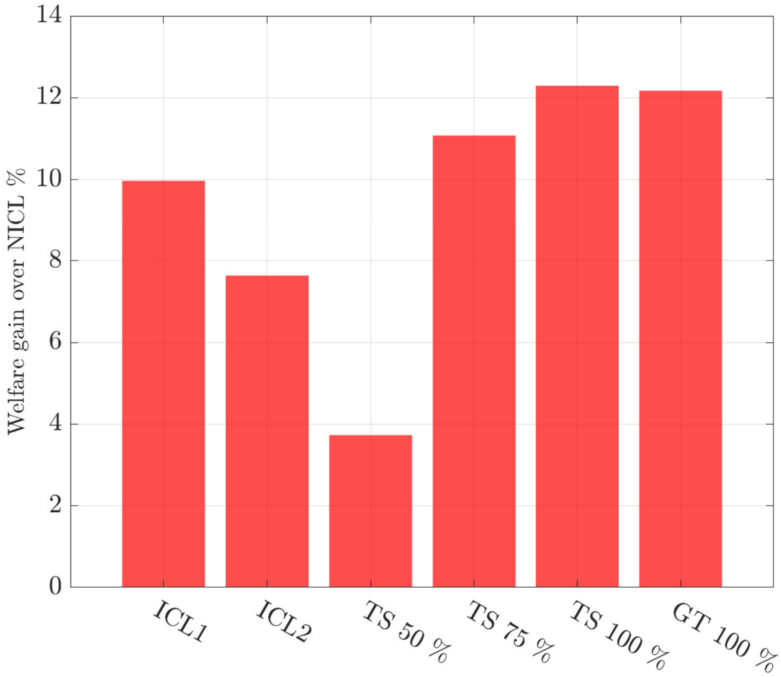

Figure 6: Consumption equivalent gain of income contingent student loans 1 and 2 (ICL1 and ICL2), tuition subsidies (TS) and graduate taxes (GT) at 50, 75 and 100 % of educational costs, relative to the baseline (non income contingent student loans - NICL).

There are four main reasons behind the relative success of TS and GT. First, we have a model environment where earnings are higher and unemployment spells are shorter in the graduate labour market. Not only is this of benefit to the individual but it also has indirect general equilibrium benefits such as a lower cost for the unemployment insurance program (although overall tax rates might increase due to education expenditures). Systems that place more mass of agents in this labour market will thus tend to have higher welfare. Second, the protections against bad outcomes provide more incentives for educational investments. Income contingent student loans do much better than the benchmark system in this regard, especially as protections against bad outcomes increase. This is patent in ICL1's superior performance over ICL2. Despite having the same debt limit and debt cancellation policies, ICL1 is more generous in its protections for bad outcomes. TS and GT go even further in that agents do not incur as many debts, or any at all, to become students. Third, although student loans deliver substantial gains, they

[28] In Table (4), the TS and GT system columns have a percentage attached to denote what fraction of education costs are covered by the state.

have debt limits. In an environment where there is college dropout and retraining risk (due to skill depreciation shocks) some agents may be stuck waiting to either pay student debt or have enough savings to join again the graduate labour market. In this regard TS and GT do better, as there is no limit in the number of times one becomes a student. Fourth, since TS and GT systems generate less debtors the economy is less susceptible to the price effects of debt. In fact, as TS and GT systems are less risky precautionary savings drop and the interest rate rises. So we end up with systems that generate relatively more lenders with higher asset income and less inequality. We elaborate on this fourth point further below in section (3.1).

Systems that generate more net debtors and more inequality have lower aggregate CEG. If a relatively high net debtor share is compounded with a larger interest rate, welfare will be depressed even further. As we can see in Table (4), systems with student loans are more prone to the negative impact on welfare coming through the price effect described in Obiols-Homs (2011). Whilst income contingent loan systems can be tailored to shield agents from the effect of interest rates, they may also be designed in a way, such as in ICL2, where debt balances accumulate during non payment spells, effectively prolonging the repayment period[29]. Remark that in NICL and ICL2, agents accrue debt when they don't have a high enough income[30]. Furthermore, even if the amount of debt is notional and may not affect those on very low incomes, the government still has to raise revenue to cover interest payments and cancellations. In equilibrium, ICL2 ends up being fiscally more expensive than ICL1, despite being less generous.

	NICL	ICL1	ICL2	TS 50 %	TS 75 %	TS 100 %	GT 100 %
K	1.842	1.856	1.833	1.858	1.880	1.868	1.834
r	4.903	5.262	5.350	4.977	5.256	5.447	5.423
Graduates %	35.000	46.272	45.228	38.458	50.054	60.234	48.035
τ %	4.922	11.018	11.810	6.312	10.027	15.080	19.395
τ_{UI} %	2.473	2.270	2.215	2.425	2.155	1.994	2.178
Gini$_{b+a}$	0.673	0.572	0.679	0.619	0.522	0.472	0.472
Net debtors % $b + a$	8.201	12.067	22.367	2.724	1.799	0.925	0.608
Net debtors % b	2.287	2.111	2.659	2.724	1.799	0.925	0.608
CWP	1.700	1.398	1.425	1.598	1.313	1.097	1.359
High ability %	48.255	50.415	50.289	48.774	50.891	52.902	50.999

Table 4: Steady state general equilibrium results. In GT we have two tax rates; * is τ_H^{GT} and τ_{UI} is τ_L^{GT}.

Changing higher education funding alters the distribution of income, education, wealth, the labour market and thus the uncertainty agents face. The feedbacks between these and prices[31] ultimately affect the level of consumption. A decomposition of aggregate welfare gains following Floden (2001) shows us that the bulk of gains of moving away from NICL stem from less inequality being generated by the alternative systems. Furthermore, Figure (7) reveals that gains from less inequality and less uncertainty increase as the system becomes more generous. As wealth inequality is intricately linked with the make up of debtors/lenders, it is natural that results in Table (4) and Figures (6) and (7) point to higher welfare in systems that generate less debtors and less inequality.

[29]See Waltmann (2022) for a recent analysis of the distributional impact of tweaking repayment thresholds and student loan interest rates in England and Wales.

[30]This is one of the main distinctions between ICL1 and ICL2. In the former the government covers student loan repayments when agents do not make contributions whereas in the latter debt may continue to accrue. Changing z_T and r_p can accentuate or soften these effects.

[31]A previous version of this paper showed that welfare rankings can easily change whether we use a partial or general equilibrium setting. In that version of the model this type of analysis was more informative. The main insight of those results is that systems that generate more lenders (and less inequality) with a higher equilibrium interest rate tend to achieve larger welfare gains. We abstain from this analysis for the sake of brevity as the insights are broadly similar to those presented herein.

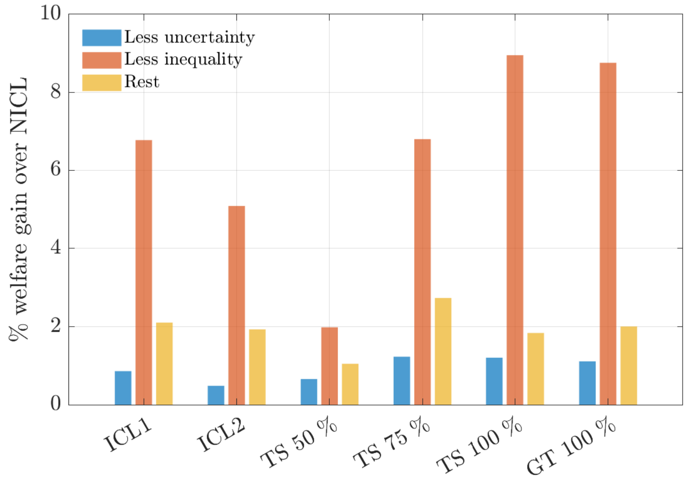

Figure 7: Floden (2001) decomposition of consumption equivalent gain of income contingent student loans 1 and 2 (ICL1 and ICL2), tuition subsidies (TS) and graduate taxes (GT) at 50, 75 and 100 % of educational costs, relative to the baseline (non income contingent student loans - NICL).

While aggregate welfare gains relative to NICL are positive for all systems, such averages mask who gains or loses and whether the majority of agents are in favour of changing higher education financing. Figure (8) reveals that most alternatives deliver non negative welfare gains to more than half of agents. Surprisingly, the exception is that of TS 100%, which also turned out to deliver the largest aggregate steady state welfare gains. Two forces are at play in this seemingly contradictory result. First, the vast majority of gains are concentrated in low wealth agents and the gains in these groups are large enough to compensate for losses of high wealth agents. Second, the results depend on the distribution that we use to weigh population shares when we ask which fraction of agents is in favour of the policy change. The distribution in NICL places more mass in low wealth (especially net debtors) groups whereas more generous public financing systems have distributions with relatively less debtors. This hints at the need for a disaggregated analysis of welfare gains[32]. Before doing so, we repeat the exercise above with over 2300 combinations of debt limits $\underline{b}$ and $\underline{a}$, subsidy rates and various values of P in order to evaluate further the price and quantity effects outlined in Obiols-Homs (2011).

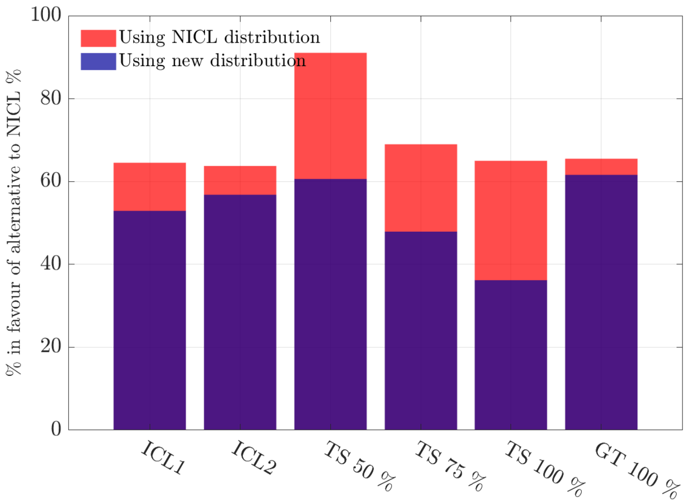

Figure 8: Total share of the population in favour of moving away from NICL. The 'new' distribution is that of the alternative regime (ICL1, ICL2, TS, GT, respectively).

[32]In section (3.2) we examine disaggregated measures of welfare.

3.1 Borrowing limits and welfare

We have seen that aggregate welfare gains appear to move in lockstep with net debtor shares and inequality. In this section we show how much aggregate CEG results change as we vary debt limits $\underline{b}$ and higher education funding policy ($\underline{a}$ and s). In a first round of experiments, we keep the NICL benchmark at the baseline calibration and compute steady state results for all other higher education financing schemes. We then repeat this exercise by also changing the cost of education P.

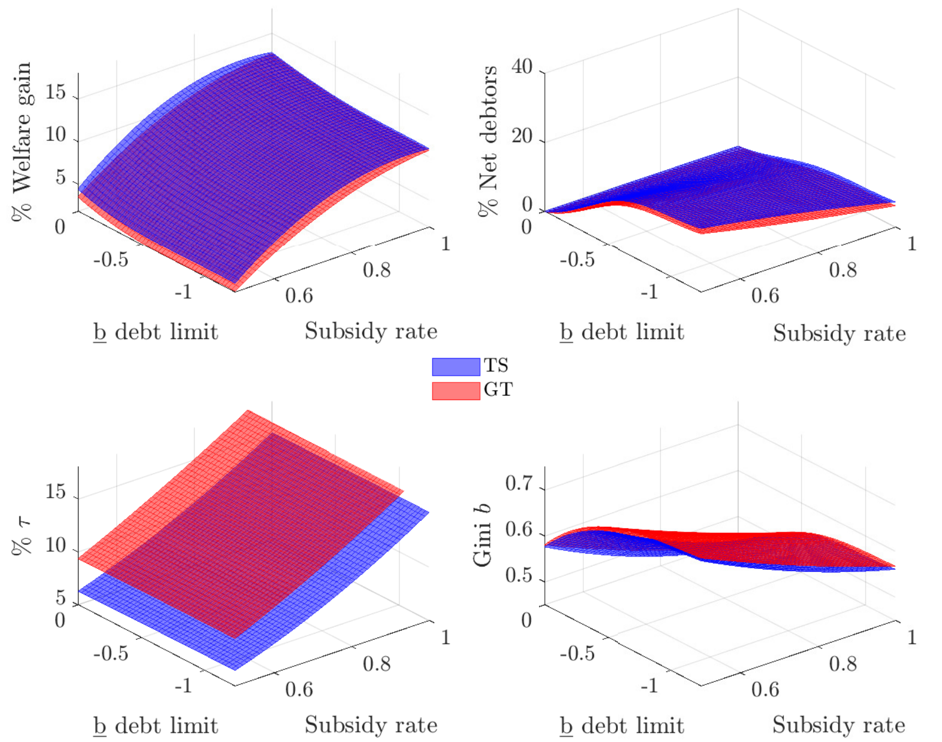

Figure 9: CEG, wealth Gini, tax rate (τ_C^{GT} in GT) and net debtor share in TS and GT

The patterns seen in Table (4) become clearer in this new experiment. We begin with TS and GT in Figure (9). Wealth inequality and the net debtor population share are closely related to the CEG welfare ranking. As seen above, there is no discernible relationship between aggregate welfare and the income tax rate. Financing tertiary education with higher tax rates is not necessarily associated with lower aggregate welfare, even when we have a flat tax rate (in ICLs and TS). However, the most striking result is how welfare is sensitive to debt limits. For each value of the subisidy rate, the optimal debt limit $\underline{b}$ is always located at zero. This result is well documented in Aiyagari economies, it is the price effect of debt described in Obiols-Homs (2011) - also described as a pecuniary externality in Dávila et al. (2012) and Nuño and Moll (2018). The negative impact of laxer debt limits is diminished by increasing the subsidy rate. As we will see further below, a higher subsidy rate compresses the wealth distribution and shifts it to the right, generating an economy with relatively less borrowers, thus decreasing the impact of the price effect of debt. Figure (10) depicts the cases of ICL1 and ICL2, where we see that the direction of results go in the same way as in the previous experiment: there is lower aggregate welfare when the economy generates a larger mass of debtors, and that higher net debtor shares go hand in hand with larger wealth inequality.

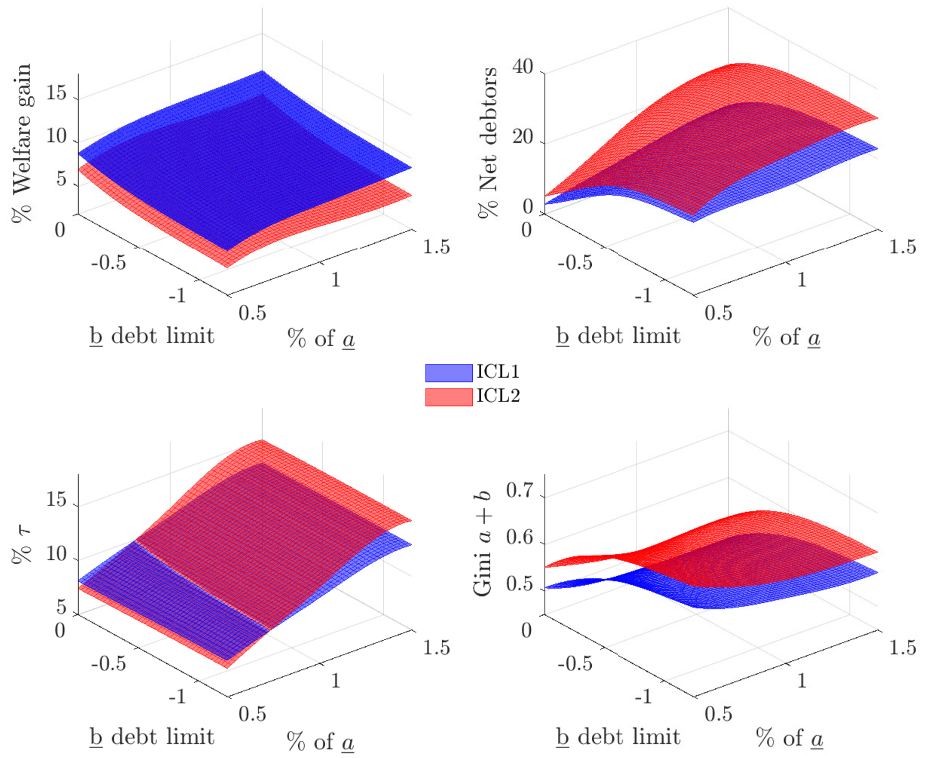

Figure 10: CEG, wealth Gini, tax rate and net debtor share in ICL1 and ICL2

The debt limit on student loans has a similar effect to that of the subsidy rate in Figure (9) with the twist that the relationship between aggregate welfare and $\underline{a}$ in ICL2 is not monotonic. The aggregate welfare gain over NICL peaks at a student loan limit that is tighter than in the baseline. Gains remain positive but then dip as we increase in magnitude $\underline{a}$. Similar results have been found in the literature, Johnson (2013), Ionescu and Simpson (2016) and Abbott et al. (2019) identify a similar effect of student debt limits on welfare: laxer limits on a can provide diminishing gains or even drag down welfare - while more generous subsidies can generate stronger gains. The novelty of this paper is making the connection with the price and quantity effects of debt and comparing systems under this light. Another remarkable result is the scale of the gains; note the magnitudes on the top left panels of Figures (9) and (10): welfare ranks consistently higher in TS and GT relative to ICLs for a large area of the parameter space.

An additional outcome that is worth noting is that the income tax rate can be higher in economies with income contingent loans, relative to that of a system relying on tuition subsidies; this is specially patent in the results for ICL2. It is surprising that the tax rate is also higher in ICL2 than in ICL1. In the latter the government provides relatively more generous income contingency support: it covers the interest *and amortisation* of agents that do not earn a high enough income. This happens for two reasons. First, note that in ICL2 agents accumulate student loan interest when they do not earn enough (when the income contingency protects low earners). Furthermore, for a vast area of the state space, the tax r_p of earnings over the threshold z_T contributes little, if at all, to pay down the student loan balance. This can pose a larger burden on the public sector. Higher taxes and lower income contingency protections in ICL2 explain why it delivers a smaller share of the population with a college degree. This takes us to the second point. ICL1 spends about the same in unemployment insurance, relative to ICL2 - see Table (4) - and yet the overall income tax rate is lower in ICL1, especially when the student debt limit is lax - see Figures (10). As ICL1 delivers a larger share of college graduates, who face a better labour market (less unemployment) and earn more than non-graduates, the cost of unemployment insurance drops, and more agents are able to pay down their student loan balances themselves, lowering the overall income tax rate. On aggregate, it is a better deal for the government to help bring down student loan balances of those that do not earn enough.

The cost of education

Rankings between systems can change depending on the cost of education. Tables (4) and (5) and Figures (27) and (28) in the appendix show that when P is low- (50 % lower than in the baseline), the market failures associated with educational investments do not matter enough to warrant government intervention. Most systems deliver roughly similar welfare gains, which once again move in lockstep with net debtor shares and wealth inequality. The exception is ICL2, which tends to do worse (yet still delivering small positive gains relative to NICL) as the student debt limit is increased, producing more net debtors and more wealth inequality[33]. In all systems, we see that as the debt limit $\underline{b}$ increases in magnitude aggregate welfare gains drop, in line with the price effect described in Obiols-Homs (2011). On the other hand, Tables (4) and (6) and Figures (29) and (30) in the appendix reveal that as the cost of education rises (by 50%), larger differences in welfare gains emerge between systems. Furthermore, we see that tuition subsidies and graduate taxes continue to dominate in terms of aggregate welfare gains, lower net debtor shares and wealth inequality, with the latter system now outperforming the former at high subsidy rates. Additionally, ICL1 clearly outperforms ICL2.

In summary, variations of the debt limits $\underline{b}$, $\underline{a}$ and public funding generosity confirm that the price effect of debt and the relationship between aggregate welfare, net debtor shares and wealth inequality are important driving forces on the relative success of higher education financing schemes. The variations in P illustrate how these effects matter more as the cost of education rises. Additionally, the sensitivity analysis shown above demonstrates that rankings between systems are quite robust. Larger values of P magnify the capital market frictions and riskiness in educational investments and welfare gains from government intervention. As for the benchmark calibration of P (and higher values) and in most of the parameter configurations considered here, it is safe to say that TS and GT often yield higher steady state consumption equivalent gains (and lower debtor shares and wealth inequality) than in student loan systems.

As Figure (32) shows, tuition inflation has outstripped growth in the CPI, healthcare and housing costs. As P rises the the capital market frictions in educational investments increase and will thus magnify welfare differences amongst the different higher education financing schemes. It is also worth noting that larger public expenditure in tertiary education relative to GDP is associated with lower income inequality[34] in the OECD - see Figure (31) in the appendix. Table (4) and Figures (9) and (10) show a similar relationship between wealth inequality and the amount of public funding for tertiary education.

3.2 Disaggregated CEG

A benefit of working with a heterogeneous agent model is that we can evaluate welfare gains due to higher education financing policies at each point of the state space. In this section we compare welfare gains by wealth deciles and by income/ability groups and asses the impact on welfare stemming from the equilibrium distributions of each system. We also look at which wealth and income/ability groups are in favour of moving away from NICL (agents that have at least a non negative welfare gain). The starting point of a disaggregate view of welfare gains

[33]Out of all the experiments considered in this paper, ICL2 is the only system where more public funding of higher education (relative to NICL) increases inequality (when P is low).

[34]Given the lack of available data that is consistent for cross country comparisons of wealth inequality in the OECD, we could only estimate this relationship for measures of income inequality.

begins by updating equation (19) so that we can get a measure of gains at each point in the state space.

$$c(b,a\tilde{,}z,m) = \left[\left(\frac{V_c(b,a,z,m)}{V_o(b,a,z,m)} \right)^{\frac{1}{1-\sigma}} - 1 \right] * 100 \tag{20}$$

With (20) we compare raw value functions (remark that these results omit the effect of changes in the distribution), delivering results such as those in Figure (11), where we compare the ICL1 welfare gains, relative to NICL, of four income/ability groups[35]. Results look broadly similar amongst all alternatives, albeit with different magnitudes of gains and losses. We begin by comparing NICL to ICL1.

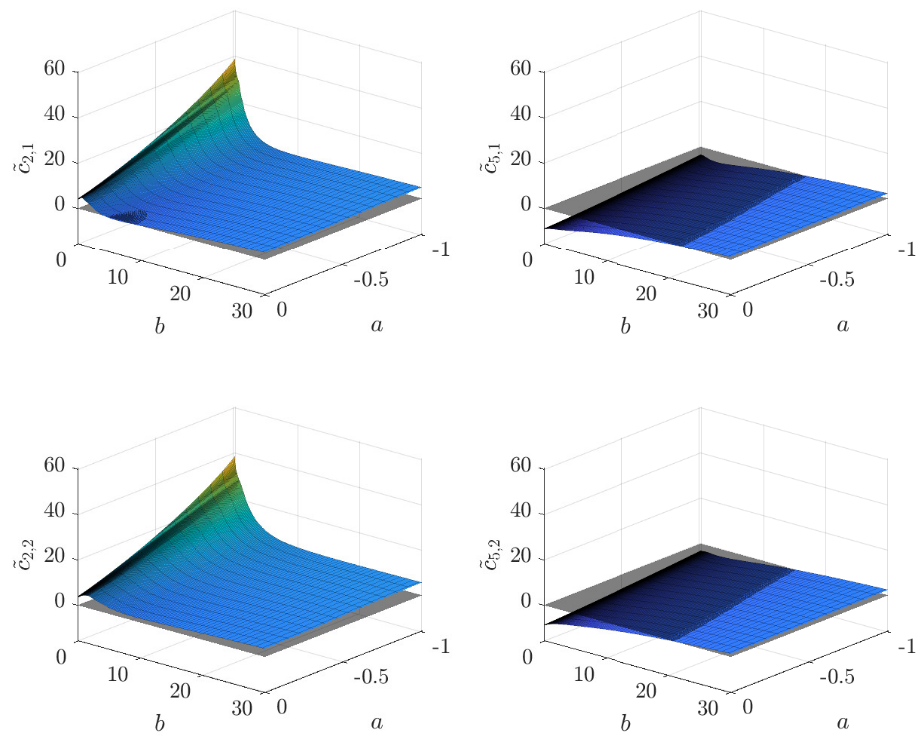

Figure 11: Disaggregate welfare gains of ICL1 vs NICL. The first row depicts welfare gains relative to NICL for high educational ability workers without and with a university degree, respectively. The bottom row shows workers (without/with a degree) with low educational ability. The black surface represents zero welfare gains.

Welfare gains are strongest for agents *without* a university degree (regardless of educational ability), with low wealth and high student debt balances. We emphasize that the agents that stand to gain the most are those that do not have a college degree as much of current debates generally consider these agents as the ones that would lose out the most. As agents are forward looking and are now more likely to afford education (for themselves or their offspring) and to influence the chances of their offspring's educational ability, non college graduates perceive the most gains from more generous public higher education financing. Additionally, the outflow of non graduates towards graduates reduces the college wage premium, raising the labour income

[35] As we have a large state space and menu of higher education systems, we focus on the four most important groups by population size (workers) for the sake of brevity. Comparisons of NICL versus TS and NICL versus ICL2 can be found in Figures (33), (34) and (35) in the appendix.

of these agents. Moreover, as agents are more likely to become college graduates, they are more likely to benefit from a labour market that has shorter unemployment spells, albeit with a lower college wage premium. Agents with large student loan balances also benefit from moving away from NICL, as income contingency protections shield them from large interest payments, skills depreciation and college dropout risk. As education becomes less risky, more agents are likely to invest in education. This is reflected in the larger share of graduates in Table (4).

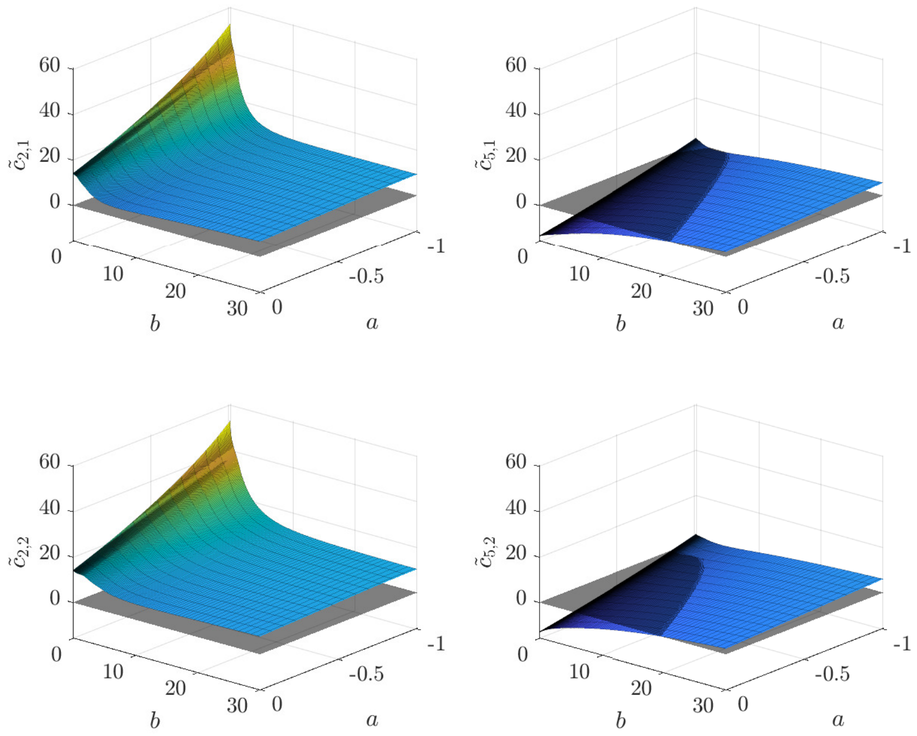

Figure 12: Disaggregate welfare gains of GT 100 % vs NICL. The first row depicts welfare gains relative to NICL for high educational ability workers without and with a university degree, respectively. The bottom row shows workers (without/with a degree) with low educational ability. The black surface represents zero welfare gains.

Agents with a college degree are the ones that experience the largest loses relative to NICL. This is because workers with a college degree have to pay back their student loans (just like in NICL) and must compete with a larger share of university graduates, facing a lower college wage premium. The larger the graduate share is the worse off low wealth university graduate workers are, as they derive relatively more of their income from labour. We also see that high wealth agents gain from moving away from NICL as they derive most of their income from wealth and as alternative systems have higher equilibrium interest rates. We find similar results when comparing NICL against ICL2, TS and GT. If the alternative systems generate more lenders (less net debtors) than NICL, then the distributions of agents in the alternative regimes reinforce aggregate welfare gains. Raw value function comparisons give us one side of the picture of the make up of aggregate welfare; ultimately, we need to see where do the new distribution of agents places more mass, in areas with positive or negative welfare gains. We elaborate on distributions further below.

Since the state space is not the same between systems without student loans (TS and GT), we compare the single asset TS/GT alternatives against each value of student loans in the benchmark. Figure (12) shows results for NICL versus GT 100 %. The magnitude of gains are overall larger than in the case of ICL1 yet we can derive the same conclusions from comparisons of NICL against any of the other higher education financing policy variants considered herein. The question is where does the new distribution places more/less mass relative to NICL (in terms of income groups and their relative position in the wealth distribution). Hence, we now consider the impact of the distribution in welfare gains.

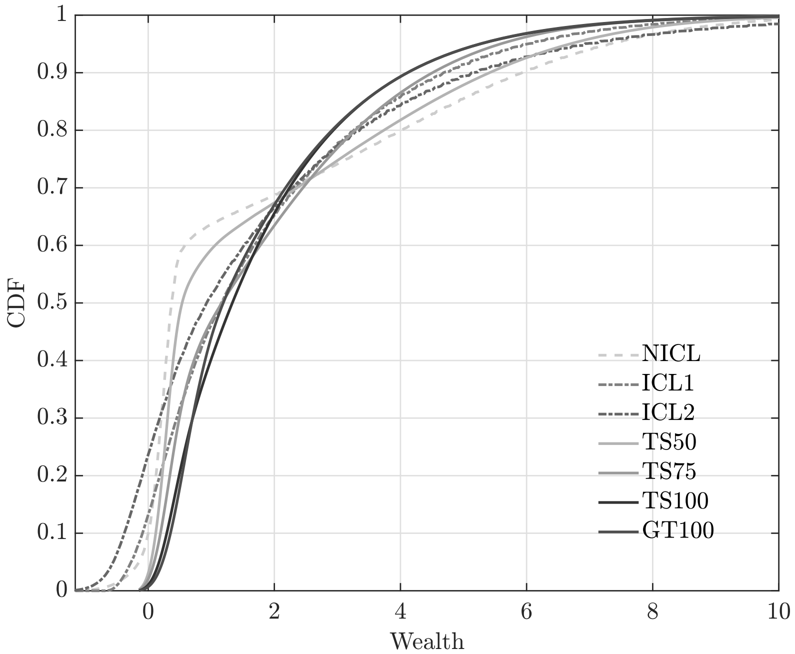

Figure 13: Wealth CDF. Each system is coloured by its equilibrium interest rate; the darker the colour, the higher the equilibrium interest rate.

Impact of the distribution: Figure (13) confirms what we saw in Table (4): TS and GT generate relatively less debtors, reduce wealth inequality and increase the capital stock. Partial and full subsidisation of tuition puts more mass in moderately high values of b and less on the high-low extremes[36]. As tuition subsidies increase (or income contingency protections), precautionary savings fall and the equilibrium interest rate goes up, as can be seen in Table (4). Since the distribution places more mass in higher income graduates, the capital stock rises, despite the fall in precautionary savings. This can be seen by an increase in mean wealth accompanied by a compression of the distribution. This becomes more patent as the subsidy rate rises, analogous to how results would look if employment and skill depreciation transition rates were even more favourable for educated agents (if the risk of falling into bad states decreased)[37]. Finally, we must consider that besides the direct impact that the distribution has in moving mass to regions of the state space with positive welfare gains, it also plays a role in changing aggregate quantities and prices. For instance, more generous higher education policies generate a larger capital stock *and* prices, as the larger graduate share increases the effective aggregate labour supply[38].

[36] Angelopoulos et al. (2017) reach similar findings in an environment where education types are determined exogenously.

[37] These results are in line with Akyol and Athreya (2005); subsidies lower risk in educational investments and in turn reduce precautionary savings while raising the equilibrium interest rate.

[38] This is reminiscent of Hanushek et al. (2003), where tuition subsidies increase welfare and reduce inequality in the presence of productive externalities from college educated agents. Even though this model does not feature direct production externalities from college graduates, we still see that increasing the graduate share relative to NICL can yield similar benefits.

We see similar results, to a lesser extent, with generous income contingent protections, as in ICL1. As the student loan program becomes more generous, precautionary savings decrease and so does the spread of the distribution. Despite the fall in precautionary savings the capital stock is bigger. Once again, this happens as more mass is placed in the educated group. Given the debt cancellation offered in the income contingent loan programs, more agents will have no student loans in the ICLs than in NICL; the distribution will place more mass in the regions where welfare gains are strongest.

Average welfare gains by income and ability groups: Next, we look at welfare gains by income and ability groups. We can use either the old (NICL) or new density (the alternative) but for the sake of brevity we use the density in NICL. We focus first in the single asset cases as the varying subsidy rates reveal how public funding generosity magnifies gains and losses across different groups. Agents without a college degree - the first three groups in Figure (14) - gain the most and more so if they have high educational ability. The gains increase as the subsidy rate rises, both in TS and GT. The opposite occurs for the last two groups, the unemployed and employed with a college degree. They perceive small gains when education is subsidised at 50% of its cost but lose at subsidy rates of 75 % and 100%, and more so in the latter case. GT appears to have gains of the same magnitude as TS although it minimises the losses for college graduates. We obtain similar results when comparing NICL versus the ICLs; the more generous the income contingent student loan program is, the more the first three groups gain on welfare. The exception is that the losses experienced by college graduates are virtually identical in ICL1 and ICL2.

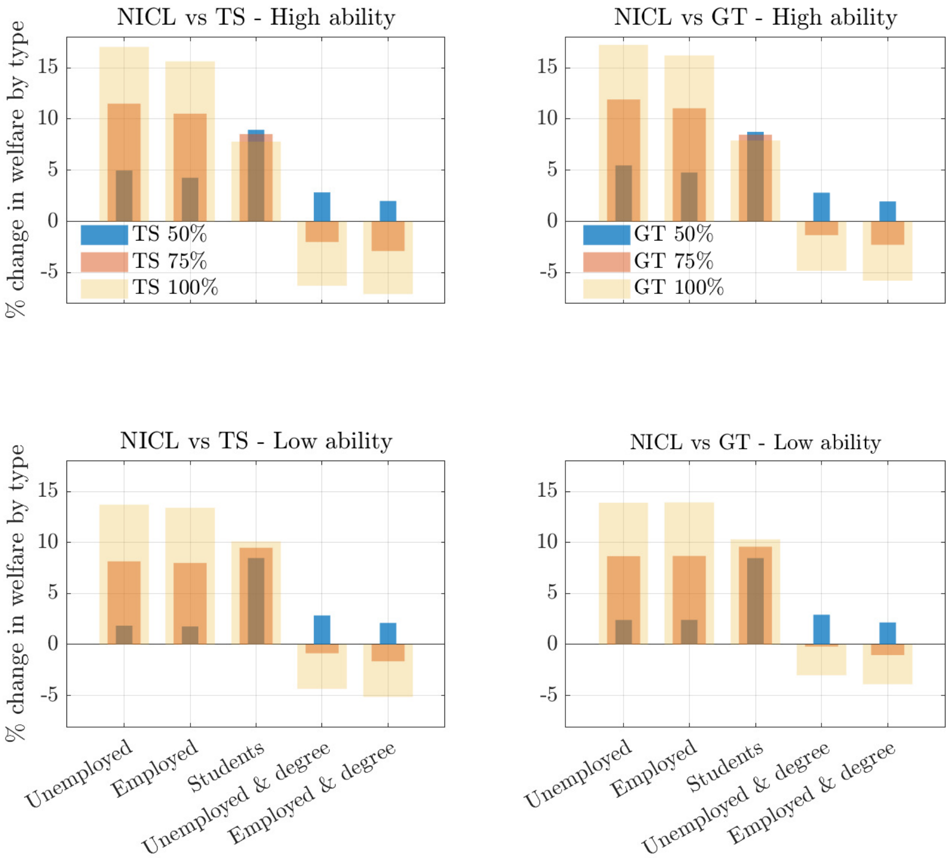

Figure 14: Welfare gains over NICL, by income and ability groups, of tuition subsidies and graduate taxes

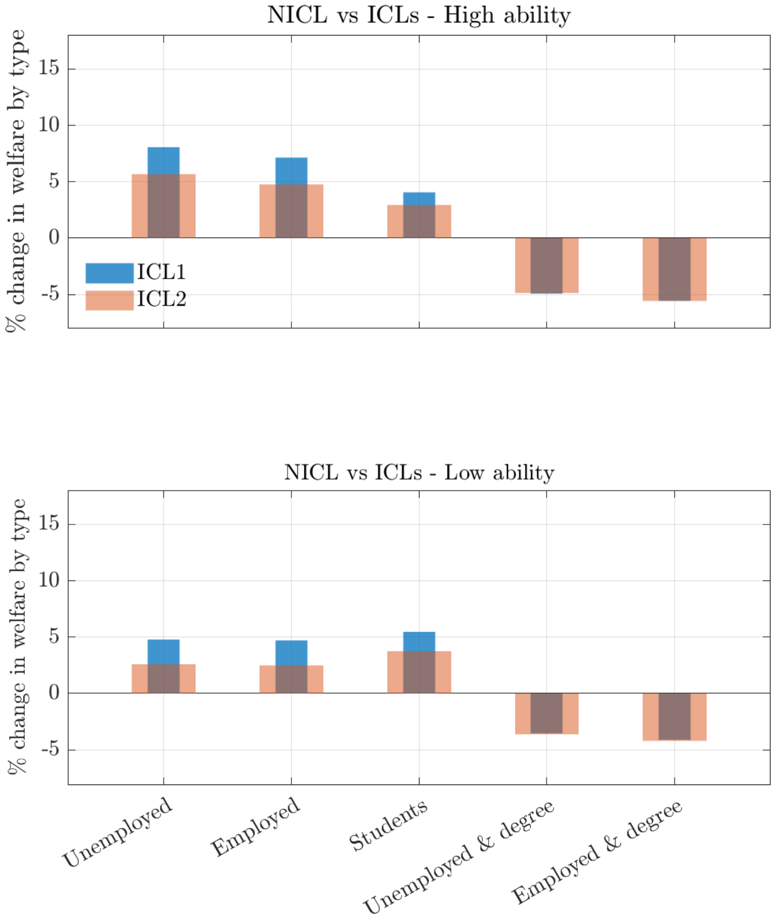

Figure 15: Welfare gains over NICL, by income and ability groups, of income contingent loans 1 and 2

Average welfare gains by wealth decile: We now look at welfare gains in each wealth (the sum of cash b and student loans a) decile. Figure (16) shows that in all systems welfare gains are strongest for those that have wealth at or below the sixth decile. Moreover, Figure (17) shows that in the first six deciles, at least 90% of agents experience non-negative welfare gains and are thus in favour of moving from NICL to any of the alternative systems. The more generous public funding is, the more the first six deciles gain. The opposite occurs for the top wealth deciles and this is true for ICLs, TS and GT. Additionally, we obtain a similar finding to the one we observed when looking at welfare gains by income and ability groups; ICL1 magnifies gains and minimises losses, when compared to ICL2. The magnitude of losses in ICLs is roughly the same as those in TS and GT although the gains are more substantial in TS and GT. The fact that wealth rich agents gain less is unsurprising as the marginal utility of consumption is lower for these groups. A more important lesson to take from here, in light of the the price effect of debt, is how low wealth agents gain more as they are shielded from the downsides of becoming debtors and failing in educational investments. In fact, the strongest gains are in the first decile, where all debtors are located[39].

[39]In NICL, zero wealth is at the ninth centile, so the first decile is mostly composed of net debtors.

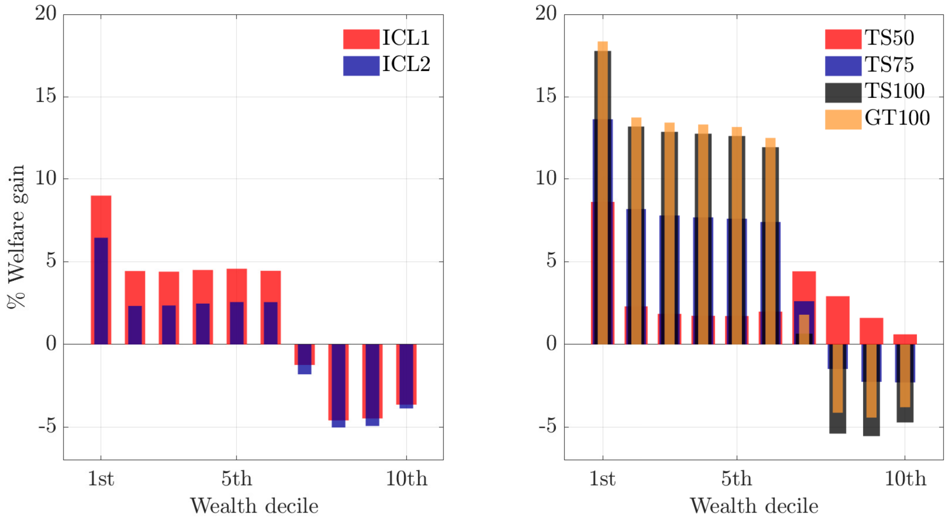

Figure 16: Welfare gains over NICL, by wealth deciles

Mass in favour of change: So far we have considered disaggregate welfare gains at each point in the state space, as averages of income/ability groups and by wealth decile. In this subsection we asses which income/ability groups and wealth deciles are in favour of moving away from NICL. That is, we consider the share of the population in each subgroup that achieves at least non-negative welfare gains from any of the alternative higher education financing policies considered herein. Figures (17) and (18) display the results from the perspective of income/ability groups and wealth deciles, respectively. Both figures lead to similar conclusions: non-college graduates overwhelmingly support more public financing of higher education, either with income contingent student loans, graduate taxes or tuition subsidies. Furthermore, the first six deciles of wealth also strongly favour such policies. The mass of agents in favour of change falls as the support becomes more generous, but this is masked when computing averages in each income/ability group (or wealth decile), as the larger gains compensate for the fall in support for more generous public funding. Nevertheless, the differences in support due to generosity in funding are very small in the first three income groups (regardless of ability type) and in the first size wealth deciles. Overall, TS and GT garner more support and larger gains than in the ICLs, with GT minimising losses when compared to TS.

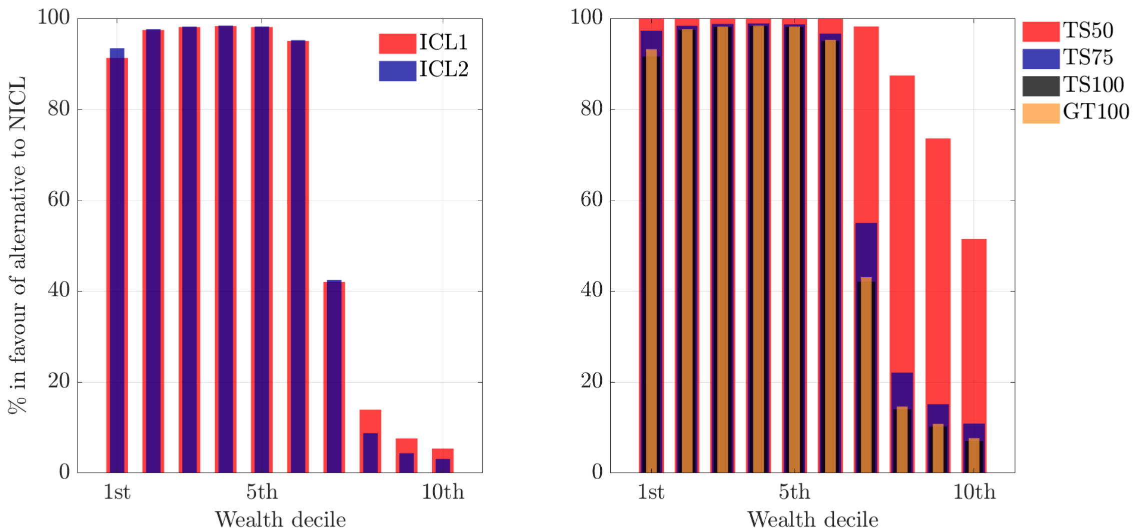

Figure 17: Mass in favour of moving to alternative system, by wealth decile.

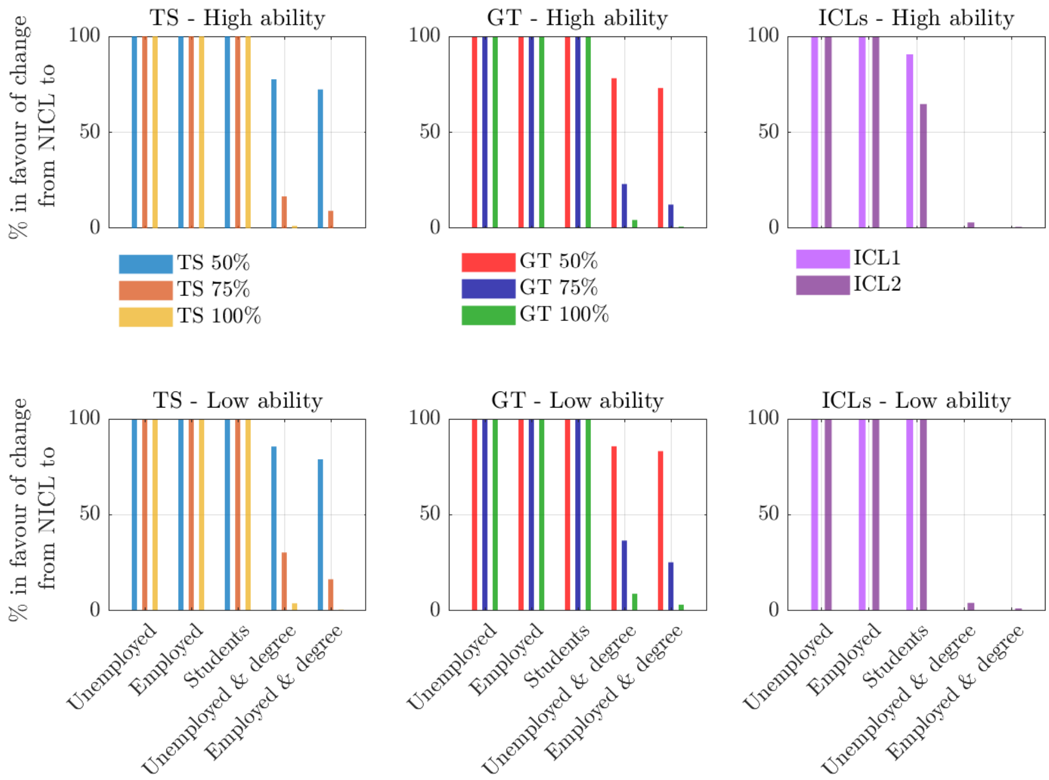

Figure 18: Mass in favour of moving away from non income contingent loans, by group, towards tuition subsidies, graduate taxes and income contingent loans

4 Transitions

Since there are large steady state welfare differences amongst the five HE systems considered in this paper, is it worth making the transition to the system yielding the largest welfare gain? This section seeks to answer that question. The next experiment considers an unexpected, immediate[40] and permanent change of the higher education financing system. There will be two variants of the experiment when we move from NICL to TS or GT, where the transitions are done with or without the government absorbing all student loan balances[41]. In the former case, the government absorbs all student loans and continues making the same amortisation and interest payments as agents used to until all balances are paid off. Figure (19) overlays the static and dynamic (gains due to the transition) welfare gains while figures (20) and (21) show the transitional dynamics of key aggregates. We compute aggregate dynamic welfare gains in a similar fashion as in (18), although this time we use aggregate welfare an instant right after the transition begins.

$$\tilde{c} = \left[\left(\frac{\hat{V_{t+\Delta}}}{\hat{V_t}}\right)^{\frac{1}{1-\sigma}} - 1\right] * 100, \tag{21}$$

where

[40]Similar experiments where policy changes are announced in advance yield lower gains since agents postpone enrolling in university until the subsidy is in place. This has negative aggregate effects given that it initially lowers θ_5. A life-cycle formulation would dampen such an effect since the education choice will probably be made once in a single life time and sooner rather than later.

[41]We do this for two reasons. Firstly, suggestions of debt cancellation along with a transition to a system without student loans generate a lot of policy interest. Second, it substantially reduces the computational cost of the transition as it allows us to drop one state variable.

$$\hat{V}_t = \sum_{m=1}^{2}\sum_{i=1}^{5}\int_{\underline{a}}^{0}\int_{\underline{b}}^{\infty} V(b,a,i,m,t)\mathrm{d}b\mathrm{d}a, \tag{22}$$

and where $\hat{V}_t$ and $\hat{V_{t+\Delta}}$ are the aggregate values before (NICL) and an instant after the policy change, respectively.

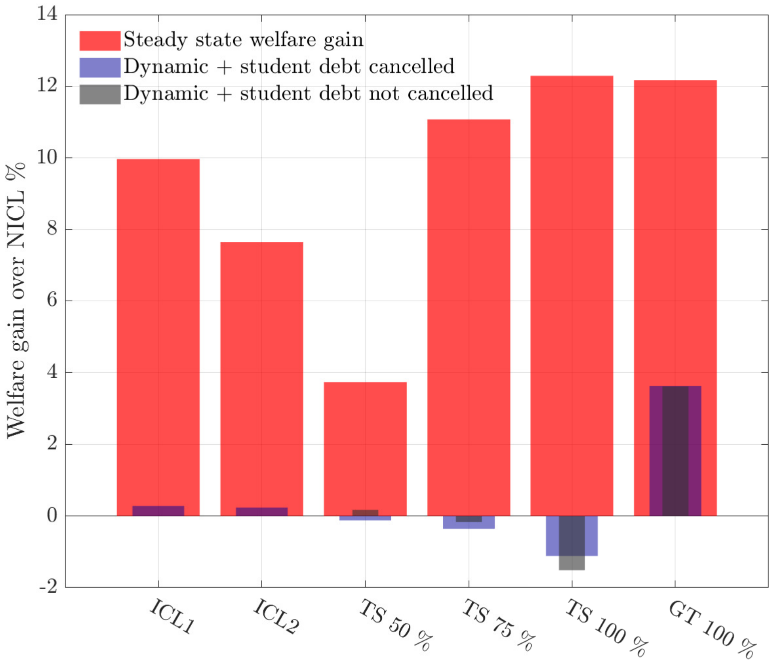

Figure 19: Consumption equivalent gain of income contingent student loans 1 and 2 (ICL1 and ICL2), tuition subsidies (TS) and graduate taxes (GT) at 50, 75 and 100 % of educational costs, respectively. The gains are relative to the baseline (non income contingent student loans - NICL). Bars in red, purple and black represent steady state, dynamic with and without student loan cancellation, respectively.

Figure (19) shows that steady state comparisons between systems can be misleading and that transitions are costly. Moving from NICL to ICLs yield a positive aggregate welfare gain, but the gains are about twenty times smaller than those from the static comparisons. The dynamic aggregate welfare gains of ICL1 and ICL2 are 0.281% and 0.234%, respectively. When moving from NICL to TS with student debt cancellation the transition gains turn out to be negative, with losses increasing by the size of the subsidy. The dynamic welfare gains of TS 50%, 75% and 100% are -0.123%, -0.357% and -1.116%, respectively. The transition that stands out is that of NICL to GT, with a gain of 3.63%. When we repeat these experiments without student debt cancellation we get somewhat similar results, with the exception of NICL to TS50 %, which delivers a small aggregate welfare gain of 0.173%. We explore further below the impact of student debt cancellation.

The transition paths of key aggregates, along with the previous results on borrowing limits and price effects of debt, illuminate why transitions deliver such rankings. Transitions are painful; all of the the price effects of debt on welfare get worse during the first years. Interest rates, the share of net debtors and inequality go up during the early phase of the transition. During such periods income falls and taxes go up as well. As the increased influx of students generates more graduates the college wage premium drops. Broadly speaking, most agents have less income during the early years and the gains of the transition do not materialise until a few years later. Given that agents discount future gains, more weight is placed on the immediate sacrifices incurred during the transition than on the distant benefits from education financing reforms. Hence, despite the substantial steady state gains, transition costs make policy changes less appealing than when we just compare steady states.

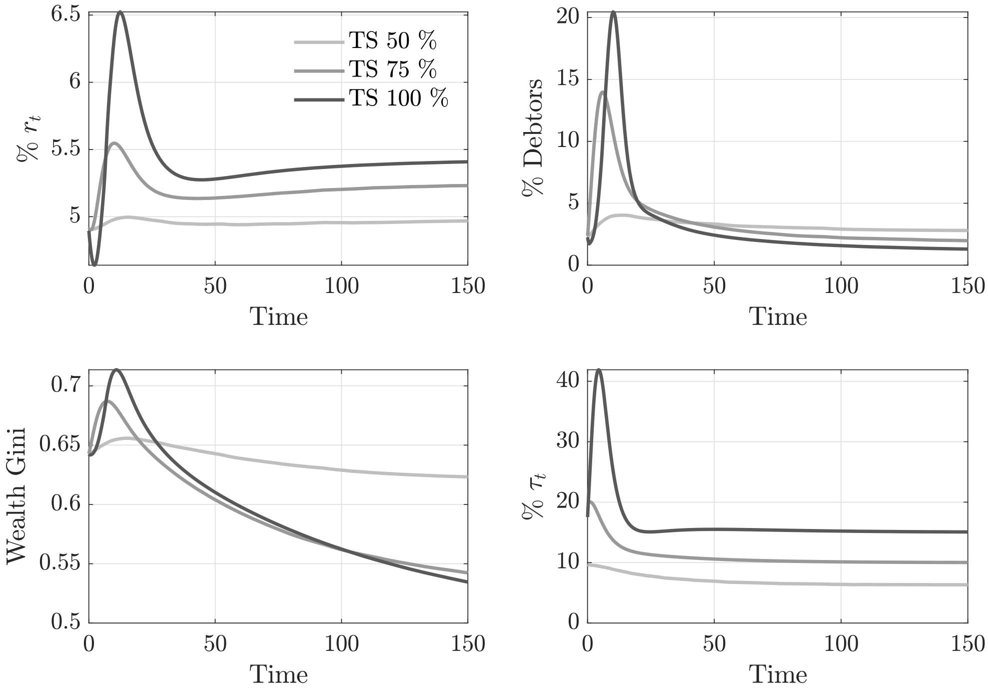

Figure 20: Transition from benchmark (NICL) to tuition subsided at different rates (TS)

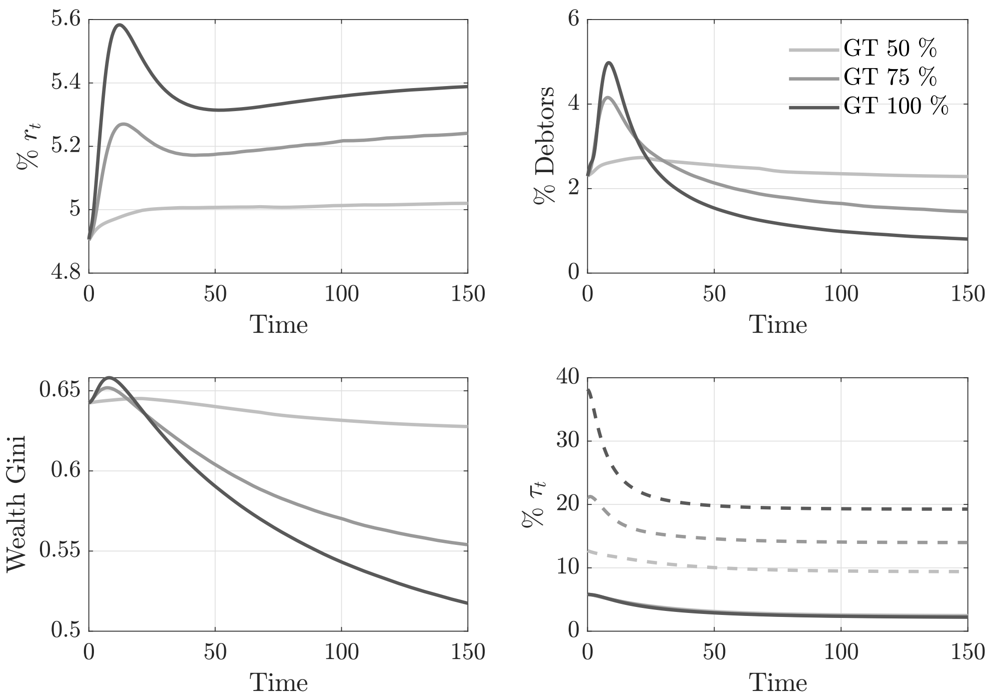

Figure 21: Transition from benchmark (NICL) to tuition subsided at different rates through graduate taxes (GT). Dashed and solid lines in the bottom right panel depict graduate labour income taxes and non-graduate labour income taxes, respectively.

The transitions from NICL towards ICLs generate moderate increases in interest and tax rates, relative to tuition subsidies. The net debtor share and inequality experience larger fluctuations. As there is a smaller influx towards graduates, the college wage premium does not fall as much. The transition towards GT goes further in mitigating the losses of the early years, by shifting taxation exclusively unto college graduates and thus moderating the influx of graduates and its effect on the college wage premium. The transition to GT experiences the smallest increase in interest rates, inequality, net debtors and taxes (for non graduates)[42]. We find, as in Heijdra et al. (2017), that while tuition subsidies and graduate taxes deliver substantial welfare gains in steady state, tuition subsidies (graduate taxes) turns out to be the worst (best) during transitions. As all systems deliver large gaps between steady state and dynamic comparisons, even changing rankings, it is clear that policy changes in higher education financing should factor the costs of transitions.

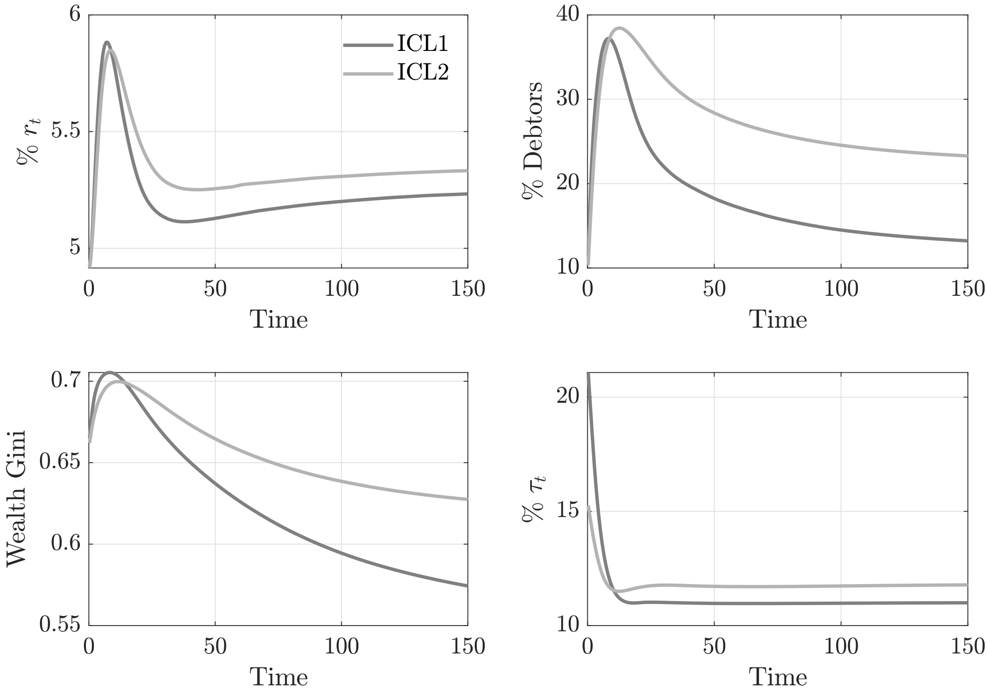

Figure 22: Transition from benchmark (NICL) to income contingent loan systems (ICL1 and ICL2)

NICL to TS with and without student debt cancellation: In this subsection we analyse the impact of cancelling student debt when moving from NICL to TS[43]. We first look at the transition to TS 100%. A few key aggregates are shown in Figure (23), comparing transitions with and without student debt cancellation. When student loans are not cancelled *and* tuition is entirely covered by the state there is a very strong incentive for non college graduates with student debt to enrol at university. This is for two reasons; they can defer student loan payments and avoid income taxation during their studies. The early years of the transition, marked by higher interest rates, make this option more attractive. The loss in income tax revenue, due to an outflow from non college grad workers towards students, coupled with the rise in education expenditure, push the income tax rate to extremely high levels. Furthermore, the larger and quicker increase in the supply of graduates delivers a more pronounced fall in the college wage premium. When student

[42]Previous versions of this paper fixed the college wage premium, which boosted steady state and dynamic welfare gains for all systems. A fixed CWP maintains a stronger incentive to become a graduate and thus places more mass in areas of the state space where there is less income uncertainty.

[43]We gain similar conclusions from analysing transitions from NICL to GT. For the sake of brevity these results are omitted but they can be reproduced upon request.

loans are cancelled there is no incentive to defer student loan payments and thus the inflow into students and the increase in the supply of graduates is more gradual. Moreover, wiping out student debt substantially reduces the share of the population in net debt and wealth inequality. As a consequence the aggregate welfare loss due to the transition is much worse when student debts are not cancelled.

The transition towards TS 50 % shifts the ranking between student debt cancellation or no cancellation. In this instance a transition with student debt cancellation triggers an initial larger outflow of non college graduate workers towards students, producing a larger loss in tax revenue and larger increase in public education expenditure along with a faster drop in the college wage premium. As the initial years of the transition carry more weight in assessing welfare, moving to TS 50% with debt cancellation delivers a smaller change in welfare (overall drop in welfare), relative to the case without debt cancellation (a small aggregate welfare gain of 0.173%).

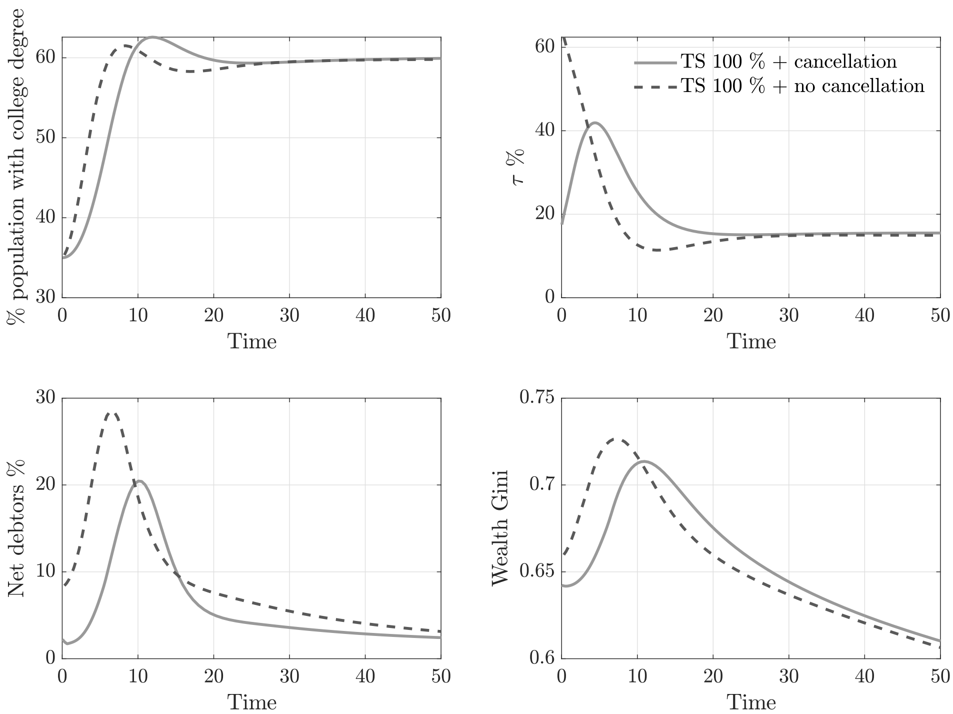

Figure 23: Transition from benchmark (NICL) to TS 100 %

5 Conclusion

In this paper we evaluate the welfare and inequality outcomes of five different higher education financing schemes, with the help of a heterogeneous agent production economy in continuous time, extended to allow for endogenous educational choices. The main contribution of this study is to evaluate tertiary education financing under the light of *the price effects of debt*. When we ignore the pecuniary externalities described in Obiols-Homs (2011), Nuño and Moll (2018) and Angelopoulos et al. (2017) we miss general equilibrium effects that are powerful enough to tilt the balance on which higher education system yields the largest aggregate and individual welfare gains. Higher education systems that deliver large shares of the population as net debtors tend to have larger inequality and lower welfare. This article also contributes in identifying when government intervention in tertiary education can increase welfare, reduce inequality and at what cost. This contribution can be broken into four findings.

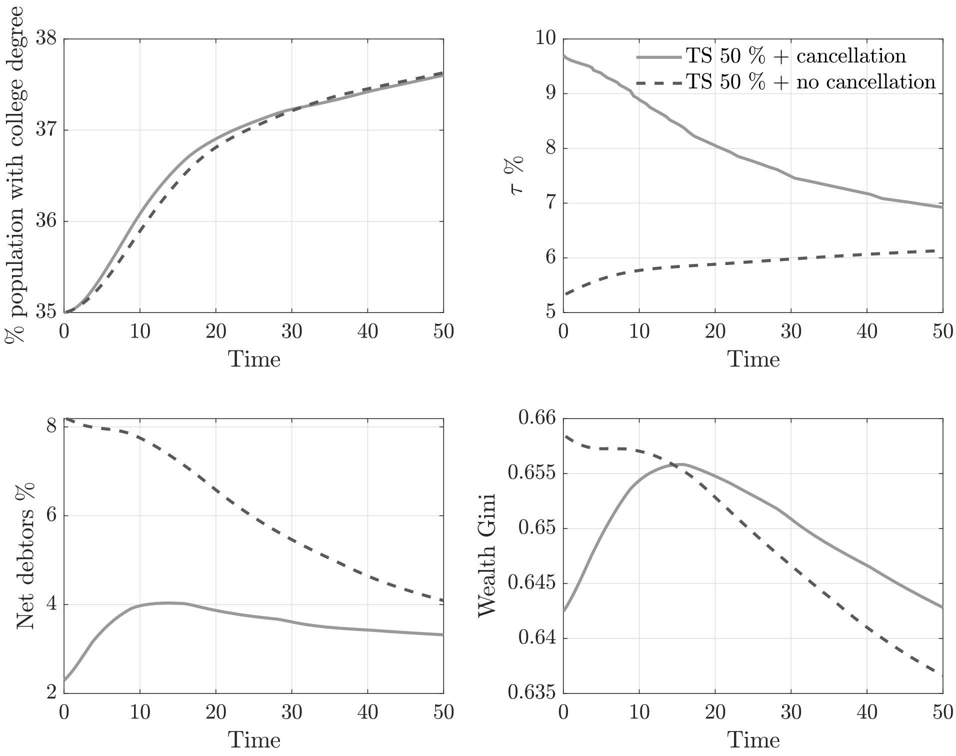

Figure 24: Transition from benchmark (NICL) to TS 50 %

First, the ranking between systems depend on the cost of education. When the cost of education is low, the market failures associated with educational investments do not matter enough to warrant government intervention. In a model calibrated to the United States, government guaranteed income contingent loans, tuition subsidies and graduate taxes can deliver substantial welfare gains and reduction in inequality over the current system. This is specially true for graduate taxes and tuition subsidies, which yield better results than two variants of income contingent loans.

Second, while we show significant steady state welfare differences between various higher education systems, large transition costs from one regime to another diminish the desirability of policy changes. That is, comparing steady states alone may be misleading for policy, transition costs must be factored in. When transition costs are taken into account, fully funded higher education with graduate taxes deliver the highest welfare gains.

Third, flat tax rates can be higher (relative to regimes with tuition subsidies) in systems relying on income contingent student loans. Fourth, public financing of higher education may increase inequality when the cost of education is low; if this cost rises to current levels or beyond, inequality falls as public sector support increases. This is particularly true with tuition subsidies; they yield the lowest inequality outcomes in all the systems considered in this paper.

This paper will be extended by disaggregating the college wage premium, as recent UK based studies - Belfield et al. (2018a), Belfield et al. (2018b) and Britton et al. (2020) - have shown that while the average premium has remained stable, if not rising, it is driven by substantial heterogeneity and large outliers, where the returns to higher education vary by subject, institution and agents' gender. Should we fund higher education generously when the distribution of returns is highly skewed? Another extension seeks to understand how increased longevity and increasing exposure to automation risk may warrant repeated educational investments and additional government support in higher education. Would longer lives lead more people to retrain and go back to university at older ages?

6 Appendix

6.1 Kolmogorov forward equations

Let g represent the density, ∂_k denote the partial derivative w.r.t. k and the subscripts in g depict the agent type. Let m and $-m$ denote the ability types.

$$\partial_t g_1^m = -\partial_a[S_{a,1}^m g_1^m] - \partial_b[S_{b,1}^m g_1^m] + \lambda_{21} g_2^m - (\lambda_{12}+\kappa) g_1^m + \lambda_{x,m}^S g_3^m + \lambda_x^U g_4^m + \chi_1^m \quad b < b_1^{m,\dagger}(a),$$
$$\partial_t g_2^m = -\partial_a[S_{a,2}^m g_2^m] - \partial_b[S_{b,2}^m g_2^m] + \lambda_{12} g_1^m - (\lambda_{21}+\kappa) g_2^m + \lambda_x^E g_5^m + \chi_2^m \quad b < b_2^{m,\dagger}(a),$$
$$\partial_t g_3^m = -\partial_a[S_{a,3}^m g_3^m] - \partial_b[S_{b,3}^m g_3^m] - [\lambda_{x,m}^S + \lambda_{34} + \lambda_{35} + \kappa] g_3^m + \chi_3^m,$$
$$\partial_t g_4^m = -\partial_a[S_{a,4}^m g_4^m] - \partial_b[S_{b,4}^m g_4^m] + \lambda_{34} g_3^m + \lambda_{54} g_5^m - [\lambda_x^U + \lambda_{45} + \kappa] g_4^m + \chi_4^m, \tag{23}$$
$$\partial_t g_5^m = -\partial_a[S_{a,5}^m g_5^m] - \partial_b[S_{b,5}^m g_5^m] + \lambda_{35} g_3^m + \lambda_{45} g_4^m - [\lambda_x^E + \lambda_{54} + \kappa] g_5^m + \chi_5^m, \tag{24}$$

where χ_i^m captures the re-injection of mass due to death (newborns), skills depreciation shocks and student loan cancellation and flows from non-college-educated agents into students[44]. In single asset regimes we drop the dependence on a. In steady state $\dot{g} = 0 \ \forall\ i, a, b$.

Before elaborating on each χ_i^m, let's recall some notation with the help of Figure (25). For each agent type and for each value of student loans, there is a free boundary $b_i^{m,\dagger}(a)$ that determines when the agent chooses to become a student. Once an agent reaches a level of wealth b larger than $b_i^{m,\dagger}(a)$ then it is immediately moved to the student type with new values of wealth and student loans $(b_i^{m,*}(b,a), a_i^{m,*}(b,a))$. We suppress the notational dependence of (b^*, a^*) on state variables for the sake of exposition, except when we describe χ_3^m.

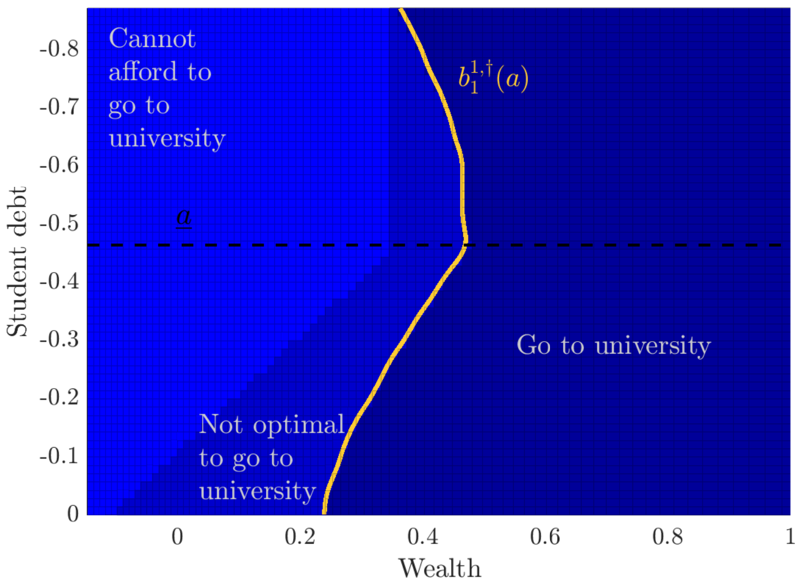

Figure 25: Decision map of unemployed agents without a college degree and with high educational ability in NICL, overlayed with $\underline{a}$ the student debt limit.

Any agent that dies or gets hit with skill depreciation or college drop out shocks, with wealth beyond $b_i^{m,\dagger}$, is immediately replaced (or substituted by newborn in the case of death) as a student with new values of wealth (b^*, a^*). We now expand on each χ^m.

[44]Note that inflows to g_1^m and g_2^m beyond their respective boundaries $b_i^{m,\dagger}(a)$ are immediately redirected to $g_3^m(b^*, a^*)$.

$$
\chi_1^m = \kappa\delta(a)\left(\sum_{i=1}^{5}\lambda_{i,m}^m g_i^m + \sum_{i=1}^{5}(1-\lambda_{i,-m}^{-m})g_i^{-m}\right)\mathbb{1}_{\{b>0\}} + \lambda_{np}g_1^m[\delta(a)-1]
$$

$$
+ \kappa\delta(b)\delta(a)\left(\sum_{i=1}^{5}\lambda_{i,m}^m g_i^m + \sum_{i=1}^{5}(1-\lambda_{i,-m}^{-m})g_i^{-m}\right)\mathbb{1}_{\{b\leq 0\}} \quad (25)
$$

The first and third terms represent the introduction of newborns to the *unemployed and no college degree* state, where they are born with non-negative assets b and zero student loan balance a. The first term captures the inflow of newborns that inherit positive values of wealth. Newborns whose parents died with debt in b are born with zero wealth (the third term). Note how the inflow of newborns takes into account the intergenerational transmission of educational ability, which is influenced by the educational status and ability of parents[45]. The Dirac deltas $\delta(a)$, $\delta(b)$ direct mass to regions where $a = 0$ and $b = 0$, respectively. The second term accounts for student loan cancellations (outflows from all a's and inflows towards $a = 0$), which direct mass towards $a = 0$.

We will need a bit more notation for χ_3. Let b' and a' denote wealth and student loan balances of an agent right before getting hit with a shock that would make her immediately enroll at university[46].

$$
\begin{aligned}
\chi_3^m = & \left[\lambda_{x,m}^S g_3^m(b',a') + \lambda_x^U g_4^m(b',a')\right]\mathbb{1}_{\{b_1^{m,*}(a^*(b',a'))\geq b_1^{m,\dagger}(a^*(b',a'))\}} + \lambda_x^E g_5^m(b',a')\mathbb{1}_{\{b_2^{m,*}(a^*(b',a'))\geq b_2^{m,\dagger}(a^*(b',a'))\}} \\
& + \lambda_{np}g_3^m(b,a)[\delta(a)-1] \\
& + \kappa\delta(a)\left(\sum_{i=1}^{5}\lambda_{i,m}^m g_i^m(b',a') + \sum_{i=1}^{5}(1-\lambda_{i,-m}^{-m})g_i^{-m}(b',a')\right)\mathbb{1}_{\{b>0\}}\mathbb{1}_{\{b_1^{m,*}(a^*(b',a'))\geq b_1^{m,\dagger}(a^*(b',a'))\}} \\
& + \kappa\delta(b)\delta(a)\left(\sum_{i=1}^{5}\lambda_{i,m}^m g_i^m(b',a') + \sum_{i=1}^{5}(1-\lambda_{i,-m}^{-m})g_i^{-m}(b',a')\right)\mathbb{1}_{\{b\leq 0\}}\mathbb{1}_{\{b_1^{m,*}(a^*(b',a'))\geq b_1^{m,\dagger}(a^*(b',a'))\}}
\end{aligned} \quad (26)
$$

Note that we now make the dependence of (b^*, a^*) on b' and a' explicit. The first terms are inflows from college dropouts and unemployed college grads that got hit with a skills depreciation shock and that have sufficient wealth to become students once again. The second term captures college grads in employment hit with a skill depreciation shock and that have enough wealth to become students once again. The third term tracks student loan cancellations (outflows from all a's and inflows towards $a = 0$), which direct mass towards $a = 0$. The fourth and fifth terms are the inflow of newborns that have sufficient wealth to become students. The fifth term is the inflow of mass coming from agents born with no wealth[47] (whose parents died with debt). The remaining $\chi_i^m, i = 2, 4, 5$ terms capture student debt cancellation.

$$
\chi_i^m = \lambda_{np}g_i^m(b,a)[\delta(a)-1] \quad \text{for } i = 2,4,5 \quad (27)
$$

6.2 Market clearing

In this subsection we develop a heuristic proof showing that $K_S = K_D$ implies $Y = C + I +$ Education costs. The same steps can be applied to any regime and will lead to the same

[45] See subsection (6.4) on the appendix, which elaborates on ability transition rates and the intergenerational transmission of ability.

[46] We use this notation since we need to track the wealth and student debt values (before tuition costs) of agents that get hit with shocks that push them to values above $b_i^{m,\dagger}(a)$.

[47] If debt limits are large enough to meet tuition costs an agent could enroll in university with zero wealth.

conclusion. For the sake of brevity, we illustrate this with the TS regime. Without loss of generality and for the sake of brevity, assume that $\underline{b} = 0$ and that there are three groups of agents: non-college-grad workers, students and graduates in employment[48]. As there is no unemployment insurance, labour income taxes are levied to cover education expenses only. Non-college-grads can become students if they have enough wealth and find it optimal to do so. It takes on average $1/\lambda_{23}$ years to graduate and college graduates receive skill depreciation shocks at rate λ_x. Students drop out of college at rate λ_s Agents die at rate κ. Let $\tilde{P} = P(1-s)$, the cost of education that an agent faces after tuition subsidies. Let g and s represent the density and drift of wealth for each agent type, respectively. The KFEs of the reduced model are shown next.

$$\partial_t g_1(b) = -\partial_b[s_1(b)g_1(b)] + \lambda_s g(b) + \lambda_x g_3(b) + \kappa(g_2(b) + g_3(b)) \quad \forall\ b < b^\dagger$$
$$\partial_t g_2(b) = -\partial_b[s_2(b)g_2(b)] - (\lambda_{23} + \lambda_s + \kappa)g_2(b) + (\lambda_s + \lambda_x + \kappa)\left[g_2(b+\tilde{P}) + g_3(b+\tilde{P})\right] \mathbb{1}_{\{b > b^\dagger - \tilde{P}\}}$$
$$\partial_t g_3(b) = -\partial_b[s_3(b)g_3(b)] + \lambda_{23} g_2(b) - (\lambda_x + \kappa)g_3(b)$$

Following Nuño and Moll (2018) we start by the aggregate law of motion of capital.

$$\begin{aligned} K &= \sum_{i=1}^{3} \int_{\underline{b}}^{\infty} b g_i(b) \mathrm{d}b \\ \frac{\mathrm{d}}{\mathrm{d}t} K &= \sum_{i=1}^{3} \int_{\underline{b}}^{\infty} b \partial_t g_i(b) \mathrm{d}b \\ 0 &= \sum_{i=1}^{3} \int_{\underline{b}}^{\infty} b \partial_t g_i(b) \mathrm{d}b \quad \text{(in steady state)} \end{aligned} \tag{28}$$

Expanding the term on the right hand side gives us the following result.

$$\begin{aligned} \sum_{i=1}^{3} \int_{\underline{b}}^{\infty} b \partial_t g_i(b) \mathrm{d}b = &- \sum_{i=1}^{3} \int_{\underline{b}}^{\infty} b \partial_b[s_i(b)g_i(b)] \mathrm{d}b + \int_{b^\dagger - \tilde{P}}^{\infty} b \left[(\lambda_s + \kappa)g_2(b+\tilde{P}) + (\lambda_x + \kappa)g_3(b+\tilde{P})\right] \mathrm{d}b \\ &- \int_{b^\dagger}^{\infty} b \left[(\lambda_s + \kappa)g_2(b) + (\lambda_x + \kappa)g_3(b)\right] \mathrm{d}b \mathrm{d}b \\ \sum_{i=1}^{3} \int_{\underline{b}}^{\infty} b \partial_t g_i(b) \mathrm{d}b = &- \sum_{i=1}^{3} b s_i(b) g_i(b) |_{\underline{b}}^{\infty} + \sum_{i=1}^{3} \int_{\underline{b}}^{\infty} s_i(b) g_i(b) \mathrm{d}b \\ &- P(1-s) \int_{b^\dagger}^{\infty} \left[(\lambda_s + \kappa)g_2(b) + (\lambda_x + \kappa)g_3(b)\right] \mathrm{d}b \end{aligned} \tag{29}$$

The first term on the right hand side is equal to zero. We now expand the second term. Let θ_i denote the share in the population of the ith type of agent.

[48]Instead of tracking the free boundary by employment state, student loan balance and ability type, which quickly gets messy, we solve a smaller version where there is only one free boundary $b^\dagger$.

$$
\begin{aligned}
\sum_{i=1}^{3}\int_{\underline{b}}^{\infty} s_i(b)g_i(b)\mathrm{d}b =& \theta_1 w_{NC}(1-\tau)+\theta_2 z_s w_{NC}+\theta_3 w_C(1-\tau)+rK^S-C \\
\sum_{i=1}^{3}\int_{\underline{b}}^{\infty} s_i(b)g_i(b)\mathrm{d}b =& w_{NC}[\theta_1+\theta_2 z_s]+\theta_3 w_C+\left(\frac{\alpha Y}{K^D}-\delta\right)K^S-C \\
&- sP\int_{b^\dagger}^{\infty}\left[(\lambda_s+\kappa)g_2(b)+(\lambda_x+\kappa)g_3(b)\right]\mathrm{d}b
\end{aligned} \tag{30}
$$

In the last step we plugged the expression of the income tax rate of the reduced model $\tau =$ Ed costs$/(\theta_1 w_{NC}+\theta_3 w_C)$. We can simply things further.

$$
\begin{aligned}
\sum_{i=1}^{3}\int_{\underline{b}}^{\infty} s_i(b)g_i(b)\mathrm{d}b =& (1-\alpha)\frac{Y}{\tilde{L}^\nu}\left(\chi[\theta_1+\theta_2 z_s]+(1-\chi)\theta_3\right)+\left(\frac{\alpha Y}{K^D}-\delta\right)K^S-C \\
&- sP\int_{b^\dagger}^{\infty}\left[(\lambda_s+\kappa)g_2(b)+(\lambda_x+\kappa)g_3(b)\right]\mathrm{d}b \\
\sum_{i=1}^{3}\int_{\underline{b}}^{\infty} s_i(b)g_i(b)\mathrm{d}b =& (1-\alpha)Y+\left(\frac{\alpha Y}{K^D}-\delta\right)K^S-C \\
&- sP\int_{b^\dagger}^{\infty}\left[(\lambda_s+\kappa)g_2(b)+(\lambda_x+\kappa)g_3(b)\right]\mathrm{d}b
\end{aligned} \tag{31}
$$

The last step requires that $K_S = K_D$, which is what was intended to be shown. Putting everything together, along with setting capital supply equal to capital demand yields the expression for law of motion of aggregate capital, which in steady state is equal to zero.

$$
\begin{aligned}
\frac{\mathrm{d}}{\mathrm{d}t}K = 0 =& Y-\delta K-C-P\int_{b^\dagger}^{\infty}\left[(\lambda_s+\kappa)g_2(b)+(\lambda_x+\kappa)g_3(b)\right]\mathrm{d}b \\
\frac{\mathrm{d}}{\mathrm{d}t}K = 0 =& Y-I-C-\text{Education costs}
\end{aligned} \tag{32}
$$

6.3 Portfolio problem and pecking order

An earlier version of this model allowed agents to maximise V_3 by choosing how to pay P with the best feasible combination of b and a in the NICL, ICL1 and ICL2 regimes. The results are very similar to the ones presented here. Agents rarely choose to pay for university with b exclusively. This motivated the use of a so-called 'pecking order' mechanism to model the decision of how to cover P. This is computationally less expensive. An example of how this works is show in Figure (26).

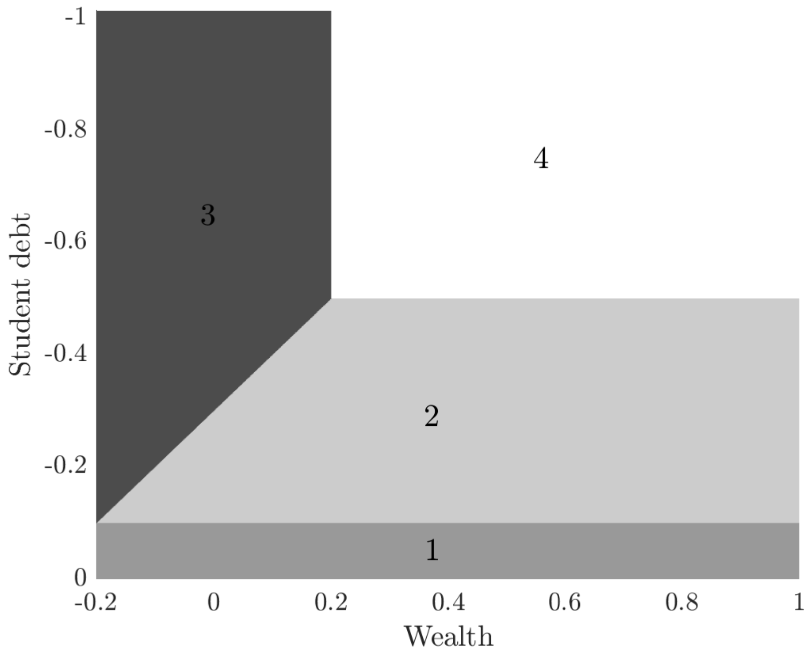

Figure 26: 1 - Cover tuition with a only. 2 - Cover tuition with mix of b and a. 3 - Cannot afford to go to university. 4 - Cover tuition with b only.

Suppose that $P = 0.4$, $\underline{a} = -0.5$, $\underline{\underline{a}} = -1$ and $\underline{b} = -0.2$. The agent will first try to cover P exclusively with student loans, a situation depicted in region 1. If the agent has more than 0.2 in student loans, it will only be able to afford tuition costs by maxing the difference between a and $\underline{a}$ in new student loans, and covering the rest with b. This case is that of region 2. If the agent has little b and a large stock of a, then it will not be able to go to university, the case of region 3. Finally, if the agent has a stock of student loans at or beyond $\underline{a}$, it will only be able to afford tuition costs with b.

6.4 Intergenerational transmission of ability

The transition matrix of the parental transmission of ability is based on a reduced (2×2) version of the ability transition matrix in Abbott et al. (2019). That is, we restrict ourselves to two ability types. The intergenerational transmission of ability depends on the parents ability and on its education status, thus yielding the following (4×2) transition probabilities.

$$\begin{bmatrix} 0.6571 & 0.3429 \\ 0.7360 & 0.2640 \\ 0.7414 & 0.2586 \\ 0.6218 & 0.3782 \end{bmatrix} \tag{33}$$

The first column is the probability that the offspring is born with the same ability as that of its parent. The second column is the probability that the child is born with a different ability. The first two rows correspond to high educational ability parents; the first and second rows depicting the probabilities by education type, no college and college education, respectively. The third and fourth rows are analogous to the first two rows, but this time recording the probabilities that a low educational ability parent will have a low or high educational ability child. We use the following notation to represent (33).

$$\begin{bmatrix} \lambda^1_{NC,1} & 1-\lambda^1_{NC,1} \\ \lambda^1_{C,1} & 1-\lambda^1_{C,1} \\ \lambda^2_{NC,2} & 1-\lambda^2_{NC,2} \\ \lambda^2_{C,2} & 1-\lambda^2_{C,2} \end{bmatrix}, \tag{34}$$

The subscripts NC and C denote no college education (agents in states θ_1^m ,θ_2^m and θ_3^m) and college education (agents in states θ_4^m and θ_5^m), respectively.

6.5 Benchmark calibration of P, debt limits and z_T

6.5.1 Cost of education P

In 2018 U.S. GDP per capita stood at $54541 according to the World Bank. We calibrate P to the average cost of attending university (tuition, room and board), taking into account, following Athreya et al. (2019), that around 60% of students enroll in public universities and the remaining 40% in private universities and also taking into account that, according to NCES (2018), about a quarter of students enroll in out of state universities[49]. The cost of public and private universities is taken from College Board (2018), where the average annual cost of an American public and private non-profit four year university in 2018 was $21370 (in-state tuition, room and board - $37430 out-state) and $48510 (tuition, room and board), respectively. The average cost of attendance, $34635, is rescaled to GDP per capita. We reweigh GDP per capita by the number of working age people in the U.S., as the model does not account for those who are not of working age. The U.S. Bureau of Economic Analysis reports that U.S. population reached 328.795 million while the OECD estimates that the working age population in the U.S. stood at 206.538 million in 2018. The benchmark calibration of P is set to:

$$P = \frac{34635\frac{1}{\Delta_{ed}}}{\frac{328.795}{206.538} * 54541} Y \tag{35}$$

Given that Y is an endogenous variable the cost of education is updated at every market clearing loop. The sensitivity analysis in P, which involves changing the numerator of (35), gives further indication of how each higher education regime fares as we change the relative cost of education.

6.5.2 Debt limits

Following the same reasoning as above and converting the debt limit figure in Athreya et al. (2019) to 2018 dollars, we compute the debt limit $\underline{b}$ as shown further below. As mentioned in the calibration section, the total aggregate student loan limit in the US for independent undergraduate students in 2018 was $57500. We use this value to compute $\underline{a}$.

$$\underline{b} = \frac{18040.81}{\frac{328.795}{206.538} * 54541} \bar{Y} \tag{36}$$

$$\underline{a} = \frac{57500}{\frac{328.795}{206.538} * 54541} \bar{Y} \tag{37}$$

Note that we use $\bar{Y}$ to indicate that this values is fixed; it is the steady state output of the baseline, NICL.

[49]The cost of education is difficult to pin down since it is not obvious to what extent we should include room and board expenses. It is not clear what percentage of students move out of home when they enrol at university. For simplicity it is assumed that all students face tuition, room and board costs. We still perform a sensitivity analysis on P, which allows us to see how results would change if we had weighed the average cost of education considering that some students do not face room and board costs.

6.5.3 ICL2 income threshold z_T

The Plan 2 student loan repayment threshold of England and Wales[50] stood at £21000 in 2018. The average GBP/USD exchange rate for 2018 is 1.3363 while the ratio of workers to population is 66.46/41.253 according to Board of Governors of the Federal Reserve System and the OECD, respectively. According to the World Bank, UK GDP per capita in 2018 stood at $42992 in 2022 dollars. After converting this figure to 2018 dollars we get an estimate of $37567.76. We thus compute z_T as follows.

$$z_T = \frac{21000 * 1.3363}{\frac{66.46}{41.253} * 37567.76}\bar{Y} \tag{38}$$

Unlike the computation of P, we do not update Y at every market iteration loop in z_T. Instead we use the baseline (NICL) steady state output $\bar{Y}$.

6.6 Results when P is lower/higher than the benchmark

When the price of education is low, the market imperfections in educational investment lessen, reducing differences amongst the various higher education financing systems considered in this paper. Also, we see smaller welfare gains relative to NICL. As shown in Table (5) the CEGs over NICL are very small when we cut the cost of education by 50%. All HE systems give more or less the same share of educated workers. We the observe the opposite result when the cost of education rises.

6.6.1 Low cost of education

	NICL	ICL1	ICL2	TS50 %	TS75 %	TS100 %	GT100 %
CEG	0	4.624	2.695	3.289	4.863	5.546	5.052
K	1.883	1.870	1.853	1.893	1.882	1.875	1.862
r	5.134	5.360	5.444	5.178	5.331	5.411	5.399
Graduates %	46.341	53.099	53.321	49.284	54.761	59.277	53.342
τ %	3.545	7.825	8.259	4.765	6.450	8.436	11.112
τ_{UI} %	2.297	2.106	2.103	2.180	2.082	2.010	2.103
Gini_{b+a}	0.574	0.505	0.635	0.492	0.480	0.454	0.459
Net debtors % $b+a$	6.814	6.974	16.986	1.323	1.223	0.890	0.733
Net debtors % b	1.056	1.320	1.717	1.323	1.223	0.890	0.733
CWP	1.396	1.246	1.241	1.330	1.210	1.117	1.241
High ability %	49.971	51.559	51.590	50.789	51.779	52.734	51.817

Table 5: Steady state general equilibrium results when the cost of education is halved

Figures (27), (28), (29) and (30) illustrate that the rankings based on steady state consumption equivalent gains (relative to NICL at the new cost of education), and its relationship with net debtors and inequality are robust to different debt limits in b and a. As the cost of education rises, welfare differences among systems become more patent.

[50] Plan 2 student loans are those issued in England or Wales from September 2012 onwards. https://www.gov.uk/guidance/previous-annual-repayment-thresholds

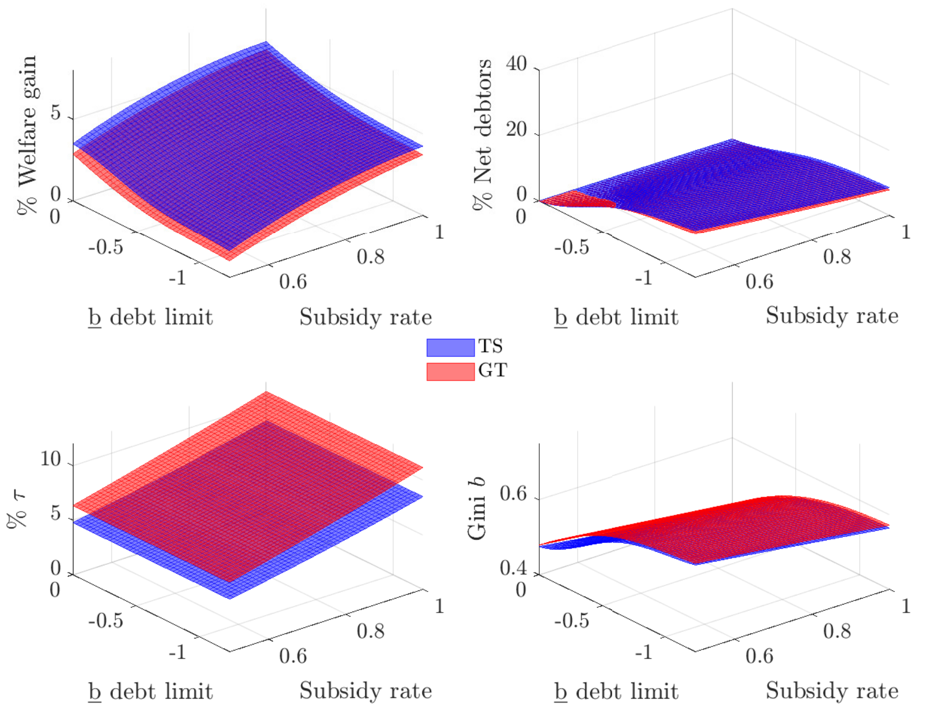

Figure 27: CEG, wealth Gini, tax rate (graduate income tax) and net debtor share in TS and GT when education costs fall by 50 %.

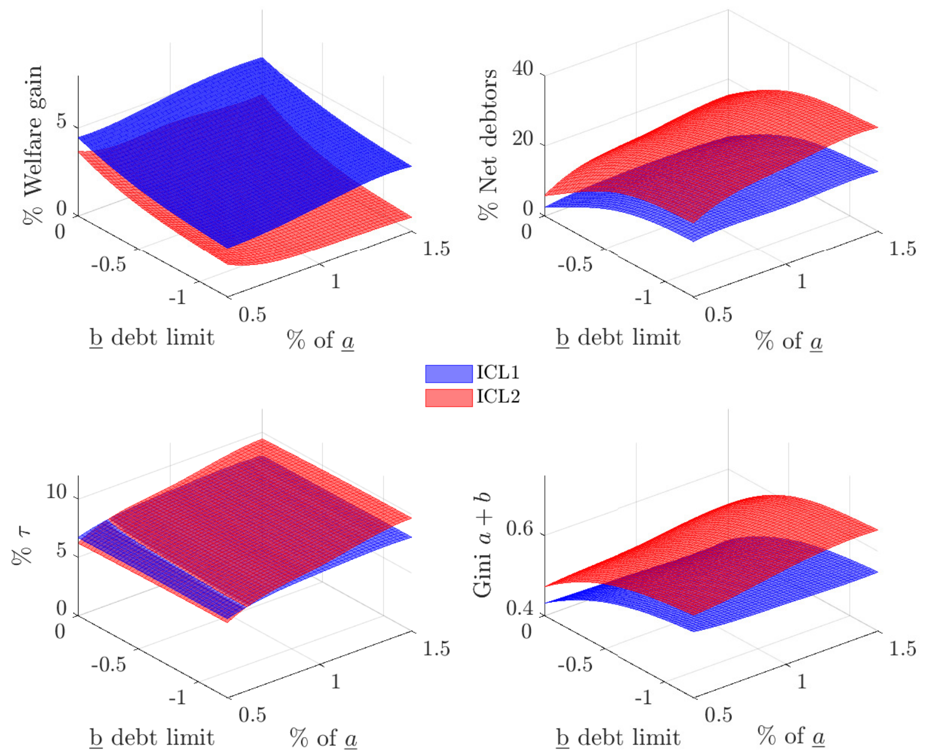

Figure 28: CEG, wealth Gini, tax rate and net debtor share in ICL1 and ICL2 when education costs fall by 50 %.

6.6.2 High cost of education

	NICL	ICL1	ICL2	TS50 %	TS75 %	TS100 %	GT100 %
CEG	0	15.222	12.553	3.832	17.153	19.018	20.764
K	1.756	1.836	1.805	1.787	1.864	1.863	1.798
r	4.712	5.081	5.193	4.766	5.172	5.474	5.440
Graduates %	24.664	38.302	37.434	27.742	44.574	61.305	42.758
τ %	5.420	11.401	11.795	6.698	12.610	22.020	26.782
τ_{UI} %	2.570	2.379	2.379	2.545	2.315	1.975	2.247
Gini_{b+a}	0.751	0.619	0.700	0.709	0.578	0.495	0.476
Net debtors % $b+a$	8.523	12.201	18.420	4.187	2.432	0.976	0.503
Net debtors % b	4.107	2.900	3.159	4.187	2.432	0.976	0.503
CWP	2.088	1.604	1.629	1.955	1.438	1.075	1.487
High ability %	46.741	49.025	48.872	47.134	49.822	53.108	50.198

Table 6: Steady state general equilibrium results when the cost of education is doubled

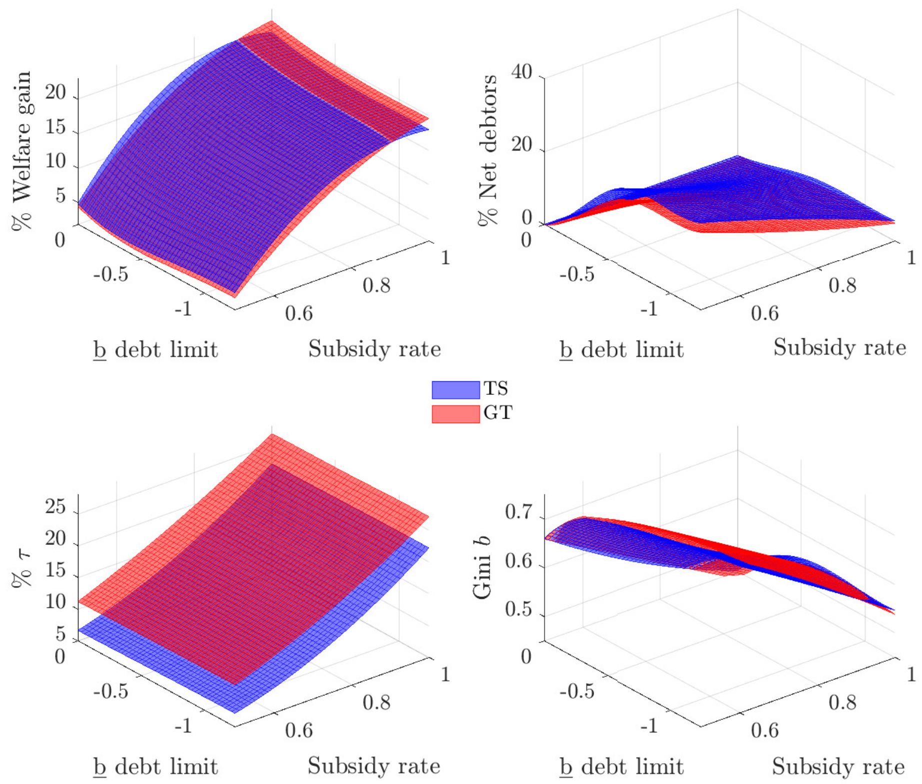

Figure 29: CEG, wealth Gini, tax rate (graduate income tax) and net debtor share in TS and GT when the benchmark cost of education rises by 50 %.

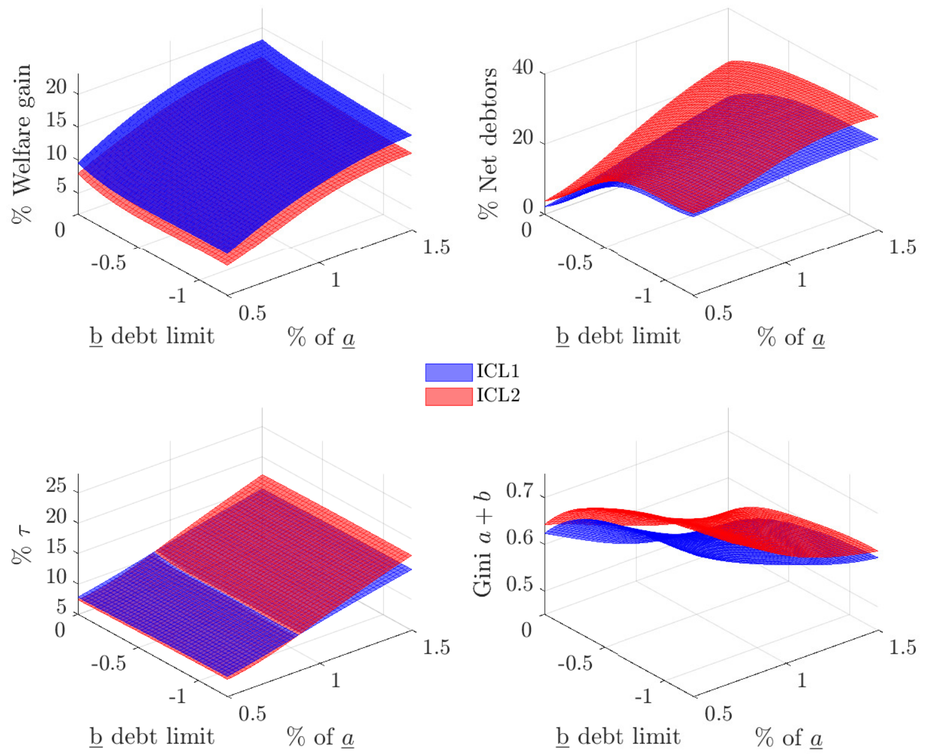

Figure 30: CEG, wealth Gini, tax rate and net debtor share in ICL1 and ICL2 when the benchmark cost of education rises by 50 %.

6.7 Additional tables and figures

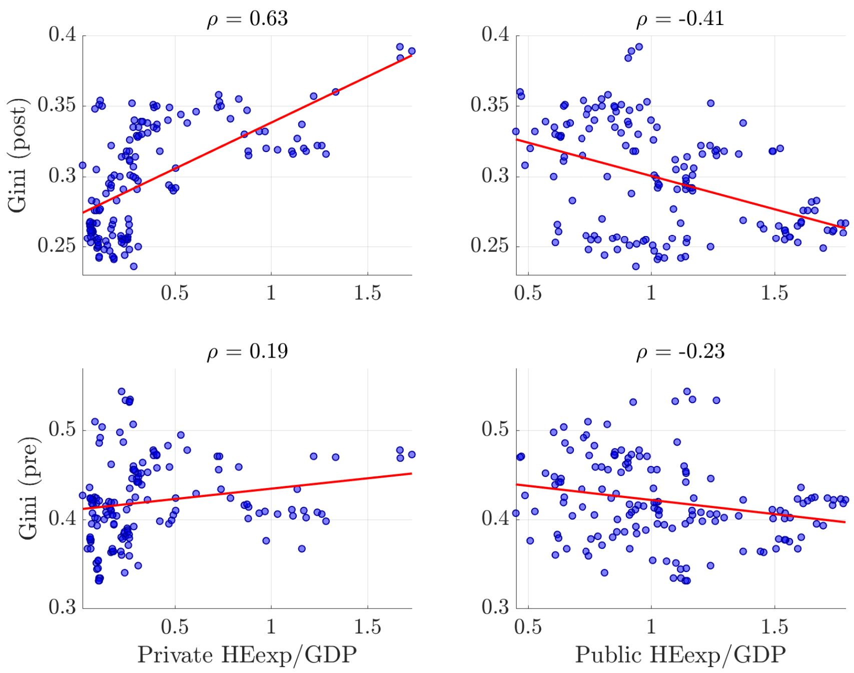

Figure 31: Working age income Gini (post and pre redistribution) vs. HE expenditure relative to GDP and its correlation during 2000-2016 in the following countries: AUS, AUT, CAN, CZE, DEU, DNK, ESP, EST, FIN, FRA, GBR, GRC, IRL, ISL, ISR, ITA, JPN, KOR, LUX, NLD, NOR, NZL, PRT, SVK, SVN, SWE, USA. Source: OECD.

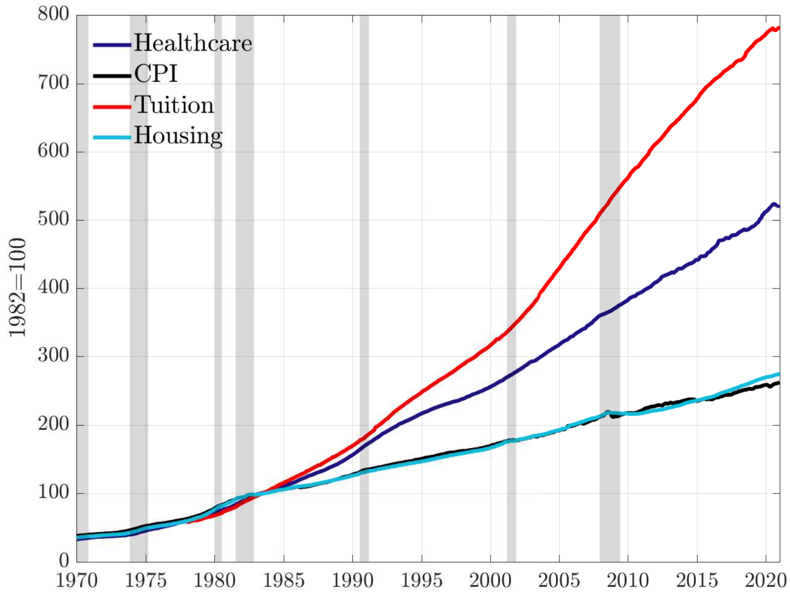

Figure 32: Tuition inflation. Source: US Bureau of Labor Statistics (2020)

6.7.1 Disaggregated welfare NICL versus TS and ICL2

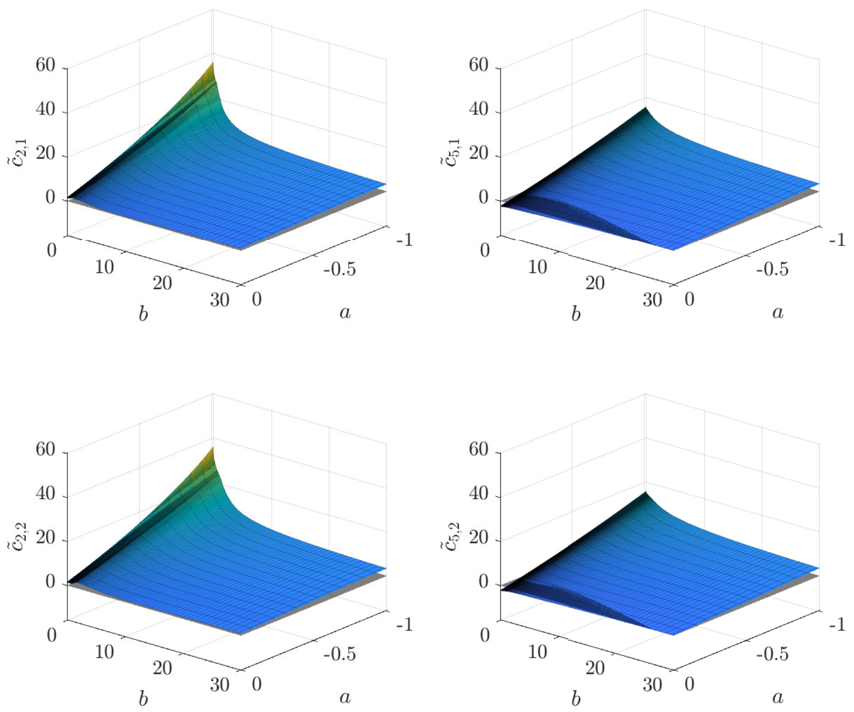

Figure 33: Disaggregate welfare gains of TS 50% vs NICL. The first row depicts welfare gains relative to NICL for high educational ability workers without and with a university degree, respectively. The bottom row shows workers (without/with a degree) with low educational ability. The black surface represents zero welfare gains.

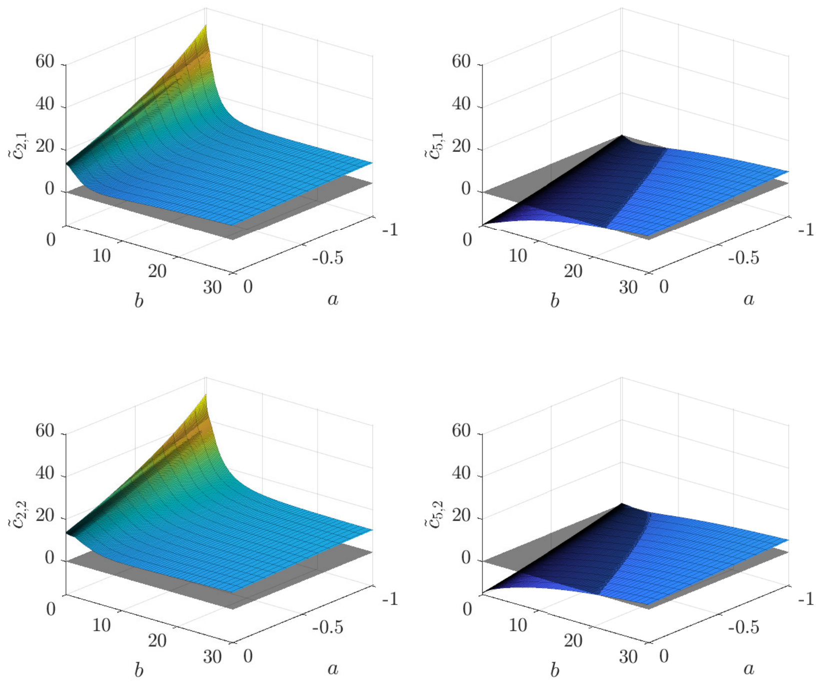

Figure 34: Disaggregate welfare gains of TS 100% vs NICL. The first row depicts welfare gains relative to NICL for high educational ability workers without and with a university degree, respectively. The bottom row shows workers (without/with a degree) with low educational ability. The black surface represents zero welfare gains.

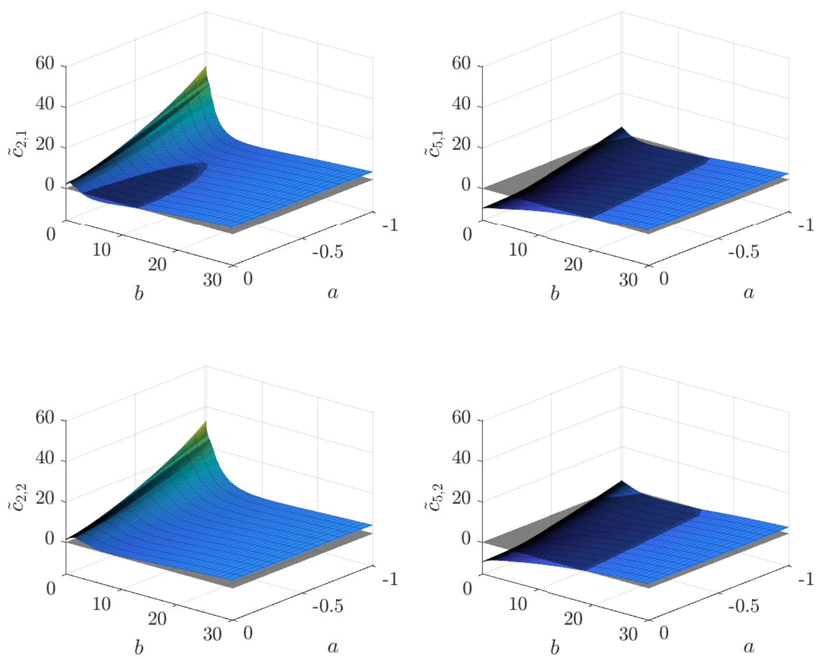

Figure 35: Disaggregate welfare gains of ICL2 vs NICL. The first row depicts welfare gains relative to NICL for high educational ability workers without and with a university degree, respectively. The bottom row shows workers (without/with a degree) with low educational ability. The black surface represents zero welfare gains.

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