Working Paper in Economics

 # 202318

Postal Platform Pricing with Limited Consumer Attention

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Abstract

We introduce limited consumer attention into a two-sided market model to investigate optimal platform pricing for the postal mail sector. Two types of senders – advertisers and non-advertisers – derive value from attention paid to their mail. Consumers pay more attention to each mail item if they receive less mail and pay more attention to advertising mail if they receive more non-advertising mail. We show that a postal monopolist subsidises non-advertising prices, which increases the value of mail to advertisers, thus inflating advertising prices. Advertisers that are most nuisance or attention-consuming for recipients face high prices. Competitive entry for delivering advertising mail cannibalises the advertising mail market and the cross-subsidisation of prices is shut down. However, if the entrant price-matches rather than competes, all postal operators, mail senders and recipients can benefit. This insight suggests that competition amongst postal operators does not necessarily benefit consumers, especially if the entrant is more efficient. Based on our findings, we argue that Universal Service Obligation policies are not as demanding as traditionally viewed.

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The authors wish to thank seminar audiences at the Centre for Research in Behavioural Economics (Nottingham, 2021), OLIGO Workshop 2021 (Maastricht/Online, 2021), 11th bi-annual Postal Economics Conference (Toulouse, 2022), OR64 Operations Research Conference 2022 (Warwick), Lancaster Game Theory Workshop (2022), and participants of the 2023 Workshop on Competition and Regulation in Digital Markets (City University, London). We are particularly grateful for discussions with Antonin Arlandis, Nursen Aydin, Anna d'Annunzio, Mateusz Cholodecki, Luis Corchon, Alexander Matros, Arina Nikandrova, Andrew Rhodes, David Ronayne, Shiva Shekhar, Soterios Soteri, and Eyal Winter.

1. Introduction

Consumers have limited attention. This fundamental insight has been established by research across disciplines such as economics, marketing, management and operations. In particular, limited consumer attention has a significant impact on the strategies of firms and the nature of competition in one-sided markets.[1] Most of the growing literature on two-sided markets, however, assumes that agents' payoffs are driven by the number of participants on each side of the platform, not the attention they pay (Armstrong, 2006; Rietveld & Schilling, 2020; Rochet & Tirole, 2003). Whilst this assumption is appropriate in some markets, e.g. the number of buyers and sellers is relevant for everyone active on auction platforms such as eBay, in many markets the attention and engagement of participants is more important. This is especially true for platforms intermediating between consumers and advertisers, where consumers' attention generates the platform's value to advertisers because, intuitively speaking, "*reaching* a consumer does not mean the message is *registered*" (Anderson & De Palma, 2012, p. 5).

In this paper, we introduce a model to explore how limited consumer attention affects pricing and competition in a two-sided market. While there exists a large literature on digital two-sided markets, our analysis is developed using the postal letter market. We thereby address recent calls from firms, regulators and EU governments to identify solutions for the systemic challenges facing this economically and socially important sector: electronic substitution has driven substantial declines in letter volumes, with profitability declining 7% across the EU during 2013-2017 (Bradley et al, 2015, Copenhagen Economics, 2019; House of Commons, 2015). Whilst total advertising spending is increasing, mail advertising revenue is declining (Ofcom, 2015). Growing demand for parcel delivery (where postal operators face stronger competition than for letters) cannot compensate for the decline in letters (Copenhagen Economics, 2019). Moreover, reports commissioned by the European Parliament find that postal operators follow cost-plus pricing, with some adjustment for senders' individual demand elasticities (Copenhagen Economics, 2012; 2019).[2]

The postal market also faces unique regulatory constraints under the Universal Service Obligation (USO) that mandates efficiency, service and price regulations, which have become financially unsustainable. Currently proposed remedies include weakening USO requirements and prioritising USO viability over competition (Copenhagen Economics, 2019).[3] Our overarching contribution is to present a new pricing approach that recognises postal operators as two-sided market intermediaries,

[1] For example, sellers may need to persuade consumers to consider their products (Eliaz & Spiegler, 2011; Falkinger, 2008; Spiegler, 2011; Manzini & Mariotti, 2018), send reminders (Calzolari & Nardotto, 2017) and engage in inefficient mass, rather than targeted, advertising (Hefti & Liu, 2020). Sellers can obfuscate product information (Chioveanu & Zhou, 2013; Ellison, 2005; Gabaix & Laibson, 2006; Persson, 2018), manipulate decision-making through their product range or framing (Apffelstaedt & Mechtenberg, 2021; Bordalo et al, 2013, 2016; Eliaz & Spiegler, 2011; Yan et al., 2022; Zhu & Dukes, 2017), and exploit consumers who neglect attributes of a product (Boyaci & Akcay, 2018; Dahremoller & Fels, 2015; Reme et al, 2021; Spiegler, 2006).

[2] See Copenhagen Economics (2012, p113) for a detailed discussion. In particular, "we observe that postal operators, to a large extent, apply volume discounts and other price incentives targeted at large mailers with high elasticity of demand to generate economies of scale and scope." Indeed, Copenhagen Economics (2012, p. 116) identify a classic inverse elasticity rule in the postal market.

[3] As a recent example in the UK, Royal Mail has requested Ofcom to remove the requirement for weekend letter deliveries. See: www.bbc.co.uk/news/business-63660320 .

with the potential to profitably meet existing USO conditions without regulators being required to weaken them.

On one side of the market, consumers receive all mail posted to them (at zero price) but possess limited attention to engage with their mail, which is determined endogenously by the perceived value of mail. More specifically, consumers receive advertising and non-advertising mail, which correspond more formally to direct and transactional mail in the postal sector.[4] Attention per letter decreases with the total quantity received, reflecting widely observed information overload, but attention to advertising mail rises as the amount of non-advertising mail increases because consumers find their mail more relevant.[5] On the other side of the market, advertisers and non-advertisers choose the quantity of mail they each send based on postal prices and consumers' attention to postal mail. Whilst mail senders compete with each other for attention, the postal operator competes for consumers' attention with other media of communication.

Evidence that consumers are attention-constrained and find advertising undesirable is widespread for many markets (Chetty et al, 2009; Crampes et al, 2009; DellaVigna, 2009; Hossain & Morgan, 2006; Wilbur, 2008). For example, Huang et al (2018) find that the number of hours a consumer listens to a radio station decreases linearly in the number of advertisements. OECD (2018) discuss how advertising irritates consumers online, resulting in 11% of internet users adopting ad-blockers in 2016, with subsequent annual increases of 30%. Eppler & Mengis (2010) review information overload across management science disciplines, whilst Belabbes et al (2022) provide a more recent conceptual analysis. Our assumption that advertising is a nuisance for consumers which reduces the attractiveness of a platform is well-established and underpins the recent theoretical frameworks of Anderson & Peitz (2020), Shekhar (2020), Jeon et al. (2022) and Chakraborty et al, (2022), amongst others, where advertising impacts the consumers' platform choice. In fact, many platforms explicitly recognise advertising as a nuisance, with TV broadcasters (e.g. ITV and Channel 4 in UK) and digital platforms (e.g. Spotify and Youtube) allowing consumers to pay a premium to remove advertising. Most recently, TV streaming platform Netflix introduced a cheaper version that introduces advertising and Amazon Prime offers some content 'Free with Ads.'[6]

At this point, it is helpful to spell out how the postal sector diverges from existing models of platform competition.[7] Firstly, a particular feature is 'involuntary multi-homing' in the sense that all consumers receive any mail sent to them, potentially from multiple postal delivery services, and recipients cannot identify the delivery operator. This matters because digital platforms (e.g. social media and price comparison websites) face competition for consumers to participate, which places downward pressure on advertising levels that otherwise degrade their service (Rietveld & Schilling, 2020; Shekhar, 2022). However, in the postal sector, operators are not incentivised to compete

[4]More specifically, EU Directive 2008/6/EC defines direct mail as consisting 'solely of advertising, marketing or publicity material and comprising an identical message, except for the addressees name, address and identifying number.' Therefore, direct mail is a specific form of advertising that competes with other advertising channels such as newspapers and radio.

[5]We discuss our consumer behaviour framework and the evidence that underpins it in detail later.

[6]www.theguardian.com/media/2022/oct/14/netflix-basic-with-ads-launches-new-cheaper-subscription-tier-ad-supported-heres-what-you-need-to-know

[7]See Belleflame & Peitz, (2021), Chapter 5 for an overview of platform competitive frameworks.

for participation and consumer attention suffers a 'tragedy of the commons'. Indeed, competing operators have a collective incentive to maximise consumer attention but a contravening individual incentive to deliver as much (advertising) mail as possible. Secondly, the cross-market network effect from the recipients to the senders does not relate to the total number of recipients participating, but to the attention they pay. Thirdly, the within-group network effect between senders is not driven by the number of senders, but the amount of mail sent. Fourthly, there are no within-group network effects on the recipient side: consumers remain unaffected if their neighbour pays more attention to mail. Finally, all revenue must derive from mail senders because charging recipients is infeasible. Overall, our analysis shifts the crucial question away from *"which side should you subsidize"* (Eisenmann et al, 2006, p.94) to *'which segment of which side should you subsidize?'*

We show that a postal operator is able to raise profit by using prices to manage the quantities and types of mail consumers receive, thereby directly affecting their attention. More specifically, a postal monopolist can optimally subsidise the price of non-advertising mail, potentially even below cost, to increase attention to adverts. This inflates advertising mail prices. Higher quantities of both mail types ensue: Non-advertising mail increases due to the subsidised price, whilst advertising mail increases (despite the higher price) because of enhanced attention of the consumers. Moreover, consumers receive a (favourable) larger proportion of non-advertising mail. Interestingly, equilibrium prices for different mail types can be inversely related to the postal operator's cost of delivery: advertising mail is less costly to sort and deliver than individual non-advertising mail items, but the latter is subsidised to increase consumer attention to mail and inflate advertising prices. This insight suggests that the currently widespread cost-plus pricing rule of postal operators is suboptimal.

Our findings also contribute a novel perspective to current debates on the financial viability of the USO. The requirement to deliver individual non-advertising letters is typically perceived to be unprofitable due to their higher processing costs. In contrast, we demonstrate that postal operators hold a significant incentive to service non-advertising mail (even below cost) to stimulate consumer attention and generate higher profit from advertisers.

We employ our model to explore how a postal operator should respond to heterogeneities between senders in terms of their mail type (advertising vs non-advertising) and further heterogeneities within mail types (e.g. heterogeneous advertisers in terms of their effect on consumer attention to postal mail). New layers of price discrimination emerge as advertisers that cause most nuisance and perturb consumer attention more, should pay higher prices, whilst non-advertising mail senders that generate particularly large positive externalities on consumer attention (e.g. a government communication), should be subsidised more. Our insights provide a formal justification for discounting prices for advertisers that are deemed to be more relevant and valuable to consumers.[8]

The European Commission and national regulators are currently reviewing the merits of competition in the postal sector following increased entry. Entrants typically focus exclusively on bulk advertising, which can be explained by the lower costs compared to non-advertising mail.[9] To inform

[8]Examples of similar mechanisms on digital platforms include LinkedIn auctions for advertising space where the winner achieves the highest score, computed as a combination of bid price and advertising relevancy metric.

[9]For example, until 2015 Whistl competed against Royal Mail in the UK for the delivery of some kinds of postal letters. See: www.ofcom.org.uk/about-ofcom/latest/features-and-news/royal-mail-whistl-competition-law

these policy debates, we extend our model to explore how competitive entry by a rival delivering advertising removes the ability for a postal incumbent to manage the mail consumers receive.[10] Intuitively, competition cannibalises the advertising mail market to marginal cost pricing. Relative to a monopoly, non-advertising mail prices increase, causing quantities to fall and in turn the value of the advertising channel diminishes, reducing total industry value and enlarging the proportion of advertising in the mail-mix. Most strikingly, entry can actually harm all market participants: recipients, mail senders, as well as the postal incumbent. However, if an entrant price matches on advertising mail rather than competing, all agents can benefit in different ways. Our analysis also uncovers a new form of semi-collusion. Accordingly, agents in one segment of one side of the market collude and this can – somewhat surprisingly – generate beneficial outcomes on this two-sided platform that would not arise in a one-sided environment.

Our framework also provides a first step in formalising the ongoing regulatory discussion on the competitive role of access operators (AO's) in the postal market, who process mail from senders but generally do use the incumbent for delivery. AO's typically focus on bulk advertising. More recently, access operators commenced delivering their own mail in densely populated areas where scale economies permit, whilst relying on the national incumbent to serve remote areas. In the UK, Ofcom (2018) requires that access charges (the prices paid by AO's to Royal Mail delivering mail on their behalf) "allow Royal Mail to charge prices to reflect its costs and investment in its network." Following the preceding lines of argument, cost-based price regulations are sub-optimal for the postal sector. This is important because the Supreme Court recently upheld a £50 million fine for abuse of dominance against Royal Mail by Ofcom for price discrimination where prices were higher for access operators who started to deliver their own mail.[11] We provide a new line of argument that charging a price premium to AO's who deliver their own mail in some regions (compared to AO's using the incumbent for delivery) can be justified by a postal operator managing the composition and quantity of mail consumers receive to preserve the value of the industry. This benefits all agents including mail senders and recipients.

Following a literature review, we present our core model of platform competition with limited attention in Section 2. In Section 2.2. we consider only one advertising mail sender and one non-advertising mail sender. This highlights the pricing incentives and regulation implications most cleanly. In Section 2.3. we explore multiple mail senders of each type, which introduces an additional layer of competition as mail senders now compete for attention against each other. Further heterogeneities amongst mail senders are also considered. Section 3. analyses the consequences of entry into the postal market for advertising mail. Notably, we uncover novel dimensions of collusion that can be welfare-enhancing. Section 4. discusses the results, policy implications, as well as managerial insights. In particular, we address the long-standing postal sector topics of access pricing regulations and the Universal Service Obligation.

[10] For example, there was competitive entry in the UK letters market by Whistl, who competed against the national incumbent Royal Mail for the delivery of some kinds of postal letters. See: www.ofcom.org.uk/about-ofcom/latest/features-and-news/royal-mail- whistl-competition-law

[11] See www.ofcom.org.uk/about-ofcom/latest/features-and-news/economic-insights-2019-royal-mail-competition-case, www.ofcom.org.uk/news-centre/2022/supreme-court-rejects-royal-mail-appeal-against-ofcom-fine

1.1. Related Literature

The theory of platforms is rooted in the well-established literature on network effects, with early work focusing on the participation decisions of agents on each side of the platform (Katz & Shapiro, 1994; Rysman, 2009).[12] Closest to our work are analyses of pricing under indirect network externalities, including Armstrong (2006), Caillaud & Jullien (2001), Parker & Van Alstyne (2005), Rochet & Tirole (2003, 2006) and Wu & Chiu (2022). However, as outlined in Section 1, the postal sector does not fit the existing models of platform competition. Our framework intends to close this gap.

We also contribute to the growing 'competition for attention' literature (Eliaz & Spiegler, 2022; Hefti, 2018; Hefti & Liu, 2020; Masatlioglu et al, 2012; Manzini & Mariotti, 2018; Wright & Barbour, 1977; Zhong, 2021). Closest to our work, Anderson & de Palma (2009) consider an environment where senders choose at most one advertisement to send to a single attention-constrained consumer. Adverts represent independent goods, removing potential business stealing effects amongst different senders. They establish the over-utilisation of consumers' scarce attention by senders. Anderson & de Palma (2012) introduce business stealing effects, where firms are grouped in product classes and consumers buy from the lowest price they pay attention to. In their framework, however, advertising prices are exogenous and non-advertising mail is excluded. Furthermore, we address a series of orthogonal questions by maintaining the case of independent senders following Anderson & de Palma (2009) but introducing optimal pricing by the intermediary, admitting price competition between multiple intermediaries, and including non-advertising messages.

Also related to our paper is Shekhar (2020) who studies advertising on digital platforms where single-homing consumers receive a product at zero price and may value other consumers using the same platform. Advertisements generate revenue but reduce the platform's quality to consumers. Shekhar (2020) uncovers how a price rise on one platform, which reduces advertising on that platform and therefore stimulates consumer switching towards it, reduces advertising demand for a rival platform. In our model, consumers implicitly multi-home and their attention decrease to mail – as the perceived quality of mail deteriorates – operates at the market rather than the firm level. Therefore, the incentive for firms to compete for attention survives even without a second postal platform due to external competition with other communication channels. Intensifying competition between platforms also drives up advertising prices in Shekhar (2020) as competition for consumers via platform quality increases. In contrast, fiercer competition amongst postal platforms leads to lower prices for advertisers in our framework.

The incentive for intermediaries to cross-subsidise 'attention grabbing' products has been identified by Pashigan & Gould (1998). They provide empirical evidence that leading stores receive favourable prices from a shopping mall operator, due to the demand generated for other stores who can then be charged a rent premium. Somewhat related to the insight by Pashigan & Gould (1998), Dinerstein et al. (2018) emphasise that platforms must maintain an inventory of attractive products to draw consumers to their platform. They identify a tradeoff for platforms between displaying greater variety and stimulating price competition between sellers.

[12] Farrell and Klemperer (2007) and Rysman (2009) provide detailed reviews, with Jullien, Pvan and Rysman (2021) providing a more recent survey, and Belleflame & Peitz (2021) offering an overview treatment. Rietveld & Schilling (2020) provide an inter-disciplinary perspective.

Furthermore, our work connects with analyses of media platforms that bring together advertisers and consumers (Anderson & Coate, 2005; Calvano & Polo, 2020; Crampes et al, 2009; Gode et al, 2009; Peitz & Valetti, 2008; Reisinger, 2012; Wu & Chiu, 2022). However, the postal sector does not at all fit the 'advertising funded' structure of many of these platforms such as Youtube.

There exists some work on firm strategies in the postal sector (De Donder et al, 2015; Bradley et al., 2015). Optimal pricing by a welfare maximising universal service provider under different competitive environments, including competition from electronic substitution, is explored by De Donder et al. (2011). They analyse the effects of asymmetric price elasticities between advertising and non-advertising (transactional) mail. However, they assume that demands for different mail types are independent, rather than the inter-dependent network structure with attention constrained consumers that constitutes the core of our framework. Besides, we also introduce heterogeneities within the groups of advertisers and non-advertisers, which leads to additional degrees of price discrimination. Bradley et al. (2015) consider a postal operator maximising profit exclusively from mass market advertising and targeted advertising. Consumers vary in their match value with the sender's product and senders compete for attention from each recipient. In contrast to our approach taken in this paper, they do not consider non-advertising mail or the network effects that arise across different mail types.

Finally, our paper proposes a new form of semi-collusion in two-sided markets with positive effects in terms of general welfare. 'Benefits' of collusive behaviour have also appeared in other contexts in the literature so far. Dewenter et al. (2011) study a newspaper duopoly and firms collude on advertising prices and/or consumer prices. Colluding exclusively on advertising prices increases advertising prices and reduces consumer prices but advertisers can benefit despite the higher price from wider newspaper circulation. Moreover, Lefouili and Pinho (2020) study the welfare effects of full and semi-collusion between two horizontally differentiated platforms. They show that one-sided collusion can generate lower (higher) prices on the collusive (competitive) side. In contrast, we consider a novel and complementary dimension of semi-collusion in a two-sided market, where firms can collude only on a segment of one side of the market. Cross-subsidisation of prices can take place between agents on the same side.

2. A Model of Platform Competition with Limited Attention

Consider a two-sided market with a single postal operator intermediating between D advertisers (sending direct mail) and T non-advertisers (sending transactional mail) on one side of the market, and a representative consumer on the other side. Each sender will choose their quantity of mail to send based on their perceived value of the postal platform and prices set by the operator. The representative consumer receives all mail sent to them and decides how much attention to allocate to each item and postal mail in total.[13] Formally, the game develops over three stages:

Stage 1: The postal monopolist chooses the price for each sender i of advertising mail $p_{i,d}$

[13] A representative consumer approach to analyse the postal sector is also used by include De Donder et al. (2011) and Dukes & Liu (2023).

and non-advertising mail $p_{i,t}$.

Stage 2: Senders of mail choose the quantity of mail that they will send to the representative consumer: $x_{i,d}, x_{i,t}$, where i indexes each mail sender and the d and t subscripts indicate the mail type: advertisers (d) and non-advertisers (t).

Stage 3: Consumers receive the mail distributed by the postal operator and devote their desired attention to each mail item. Senders receive their payoff based on the attention that consumers pay to their mail.[14]

For mail senders, the value of the postal platform increases when their mail receives more attention. The attention to mail from the representative consumer follows two empirically grounded assumptions, for which we discuss the evidence in the next section.

Assumption 1: Mail Volume. For a given type of mail, the attention a consumer allocates to each item received of that type is decreasing in the total quantity of that type of mail.

Assumption 2: Mail Composition. A consumer pays more attention to their advertising mail when there is a higher quantity of non-advertising mail.

Let A_d and A_t denote the amount of attention paid to each unit of advertising and non-advertising mail, respectively. $X_d = \sum_{i=1}^{D} x_{i,d}$ and $X_t = \sum_{i=1}^{T} x_{1,t}$ represent the total amount of advertising and non-advertising mail received by the consumer. For simplicity, we develop our core framework using the linear attention function outlined below, which satisfies assumptions 1 and 2. However, our results are robust to more general functional forms for attention.

$$A_t(X_d, X_t) = u_t - \frac{1}{2}X_t \quad \text{and} \quad A_d(X_d, X_t) = u_d - \frac{1}{2}X_d + \alpha X_t \tag{1}$$

The marginal cost for the postal operator to deliver mail is constant ($c_d,\ c_t$) and without loss of generality fixed costs are excluded.

Assumption 3. $c_d \leq c_t$.

Let u_t, u_d be demand parameters for the two mail types. Without loss of generality we assume that $u_t = u_d = u$ unless specified otherwise. The following assumption is imposed to ensure positive demand.

Assumption 4. $c_t < u$.

The attention function captures a range of network effects. $\alpha \in [0, 1]$ captures the strength of the positive externality from non-advertising mail on the attention consumers pay to advertising. If $\alpha = 0$, then attention (per mail item) decreases in the quantity of that mail type, capturing Assumption 1. If $\alpha > 0$, then non-advertising mail exerts a positive externality on the attention paid to advertising mail, capturing Assumption 2. In our baseline model, α is common knowledge among all the agents. Without loss of generality, define the baseline value of mail to each type of sender solely as a function of the attention consumers pay: $V_t(A_t) = A_t$ and $V_d(A_d) = A_d$.

[14] We do not expect that a consumer disposes of mail without paying *any* attention to it (in Stage 3). The distinction lies in whether the consumer pays sufficient attention to potentially act upon the information or not.

2.1. Discussion of Model Assumptions

Assumption 1 implies that attention to each unit of mail decreases in the amount of that mail type received by consumers. This reflects the commonly observed psychological effect of information overload. Assumption 2 embeds the core interdependence between advertising and non-advertising mail, where the mail a consumer receives becomes more valuable whenever a higher quantity is personalised non-advertising content.

Whilst we focus on a specific linear attention function, our results do not at all rely on this environment. Indeed, also a general attention function could be used (which we choose not to, so that the model and all subsequent results are as simple and transparent as possible:

$$A_t(X_d, X_t) = u_t - \Lambda X_t \quad \text{and} \quad A_d(X_d, X_t) = u_d - \Theta X_d + \alpha X_t \tag{2}$$

The parameter $\alpha \in [0, 1]$ captures the positive externality from non-advertising mail on the attention to advertising mail. Λ (> 0) and Θ (> 0) capture the marginal effect of an additional unit of each mail type on the attention paid to that mail type. Simplifying to $\Lambda = \Theta = \frac{1}{2}$ replicates the standard linear demand model. If the attention function is plotted in terms of the mail received, the demand parameters correspond to the vertical intercepts (whenever $\alpha = 0$). Therefore, the elasticity of attention (per unit of mail) with respect to the total quantity of each mail type, decreases as the quantity increases.

Assumption 3 reflects that advertising mail is typically bulk mail of a standardised size and provided to the postal operator pre-sorted into delivery regions, minimising processing costs. Non-advertising mail is more heterogeneous and requires greater sorting, increasing the marginal cost.[15] The requirement that $u > c_t$ is only a minimal technical restriction to ensure positive demand when non-advertising mail is priced at cost, which will not be binding in equilibrium anyway.

Assuming equal demand parameters for advertising and non-advertising mail has no bearing on our results but simplifies the technical details. In Section 2.2..3 we explore how changes to the underlying demand for either mail type impacts the market whenever $u_t \neq u_d$ holds.

We have in mind that advertising increases the probability that a consumer will purchase the product or service. Non-advertising mail is used by firms or private senders to communicate information (e.g. a bank statement or a private letter). Consistent with the 'competition for attention' literature, we assume attention to be valuable to senders. However, there exists a minority of cases, where attention is undesirable to mail senders. For example, if a firm is mandated to advise consumers of higher prices through letter mail, the sender may prefer less attention from their clients, which our baseline model excludes.

We do not consider heterogeneities amongst recipients. However, recent theory and evidence indicates that even when recipients vary, and advertisers can target their preferred consumers at low cost, advertisers still continue to use mass market campaigns that address all consumers (Hefti & Liu, 2020). Intuitively, advertisers target consumers with low match values for their products because the consumer possesses insufficient attention to consider the full range of market alternatives. This line of argument is consistent with our approach.

[15] Assumption 3 is consistent with information from discussions with industry postal operator executives.

2.2. Analysis with One Sender of Each Mail Type

To explore the relationship between pricing and consumer attention most cleanly, suppose there to be a unique sender for each mail type ($D = T = 1$) who independently choose their quantity of mail. The general form of the two senders' profit maximization problems are:

$$\max_{x_t} \pi_t = [V(A_t) - p_t]\, x_t \quad \text{and} \quad \max_{x_d} \pi_d = [V(A_d) - p_d]\, x_d \tag{3}$$

The functions $V(A_d)$ and $V(A_t)$ represent the gross values per unit of advertising and non-advertising mail, respectively. Using the linear attention function from Section 2., the profit maximization problems for each mail sender become:

$$\max_{x_d} \pi_d = \left(u - \frac{1}{2}x_d + \alpha x_t\right) x_d - p_d x_d \tag{4}$$

$$\max_{x_t} \pi_t = \left(u - \frac{1}{2}x_t\right) x_t - p_t x_t, \tag{5}$$

The ensuing optimal demands are: $x_d^* = (1+\alpha)u - p_d - \alpha p_t$ and $x_t^* = u - p_t$. Anticipating these optimal quantities of the two senders, the postal operator faces the following profit maximization problem:

$$\max_{p_d,\, p_t} \pi_{PO} = p_d x_d^* + p_t x_t^* - c_d x_d^* - c_t x_t^* \tag{6}$$

The optimal prices and quantities for the two mail types obtain as follows, where superscript, M, refers to a monopoly postal market:

$$p_d^M = \frac{(2+\alpha)u + (2-\alpha^2)c_d - \alpha c_t}{4-\alpha^2} \quad \text{and} \quad p_t^M = \frac{(2-\alpha-\alpha^2)u + \alpha c_d + 2c_t}{4-\alpha^2} \tag{7}$$

$$x_d^M = \frac{(2+\alpha)u - 2c_d - \alpha c_t}{4-\alpha^2} \quad \text{and} \quad x_t^M = \frac{(2+\alpha)u - 2c_t - \alpha c_d}{4-\alpha^2} \tag{8}$$

Result 1. *In a monopoly postal market with one sender of each mail type, mail prices and quantities follow (7) and (8) in subgame-perfect equilibrium. The main comparative statics (with $\alpha > 0$) are as follows:*

(i) The price of advertising mail, quantity of advertising mail, and quantity of transactional mail increase with the positive externality (α). The price of non-advertising mail decreases with the positive externality (α).

(ii) Mail prices are increasing in their own cost of delivery. However, the price of advertising mail decreases in the cost of non-advertising mail, and the price of non-advertising mail increases in the cost of advertising mail.

(iii) The quantity of each mail type is decreasing in their own cost and the cost of the other mail type.

(iv) If the cost of advertising mail is less than (equal to) the cost of non-advertising mail, the decrease in the price of non-advertising mail created by the positive externality is larger than (equal to) the increase in the price of advertising mail.

(v) Total profit for the postal operator is increasing in the postive externality.

If $\alpha > 0$, then non-advertisers exert a positive effect on the value of advertising mail to the sender, which is extracted through a higher advertising price by the postal operator. The externality incentivises the operator to reduce the price of non-advertising mail to stimulate the demand for advertising further, yielding an effective cross-subsidisation between the mail types. In the case of servicing costs being equal ($c_t = c_d$), the value of the cross-subsidisation are exactly matched. However, whenever non-advertising mail is more costly to service ($c_t > c_d$), the non-advertising price decreases by a larger amount than the increase in the price of advertising. The intuition is that $c_t > c_d$ implies the price of non-advertising mail to be higher for $\alpha = 0$. Therefore, as α rises, larger price cuts are required to generate the same increase in the quantity demanded of non-advertising mail.

If $\alpha > 0$ and $c_t > c_d$, then the ranking of prices is driven by the relative strength of the cost differential ($c_t - c_d > 0$) and the positive externality (α). When the positive externality is sufficiently small relative to the cost difference, non-advertising mail continues to be most expensive. As α increases, there exists a critical $\hat{\alpha}$ such that the higher cost of servicing non-advertising mail is exactly offset by the value of the positive externality generated on advertising mail, which leads to uniform pricing: $p_t = p_d$. Therefore, the absence of price discrimination by the postal operator could indicate: (a) The postal operator has neglected the positive externality and the cost differences when setting prices, (b) $\alpha = \hat{\alpha}$. For $\alpha > \hat{\alpha}$, advertising is more expensive than non-advertising mail even though advertising is cheaper for the operator to service. Non-advertising mail prices can be subsidised below cost. This result helps to explain several 'free post' schemes offered by postal operators. For example, the 'free postcards' scheme operated by Swiss Post to stimulate the quantity of non-advertising mail. More generally, below cost pricing of non-advertising mail fits with the established literature on loss-leader pricing (Tirole, 1988).

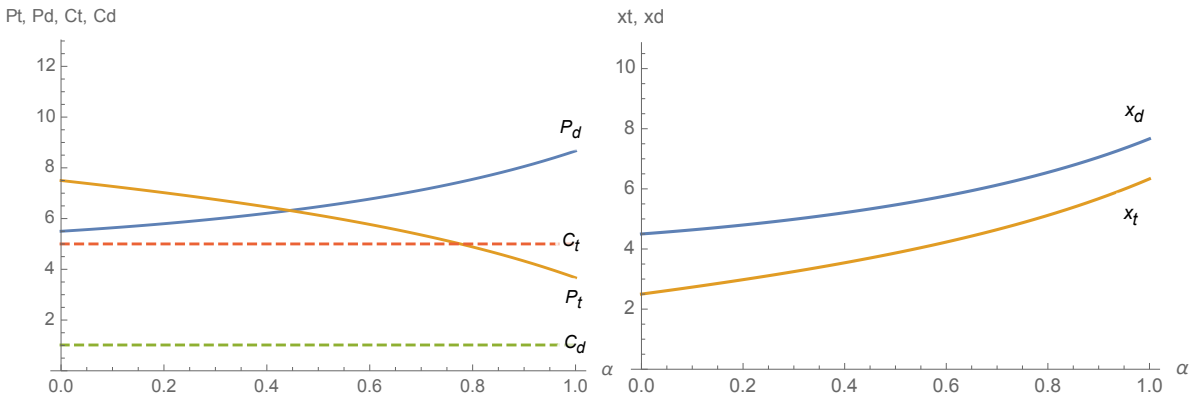
In the special case of $\alpha = 0$, the mail types are priced independently at their respective monopoly prices. If $c_t > c_d$, non-advertising mail has a higher price derived from servicing costs.

The result that the postal operator's profit varies with the strength of the positive externality yields at least two direct implications for its strategy. Firstly, neglecting the positive externality 'leaves money on the table,' which we investigate in Section 2.2..2. Secondly, if the postal operator is able to stimulate the strength of the positive externality, profit actually increases further.

2.2..1 Numerical Example

To illustrate the mechanisms underlying Result 1, consider a numerical example with parameters $u = 10$, $c_t = 5$, and $c_d = 1$. Figure 1 illustrates the optimal prices and resulting equilibrium quantities for varying α.

Whenever $\alpha = 0$, non-advertising mail is more expensive due to the higher marginal cost. Therefore, consumers receive more advertising than non-advertising mail. As α rises, non-advertising



(a) Pricing

(b) Mail Quantities

Figure 1: Optimal Prices & Quantities for Varying α.

mail becomes cheaper, increasing the quantity of non-advertising mail, which stimulates the value to advertisers. It is interesting that both the price and quantity of advertising go up with α in equilibrium. Intuitively, the reduction in non-advertising mail price causes a sufficiently large increase in the value of postal advertising that advertisers choose to send more mail as α grows, despite the postal operator charging higher non-advertising mail prices. The total quantity of mail consumers receive is larger but a higher proportion is non-advertising mail.

2.2..2 The Cost of Neglecting Externalities

The cost-plus pricing approach currently adopted by postal operators neglects the positive externality between mail types. We now explore the consequences when the postal operator and mail senders behave in this way. We refer to this setting as Fully Neglected Interdependencies (nI). All parties behave as if $\alpha = 0$:

$$p_d^{M,nI} = \frac{u + c_d}{2}, \quad p_t^{M,nI} = \frac{u + c_t}{2}, \quad x_d^{M,nI} = \frac{u - c_d}{2}, \quad x_t^{M,nI} = \frac{u - c_t}{2}. \tag{9}$$

> ***Result 2:*** *If the postal operator and mail senders neglect the externality between mail types, advertising mail becomes cheaper and non-advertising mail more expensive. Both mail quantities decrease, the postal operator earns lower profit, and consumers receive a higher proportion of advertising mail.*

Neglecting the network effects on the postal platform has economically significant price and non-price effects. Firstly, consumers receive a higher proportion of advertising mail, which reduces the perceived relevance of postal mail and attention to it. Secondly, the non-advertiser pays a higher price and reduces their quantity. Thirdly, the advertiser suffers from postal mail being less attractive. Despite a lower advertising price, the quantity of advertising falls. Finally, the postal operator earns lower profit.

In practice, mail senders may be aware of the network effects present on the platform, even if they are not internalised through pricing by the postal operator.[16] We refer to this environment as Partially Neglected Interdependencies (nI'). The prices and the quantity of non-advertising mail mirror the case of no externalities ($\alpha = 0$). However, the quantity set by the advertiser is higher as they recognise the additional value of the postal channel. Formally:

$$p_d^{M,nI'} = \frac{u + c_d}{2} \quad \text{and} \quad p_t^{M,nI'} = \frac{u + c_t}{2}. \tag{10}$$

$$x_d^{M,nI'} = \frac{(1+\alpha)u - c_d - ac_t}{2} \quad \text{and} \quad x_t^{M,nI'} = \frac{u - c_t}{2}. \tag{11}$$

> ***Result 3:*** *If only the postal operator neglects the externality between mail types, advertising mail becomes cheaper and non-advertising mail more expensive than when the externalities are internalised, replicating Result 2. The advertiser chooses a higher quantity than in Result 2 and appropriates the value of the positive externality. Profit for the postal operator is lower than Result 1 but higher than Result 2.*

A fundamental difference from the full information case is that the surplus generated by the positive externality is appropriated by the advertiser not the postal operator. The operator benefits from the sender recognising the externality as the quantity of advertising mail increases. However, internalising the externality through prices would boost the postal operator's profit even further. Consumers also receive the highest proportion of advertising mail when only the postal operator neglects the externality.

2.2..3 Electronic Substitution of Postal Mail

Our model can be applied to capture the long-run effects of electronic substitution of postal mail. If electronic substitution affects both mail types equally, then u decreases, which represents an (exogenous) decrease in demand for postal mail by senders, or equivalently a decline in attention to mail by consumers. The following result is therefore intuitive:

> ***Result 4a:*** *An exogenous decrease in mail demand causes mail prices and quantities of mail to fall in subgame-perfect equilibrium. Postal operator profit decreases.*

An asymmetric fall in mail demand across types is more interesting and consistent with current postal market conditions where non-advertising has been most affected (Copenhagen Economics, 2019). We now allow $u_t \neq u_d$.[17].

> ***Result 4b:*** *An exogenous decrease in demand for non-advertising mail leads to a reduction of the price and quantity of advertising mail in subgame-perfect equilibrium. For $\alpha > 0$, advertising price and quantity also decrease.*

[16] This is formally equivalent to the postal operator facing prohibitively high transaction costs or regulatory restrictions preventing price discrimination between sender types.

[17] We defer the optimal prices and quantities to the Appendix.

An exogenous fall in demand for non-advertising mail causes the postal operator to drive down the price of non-advertising mail. Despite the price reduction, the quantity decreases. Moreover, when $\alpha > 0$, a decline in non-advertising mail erodes its value, and therefore demand. This causes the postal operator to reduce the price of advertising and the quantity declines. It follows that the postal operator is unambiguously worse off from both the direct and indirect consequences of falling demand for non-advertising mail.

> ***Result 4c:*** *An exogenous decrease in demand for advertising mail leads to a reduction of the price and quantity of advertising mail in subgame-perfect equilibrium. For $\alpha > 0$, non-advertising mail prices increase and quantity decrease.*

An exogenous fall in demand for advertising mail (e.g. due to new online advertising opportunities) also generates intricate externalities. Firstly, advertising mail price and quantity both decrease. Secondly, if $\alpha > 0$, then the price of non-advertising mail increases because cross-subsidising these prices to stimulate demand for advertising mail becomes less profitable. This causes the quantity of non-advertising mail to drop, with a third-order effect of further depressing both the price and quantity of advertising mail as the value of mail falls for advertisers.

2.2..4 The Effect of a Single Decision-Maker Sending Both Mail Types

Some mail senders post both advertising and non-advertising mail. For example, retail banks send individual financial statements and adverts for new products. What is the effect of such senders on mail prices, mail quantities, and postal operator profit? How do the externalities between mail types affect this market environment? Without any loss of generality, assume that $u = u_d = u_t$.

> ***Result 5:*** *If a single decision-maker chooses the quantity of both mail types:*
>
> (i) *Postal pricing follows the case without mail externalities ($\alpha = 0$). Both mail quantities are higher if there is a single decision maker.*
>
> (ii) *The postal operator earns higher profit if there is one decision maker.*
>
> (iii) *The positive externality stimulates both mail quantities and increases postal operator profit but has no impact on prices.*

Whenever a single decision-maker sends both advertising and non-advertising mail, the positive externality from non-advertising mail to advertising mail is implicitly internalised by the sender. Therefore, the postal operator needs not to engage in cross-subsidisation. As the strength of the positive externality (α) increases, the sender chooses to send more of both mail types: non-advertising mail increases as items which were not previously marginally profitable become profitable to send due to the large positive effect on advertising mail. As a consequence, advertising quantities go up. The postal operator benefits from the additional letters, without making any price adjustments compared to the case where $\alpha = 0$.

The result that the postal operator benefits from a single decision maker sending mail is initially surprising as it loses market power. Interestingly, if servicing costs are uniform between mail types

$(c_t = c_d)$, the prices are also uniform even if $\alpha > 0$. Therefore, limited price discrimination between mail types can be optimal in the presence of cross-market externalities ($\alpha > 0$), if a single decision maker is choosing the quantities of both mail types.

2.3. Analysis with Multiple Mail Senders of Each Type

In this subsection we explore the consequences of competition amongst two senders of the same mail type and, with reference to the case with one sender of each type, isolate these from the cross-market externalities that arise in the absence of competition. This also enables us to introduce heterogeneity across senders of the same mail type in terms of their impact on recipients' attention to mail.[18] The advertisers are assumed to sell independent products yet they compete for the consumers' attention. The optimisation of each sender reads as follows:

$$\max_{x_{i,d}} \pi_{i,d} = \left(u - \frac{1}{2}X_d + \alpha X_t\right) x_{i,d} - p_d x_{i,d} \tag{12}$$

$$\max_{x_{i,t}} \pi_{i,t} = \left(u - \frac{1}{2}X_t\right) x_{i,t} - p_t x_{i,t} \tag{13}$$

where $X_d = \sum_{i=1}^{D} x_{i,d}$ and $X_t = \sum_{i=1}^{T} x_{i,t}$. Each sender incorporates the quantity of mail sent by the other senders of the same type, which introduces an additional intra-network externality: the attention consumers pay to each unit of sender 1's mail is decreasing in the amount of mail sent by another sender of the same type. The ensuing optimal prices for the postal operator are given below:

$$p_d = \left[\frac{9}{9 - 4\alpha^2}\right]\left(\frac{u + c_d}{2} + \frac{2\alpha}{3}\left[\frac{u - c_t}{2} - \frac{2\alpha c_d}{3}\right]\right) \tag{14}$$

$$p_t = \left[\frac{9}{9 - 4\alpha^2}\right]\left(\frac{u + c_t}{2} - \frac{2\alpha}{3}\left[\frac{u - c_d}{2} + \frac{2u\alpha}{3}\right]\right) \tag{15}$$

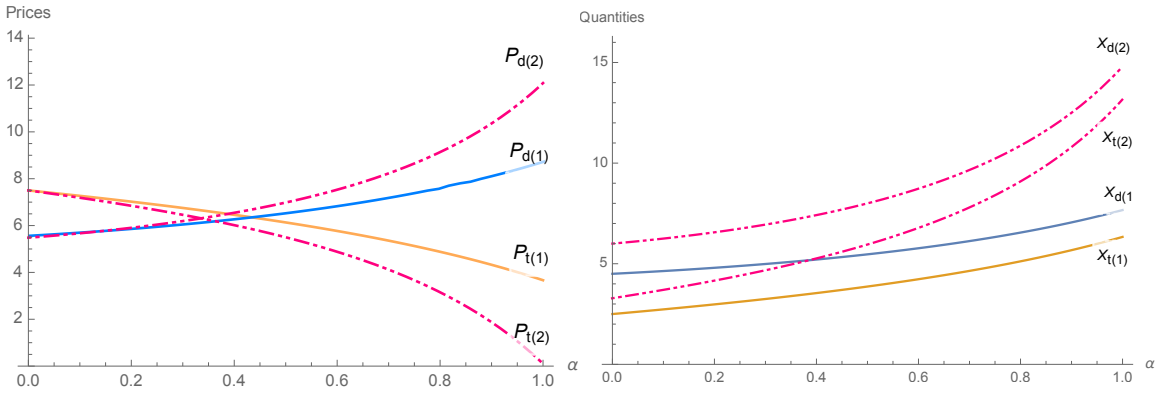
> ***Result 6:*** *With two senders of the same mail type, prices and quantities adjust more significantly to the positive externality between different mail types, than in the one sender case.*

The main insights developed with one sender of each mail type are strengthened with multiple senders of the same type. Reductions in the price of non-advertising mail now generate larger increases in the total quantity, and therefore the value of advertising mail. This leads to greater cross-subsidisation between mail types as illustrated in Figure 2, where the number of senders of each type is given in brackets.[19]

In practice, some non-advertising mail items are more effective in drawing consumers' attention to mail. Important bank statements or government communications can make the postal channel

[18] The idea that all senders are supplying independent goods and each sender uses only one type of mail is maintained. This removes the potential additional competitive dimension of a 'winner takes all' kind of sale between multiple advertisers.

[19] The parameters are set to $u = 10$, $c_t = 5$, and $c_d = 1$.



(a) Pricing of Mail

(b) Total Quantities of Mail.

Figure 2: Prices & Quantities with Two Senders of Each Type

more salient. Conversely, some advertisements are a particular nuisance to consumers. We develop our model to explore how a postal operator should respond to such heterogeneities.

2.3..1 Heterogeneous Non-advertising Mail Senders

Sender-specific weights are inserted into the attention function in terms of the positive externality on consumers' attention to advertising mail. Define the consumer attention to each advertising mail item as:

$$A_d(X_d, X_t) = u - \frac{1}{2}X_d + \alpha X_t^{\gamma} \tag{16}$$

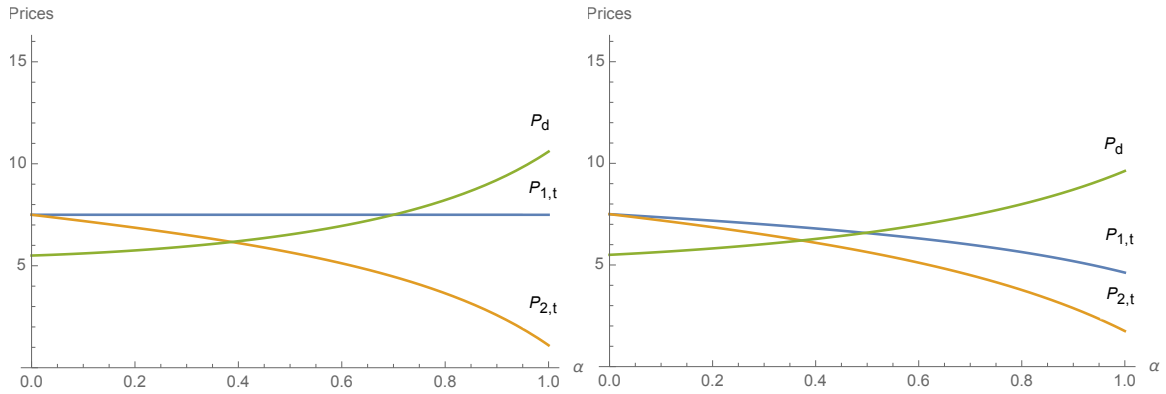
where $X_t^{\gamma} = \sum_{i=1}^{2} \gamma_i x_{i,t}$. The parameter $\gamma_i \in [0,1]$ indicates the strength of additional attention created by non-advertising mail from sender i, where $X_t^{\gamma} \leq X_t$. Adjustments to α reflect the magnitude of the positive externality from non-advertising mail on advertising mail, whereas adjustments to γ_i capture a change in the relative contribution of each non-advertising mail sender to the positive externality. More specifically, if $\gamma_1 = \gamma_2 = 1$, then all non-advertising mail has an equal positive externality on the value of advertising. If $0 \leq \gamma_i < \gamma_j \leq 1$ with $i, j \in \{1,2\}$ such that $i \neq j$, then the positive externality generated by non-advertising mail sender i is lower than that of j.

> **Result 7:** Suppose that non-advertisers vary in the strength of their positive externality on attention to advertising.
>
> (i) The non-advertising mail sender that exerts a larger positive externality on attention to advertising mail pays a lower price.
>
> (ii) Non-advertising mail prices decrease in the positive externality but the price charged to the non-advertiser with the larger externality diminishes at a faster rate.

The postal operator sets a lower price for the non-advertiser that induces the largest positive externality. This stimulates their mail quantity and generates more value for the postal operator to

capture through higher advertising prices. Consumers therefore receive more non-advertising mail from senders with larger positive attention externalities.

More subtle results emerge in numerical examples. Figure 3a illustrates the case where only sender 2 generates a positive externality on advertising mail, whilst Figure 3b illustrates the case where sender 1 exhibits a smaller positive externality than sender 2.[20] If $\gamma_1 = 0.5$, then the postal operator is less reliant on non-advertising mail sender 2 to stimulate the value of advertising compared to the case of $\gamma_1 = 0$. Therefore, the price charged to sender 2 is higher if $\gamma_1 = 0.5$. Intuitively, the postal operator seeks to maximise the value of the mailstream as efficiently as possible and the quantity sent by each sender becomes less elastic with successive price cuts. It follows that the monopolist deems it optimal to reduce the cross-subsidisation through sender 2. Instead the cross-subsidisation is achieved through sender 1 as γ_1 grows from 0 to 0.5.



(a) $\gamma_1 = 0, \gamma_2 = 1$ (b) $\gamma_1 = 0.5, \gamma_2 = 1$.

Figure 3: Pricing with Heterogeneous Non-advertising Mail Senders

2.3..2 Heterogeneous Advertising Mail Senders

Advertisers can also have asymmetric effects on consumers' attention to postal mail and the attention function is now adapted to include such heterogeneities (with symmetric non-advertising senders):

$$A_d(X_d^\beta, X_t) = u - \frac{1}{2}X_d^\beta + \alpha X_t \tag{17}$$

where $X_d^\beta = \sum_i^2 \beta \cdot x_{i,d}$ and $\beta \in [0, 1]$. A larger β_1 corresponds to a greater reduction in attention to advertising mail per unit of $x_{1,d}$. The parameter configuration $\beta_1 = \beta_2 = 1$ captures the standard case of all advertising exerting an equal effect. The following result is established using numerical simulations:[21]

> ***Result 8.*** *An advertiser that causes a larger (smaller) decrease in consumer attention to advertising from their mail faces a higher (lower) price.*

[20] In each case: $u = 10$, $c_d = 1$, and $c_t = 5$.

[21] Existing related work that also draws on numerical simulations include Boldron et al. (2011), Bradley et al. (2015), De Donder et al. (2011) and Wu & Chiu (2020).

Advertisers who cause consumers to reduce their attention to mail more, face higher prices. This internalises the externality they impose on the value of postal mail to other senders. Importantly, this result arises if $\alpha = 0$. Therefore, the idea that mail prices should be higher for advertisers who cause consumers to reduce their attention more appears even in the absence of the attention externality between different mail types.

3. Entry into the Postal Market for Advertising Mail

In recent years, incumbent postal operators have faced new competition from entrants who do not face USO's and focus primarily on advertising (direct) mail (Copenhagen Economics, 2019).[22] The traditional explanation for entrants to focus exclusively on advertising stems from lower servicing costs compared to non-advertising mail. However, our analysis indicates an additional motive driven by the inflated margin on advertising. In this section we explore the consequences of this kind of selective entry on pricing decisions, mail quantities, the mail consumers receive, and the viability of the incumbent's USO.

For simplicity, we return to one sender of each mail type.[23] Define all price, quantity, cost, and profit terms relating to the entrant using $\hat{p_d}$, $\hat{x_d}$, $\hat{c_d}$, and $\hat{\pi}$ as notation. It also needs to be specified how demand is allocated if multiple postal operators tie at the lowest price. We permit any deterministic sharing rule: $g, \hat{g} = (1-g)$ with $g \in (0,1)$ and where g ($\hat{g}$) specifies the proportion of forthcoming demand received by the incumbent (entrant) if the firms tie at the same lowest price.[24] Initially neither the entrant nor the incumbent is assumed to have a cost advantage and thus cost symmetry is imposed:

Assumption 5: $c_d = \hat{c_d}$.

3.1. Competitive Entry

Result 9a. *Following entry into advertising mail delivery by an equally efficient entrant, the unique Nash Equilibrium involves:*

(i) *Marginal cost pricing for advertising mail. Non-advertising mail priced at the monopoly level associated with no externalities ($\alpha = 0$).*

(ii) *Entry reduces (increases) the quantity of non-advertising (advertising) mail received by consumers. Incumbent earns positive profit below the pre-entry level. Entrant earns zero profit.*

[22] For a recent discussion in the UK market, see www.ofcom.org.uk/about-ofcom/latest/features-and-news/royal-mail-whistl-competition-law

[23] The subsequent results are not sensitive to this restriction.

[24] The standard equal sharing rule is a special case whenever $g = 0.5$. However, our general rule extends readily to postal markets with a legacy provider that enjoys greater market prominence. Our results also hold for a random sharing rule, such as the 'winner takes all' approach where one firm from those charging the lowest price is chosen randomly to serve the entire market demand forthcoming at that price. However, this appears less practically relevant to the postal sector.

Following entry, advertising mail replicates a Bertrand duopoly with marginal cost pricing and the most profitable market segment is cannibalised. Non-advertising mail becomes the only profitable segment and the incumbent loses control of the composition of mail. Moreover, the incentive for the incumbent to subsidise non-advertising mail disappears because it cannot extract the value from the advertising channel any more. There are also distributional effects: the surplus from advertising is now captured by the sender, rather than the postal operator and prices of all mail become independent of α. Nonetheless, the quantity of advertising mail continues to be stimulated by the quantity of non-advertising mail through α.

The effect on the total quantity of advertising mail is multi-faceted. Non-advertising mail decreases as its price rises, which reduces the value of advertising to senders, equivalent to a leftward shift of the advertising mail demand curve. However, the diminishing price of advertising mail causes an increase in the quantity, equivalent to a movement down the advertising mail demand curve. Overall, the effect of the price cut dominates and the quantity of advertising increases.

These results are robust with respect to many firms simultaneously joining the market. The core difference with multiple entrants is that there exists an infinite number of pure strategy price equilibria for advertising mail. Provided that at least two firms set marginal cost pricing, a third (or more) firms can set any price above marginal cost and this constitutes a Bertrand equilibrium. There is no impact on non-advertising mail pricing or profit for any firm but the incumbent may become inactive in the advertising mail market. Intuitively, the incumbent becomes indifferent between pricing advertising at marginal cost and sharing the forthcoming demand whilst earning zero profit per unit, or pricing above marginal cost and attracting zero demand.

3.2. Tacit Price Coordination

Can the entrant avoid the advertising mail market from being cannibalised by matching the price set by the incumbent, instead of competing? Is this an equilibrium strategy and what are the welfare effects? We first consider the case where an entrant can price-match and the incumbent cannot adjust the price without invoking a price war, following a standard grim trigger strategy. We subsequently consider the optimal collusive price.

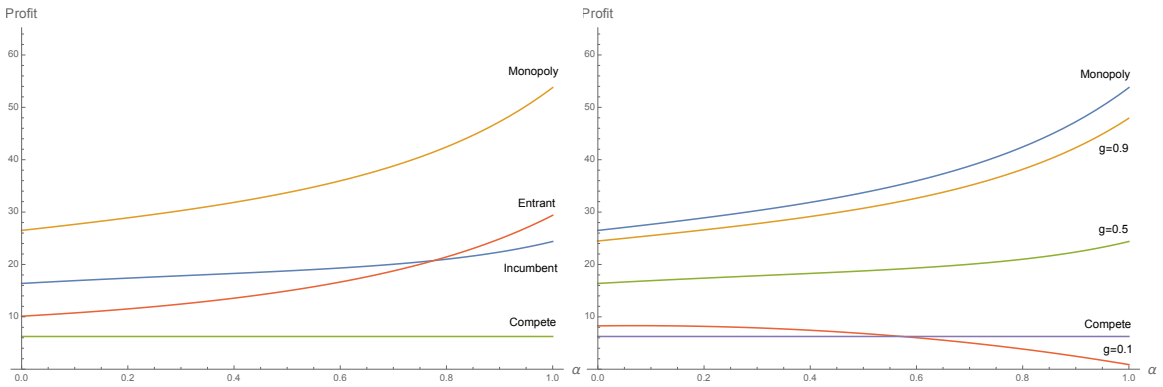
> ***Result 9b.*** *If the entrant matches the monopoly price of advertising set by the incumbent:*
>
> (i) *The entrant earns higher profit than in the competitive outcome. The entrant earns a higher or lower profit than the incumbent, depending on the positive externality.*
>
> (ii) *The incumbent prefers price matching or competing, depending on the positive externality and the incumbent's share of advertising mail. Incumbent profit can be decreasing in the positive externality.*
>
> (iii) *Consumers receive more non-advertising mail and less advertising mail.*

Price coordination is the weakly dominant strategy for the entrant because only if both firms coordinate can the entrant earn positive profit. Importantly, the motivation for the entrant to

coordinate differs from traditional price collusion, where prices are inflated to extract higher profit from a given demand curve. Instead, coordination on advertising prices incentivises the incumbent to continue engaging in cross-subsidisation of non-advertising mail. This in turn stimulates the value of the market to senders and shifts the advertising mail demand curve rightwards.

The incumbent's incentives are more intricate. The entrant steals a share of the profitable advertising mail, whilst the incumbent bears the cost of stimulating its value through subsidised non-advertising mail prices. On one hand, the incumbent can coordinate and accept the loss of a portion of advertising mail demand. On the other hand, it can compete, earn zero profit from advertising mail, and extract the monopoly profit from non-advertising mail. Therefore, the share of the advertising mail market taken by the entrant and the degree to which non-advertising mail prices are depressed prior to entry are critical. The latter is driven by the positive externality from non-advertising mail to advertising mail (α). The reason incumbent profit can decrease in α stems from the inability to adjust prices following entry without generating competition. Prior to entry, the incumbent sets the monopoly price and cross-subsidises non-advertising mail potentially below cost. Following entry, the incumbent would prefer to reduce the degree of cross-subsidisation but cannot do so without instigating competition.

The welfare incidence of coordination is multi-faceted: postal operators gain higher profit and non-advertisers enjoy a lower price. Advertisers benefit from a more valuable mail channel, although this is appropriated by the postal operator(s) in the form of higher price. Most surprisingly, consumers are made better off since the mailmix improves: they receive more non-advertising mail and less advertising mail under coordination than competition.



(a) Coordination vs Competition Profit.

(b) The Role of the Sharing Rule.

Figure 4: Coordination vs Competition Incentives

Figure 4a illustrates the main insights[25] 'Incumbent' and 'Entrant' correspond to price coordination profits for each postal operator, whereas 'Monopoly' and 'Compete' correspond to the profit of the incumbent pre-entry and post-entry following competition, respectively. In this example, incumbent profit is always higher under coordinated pricing than competing. The value of α at

[25]The parameters are set as$u = 10$, $c_d = 1$, $c_t = 5$, and $g = 0.5$.

which entrant profit exceeds incumbent profit reflects the point at which the incumbent is pricing non-advertising mail below cost. Under price coordination, the entrant's profit increases more sharply with α than the incumbent's and the entrant can earn higher profit. Intuitively, the entrant benefits from the elevated price of advertising as α increases, without bearing the cost of the cross-subsidisation. In this example, the incumbent retains a sufficiently large share of the advertising mail to prefer coordination.

Figure 4b illustrates the monopoly pre-entry profit, competitive profit post-entry, and coordinated profits post-entry, when the incumbent receives 90%, 50%, or 10% of the advertising mail demand. If the incumbent receives the majority of the advertising mail demand (90%) at the coordinated price, then coordination dominates the incentive to compete and profit increases with α. In other words, the benefit of the cross-subsidisation outweighs the cost. If the incumbent receives a minority share of advertising (10%), then the cost of cross-subsidisation outweighs the benefit of the higher advertising prices/quantities. Profit decreases with α as the incumbent loses such a large share of the profitable advertising mail to the entrant and competition can become preferable. Therefore, the positive externality across the mail types can impact market structure as well as coordination incentives.

Beyond accepting price coordination or engaging in competition, the incumbent may adjust prices in anticipation of entry and accept coordination at the new adjusted prices.

> ***Result 9c.*** *If the incumbent anticipates entry by a price-matching competitor on advertising mail, the incumbent increases (decreases) non-advertising (advertising) mail prices.*

Price cuts prior to potential entry are traditionally interpreted as entry deterrence but this is not the motivation here. In this environment, the incumbent's incentive to cross-subsidise non-advertising mail is weakened by the loss of a share of advertising mail to the entrant. Therefore, the price of non-advertising items increases and the price of advertising items decreases.

3.3. Entry by a More Efficient Entrant

It seems reasonable to ask whether the potential negative consequences of postal competition persist if the entrant is more efficient. Cost asymmetries generate more intricate pricing dynamics, which depend on the extent of the cost advantage. Define the entrant's monopoly price as:

$$\hat{p_d^{**}} = argmax\{(\hat{p_d} - \hat{c_d})x_d^{**}\}$$

where x_d^{**} is the resulting demand for advertising mail in the case of only the entrant delivers advertising. We call the cost advantage large (small), whenever the monopoly price of the entrant is below (above or equal to) the incumbent's marginal cost $\hat{p_d^{**}} < c_d$ ($\hat{p_d^{**}} \geq c_d$). The following result ensues:

> ***Result 9d:*** *If the entrant holds a large cost advantage, there exists a unique Nash Equilibrium such that:*

(i) The incumbent focuses exclusively on non-advertising mail. Non-advertising price and quantity follow the case with no externality ($\alpha = 0$). As with symmetric costs between the operators, entry increases non-advertising mail price and reduces quantity.

(ii) The entrant monopolises advertising mail and chooses the monopoly price.

(iii) The quantity of advertising mail can be higher or lower following entry.

Entry by a firm with a *large* cost advantage leads to market separation: the incumbent (entrant) focuses only on non-advertising (advertising) mail. Following entry, the incumbent increases the non-advertising mail price as there is no incentive to cross-subsidise. Therefore, equilibrium quantity of non-advertising mail and incumbent profit decrease.[26] The entrant sets a lower price for advertising than the incumbent would choose in the absence of entry, for two reasons. Firstly, the incumbent would have reduced non-advertising prices to stimulate demand and captured this value through higher advertising prices. Secondly, the cost advantage of the entrant reduces the entrant's monopoly price.

The entrant earns a higher profit than the incumbent. This stems from two drivers. Firstly, the marginal cost of servicing advertising are lower, which is exacerbated by the cost advantage of the entrant. Secondly, the profitability of advertising mail is enhanced by the positive externality from non-advertising mail.[27]

The equilibrium quantity of advertising can be higher or lower following entry, depending on the cost difference $(c_d - \hat{c_d})$ relative to the value of the positive externality from non-advertising mail (α).[28] If α is small (large) relative to the cost difference $c_d - (\hat{c_d})$, then the quantity of advertising increases (decreases) following entry. The reduction in price from the entrant's cost advantage more than (less than) compensates for the decline in the value of the advertising channel due to the diminishing quantity of non-advertising mail.

Result 9e. *Whenever an entrant holds a* ***small*** *cost advantage, there exists no Pure Strategy Nash equilibrium.*

The non-existence of Pure Strategy Nash Equilbrium relates to the well-known open-set problem created by the possibility of infinitesimally small undercutting deviations in price games.[29] Whilst the equilibrium does not formally exist, the implication for practitioners is that the entrant would

[26] The optimal price and quantity for the entrant are: $\hat{p_d^*} = \frac{u+\hat{c_d}}{2} + \frac{\alpha(u-c_t)}{4}$ and $\hat{x_d^*} = \frac{(u-\hat{c_d})}{2} + \frac{\alpha(u-c_t)}{4}$.

[27] These two drivers are most clearly visible by comparing the profit expressions of the incumbent and the entrant: $\pi = \left(\frac{u-c_t}{2}\right)^2$ and $\hat{\pi} = \left(\frac{u-\hat{c_d}}{2} + \frac{\alpha(u-c_t)}{4}\right)^2$.

[28] For the quantity of advertising mail to be higher before entry, it needs to be the case that $2\alpha^2(u-\hat{c_d}) + \alpha^3(u-c_t) > 8(c_d - \hat{c_d})$.

[29] See Vives (1999) for a detailed discussion of this problem. If both firms price above the incumbent's marginal cost, either firm can profitably undercut to secure the entire demand at a negligibly lower price. If both firms price below the incumbent's marginal cost, the incumbent would deviate to a higher price to earn non-negative profit. If both firms price at the marginal cost of the incumbent, the entrant can profitably undercut to a marginally lower price. However, this resulting position cannot be an equilibrium because the price of the more efficient seller: $p_d = c_d - \epsilon$ for some $\epsilon > 0$, is undefined.

undercut at the highest price possible without allowing the incumbent to profitably service advertising mail. Therefore, market separation reappears. Notably, increasing α and/or decreasing c_t can enable the incumbent to contest the advertising mail market: this moves the situation from the large to the small cost advantage case. The intuition is that as α rises (or c_t declines), the entrant's monopoly price for advertising mail grows and competing becomes viable for the incumbent.

4. Discussion: Managerial Implications, Policy & Conclusion

In a two-sided postal market with limited consumer attention, a postal operator can increase profit by optimising prices to manage the quantity and composition of mail consumers receive. Competitive entry by postal operators focused on advertising mail can harm all market participants. Most strikingly, price coordination in this context can benefit all market participants. In this section, we spell out the implications for the current debate on USO and price regulations, and demonstrate how our results provide a new line of argument in recent competition cases by UK and European regulators.

4.1. Universal Service Obligation and Price Regulations

The Universal Service Obligation, which enforces efficiency expectations and mandates geographically uniform and affordable prices, is becoming increasingly difficult to sustain (House of Commons, 2015). Existing policy debates highlight a strategic tradeoff between enhancing competition and preserving the financial viability of the universal service provider (Copenhagen Economics, 2019, p. 35). A recent report for the European Parliament discusses possible remedies including adapting the degree of state intervention in response to changes in postal users needs, weakening the requirements of the USO, and prioritising the financial viability of the USO above enhancing competition (Copenhagen Economics, 2019, p. 10.).

We have two main contributions here. Firstly, we have shown that in the absence of the USO it can be optimal for a postal operator to continue to service non-advertising mail at a price below cost, due to the positive externalities on other revenue streams. Therefore, the requirement is weaker than initially appears. Secondly, we present a novel rationale for competition from entrants who specialise in delivering only advertising to be harmful to stakeholders across the postal industry, especially once non-price factors are considered for mail senders and recipients.

In many countries, postal operators are also constrained by price regulations. For example, in the UK, Ofcom imposes a price cap on second class mail to ensure a basic, affordable service for all consumers.[30] Recent policy debates surrounding the European Parliament discuss the need for more flexible pricing, partially to sustain the USO. We highlight an additional consideration that does not currently feature in the policy debate: price regulation on some services can constrain the postal operator from internalizing market externalities around the platform. For instance, suppose the price of advertising mail faces a price ceiling below the optimal level identified in Figure 1. This prevents the postal operator from engaging in full cross-subsidisation between mail types, which leads

[30]Further details: www.ofcom.org.uk/about-ofcom/latest/media/analysts/regulated-prices.

to lower quantities of both mail types and higher prices for non-advertising mail. Therefore, policy regulation and antitrust more broadly should be mindful of the network of externalities present on the postal platform.

4.2. Semi-Collusion

Our analysis contributes further evidence that classic results regarding the harm deriving from collusion in traditional markets need not survive in two-sided environments. On our two-sided postal platform, advertisers who face collusive prices, pay above the competitive level. However, there is another type of agent on this side of the platform: the senders of non-advertising mail, who benefit indirectly from the collusive price. Specifically, collusion on advertising mail leads to subsidised prices for non-advertisers. Moreover, consumers are rewarded by a lower ratio of advertising to non-advertising mail. Finally, despite the inflated price, advertisers enjoy increased attention from consumers.

4.3. Implications for Access Pricing Regulations

In 2018, UK communications services regulator Ofcom fined Royal Mail (RM, the UK's former publicly owned postal monopolist) £50 million for breaching competition law through abuse of dominance. Access operators process and sort bulk (predominantly advertising) mail before paying RM to complete the final delivery in some or all geographical areas.[31] In 2014, one third of RM's letters came via the largest access operator, Whistl. Whistl intended to bypass RM in some geographical locations delivering their own mail, typically in low cost (densely populated) areas. In response, RM increased the tariff price for access operators servicing their own mail in some areas, with two justifications: (i) higher average costs when access operators deliver some of their own mail, (ii) a price premium for access operators requiring flexibility to deliver their own mail in some areas (Ofcom, 2018). Neither explanation was accepted: "We have concluded that Royal Mail did not have a legitimate justification for discriminating in this way against its access customers which chose to compete with it " (Ofcom, 2018, p. 4).

Whilst a full formal analysis of access pricing lies beyond the scope here, our findings provide the first step for a new defence for incumbents to charge higher tariffs to access operators that deliver their own mail. Specifically, these operators generate a negative industry externality by removing the postal operators' ability to manage the composition and quantity of mail. They reduce consumer attention to mail and the value of postal services. Importantly, this problem does not arise in models of digital platform competition (e.g. social media), where consumers identify which platforms to participate in and operators compete for their membership (Shekhar, 2020). In the postal market consumers receive all mail sent to them and they cannot distinguish between (or unsubscribe from) particular operators.

In the UK, Ofcom also requires that access prices "allow Royal Mail to charge prices to reflect its

[31] "Bulk mail does not have a precise definition, but the term is used to refer to high volume mailings of often similar or identical mailing items being sent to addresses across the whole of UK or at least a substantial part of it." (Ofcom, 2018, p. 10)

costs and investment in its network..." However, cost-driven price regulations neglect externalities within the postal sector. Postal operators such as RM also require that to access some tariffs, access operators maintain a specific geographical mail delivery profile to RM, to ensure capacities and demand are aligned (Ofcom, 2018). However, there is currently no equivalent requirement on the composition of mail or the quality and relevance of the mail sent through access operators.

In this direction, LinkedIn.com - a platform connecting job seekers with recruiters - sells advertising space through an auction where the winning bidder has the highest 'price multiplied by 'advertising relevancy' score.' Our results help to explain how this auction mechanism ensures advertisers pay a premium when their content is less relevant and engaging for members, due to the negative externality on the platform. In addition, the mechanism incentivises advertisers to enhance the quality of their advertising.

4.4. Concluding Remarks

This paper has explored the postal mail sector as a platform connecting different types of mail senders with recipients. We have highlighted the importance of incorporating limited consumer attention into the price determination process on platforms and the role of pricing for platform operators to manage consumer attention. The main mechanism we uncover is due to externalities between mail types affecting the postal platform. Accordingly, discounting non-advertising mail prices stimulates the quantity of non-advertising mail sent. This increases the attention recipients pay to their received advertising, which in turn renders the mailstream more valuable for advertisers. As a consequence, the postal operator can extract this additional value by inflating advertising prices. In equilibrium the overall mail volumes increase for both mail types.

We provide a new rationale for entry into the servicing of advertising, not driven by the lower cost of processing. Instead, advertising mail is attractive due to the higher mark-ups, generated by the cross-subsidisation from non-advertising prices, which invites entry. However, competitive entry cannibalises the advertising mail market and non-advertising mail becomes the profitable segment.

We highlight a new form of semi-collusion, where only a subset of agents on one side of a two-sided market face collusive prices. Unusually, tacit coordination between an incumbent and entrant can benefit all market participants due to the two sided market. Postal operators earn higher profit from the cross-subsidisation of non-advertising prices, which is shut down by competitive entry. Relative to the case of competitive entry, advertisers benefit from recipients paying more attention to their mail, non-advertisers benefit from lower mail prices and recipients benefit from an improved composition of the mail they receive as the proportion of advertising mail decreases.

For regulators, we show that the Universal Service Obligation imposed on postal sector incumbents is less demanding than traditionally viewed: whilst non-advertising mail is more costly to process and deliver, the incumbent can extract additional value from the advertisers by servicing non-advertisements. The postal incumbent can find it optimal to deliver all mail types without regulatory mandates. However, pricing regulations including access pricing based on costs can harm the postal industry.

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Appendix

This appendix presents the full details and proofs of the results contained within the paper.

Proof of Result 1. The main result follows from the analysis in the main text. The comparative statics are as follows:

(i)

$$\frac{\partial p_d^M}{\partial \alpha} = \frac{4(u-c_t) + 4\alpha(u-c_d) + \alpha^2(u-c_t)}{(4-\alpha^2)^2} > 0$$

Note that: $u > c_t, u > c_d$. Hence the numerator is positive.

$$\frac{\partial p_t^M}{\partial \alpha} = \frac{4(c_d-u) + \alpha^2(c_d-u) + 4\alpha(c_t-u)}{(4-\alpha^2)^2} < 0$$

$$\frac{\partial x_d^M}{\partial \alpha} = \frac{(4+\alpha^2)(u-c_t) + \alpha^2(u-c_t)}{(4-\alpha^2)^2} > 0$$

$$\frac{\partial x_d^M}{\partial \alpha} = \frac{(4+\alpha^2)(u-c_d) + 4\alpha(u-c_t)}{(4-\alpha^2)^2} < 0$$

(ii) The following comparative statics are clear by inspecting the expressions in the main text: $\frac{\partial p_d^M}{\partial c_d} > 0, \frac{\partial p_d^M}{\partial c_t} < 0, \frac{\partial p_t^M}{\partial c_t} > 0, \frac{\partial p_t^M}{\partial c_d} > 0$.

(iii) The following comparative statics are clear by inspecting the expressions in the main text: $\frac{\partial x_d^M}{\partial c_d} < 0, \frac{\partial x_d^M}{\partial c_t} < 0, \frac{\partial x_t^M}{\partial c_d} < 0, \frac{\partial x_t^M}{\partial c_t} < 0$.

(iv)

$$\frac{\partial p_d^{M,I}}{\partial \alpha} + \frac{\partial p_t^{M,I}}{\partial \alpha} = \frac{(-4+4\alpha-\alpha^2)(c_t-c_d)}{(4-\alpha^2)^2} < 0$$

Note that by definition: $c_t \geq c_d$.

(v) *Claim: Postal operator profit increases with α.*

We establish this result by means of Subgame-Perfect Equilibrium.

Step 1: Suppose to the contrary that postal operator profit does not increase with α. If this were the case, the postal operator could price as if $\alpha = 0$. In this case, prices become independent of α, the quantity of non-advertising mail is independent of α but the quantity of advertising mail sent increases with α. To see this, note that the best response function of the advertising mail sender is: $x_d^* = (1+\alpha)u - p_d - \alpha p_t$. Therefore, if the postal operator sets prices as if $\alpha = 0$, profit increases with α.

Step 2: We have established that profit for the postal operator increases with α even when prices do not vary with α. By definition of the Nash equilibrium, the postal operator will only adjust prices with α if profit increases further by adjusting prices with α. Therefore, by reducing the price of non-advertising mail and increasing the price of advertising mail, as α increases, this implies that profit must be increasing with α. The level of profit associated with a lower value of α could always be achieved when α is higher.

Proof of Result 2. The proof follows from the main model when $\alpha = 0$. The result that profit for the postal operator decreases when all parties neglect α follows from Result 1, where profit is increasing in α.

Proof of Result 3. The two senders' profit maximization problems become:

$$\max_{x_d} \pi_{dms} = (u + \alpha x_t)x_d - \frac{1}{2}x_d^2 - p_d x_d$$

$$\max_{x_t} \pi_{tms} = ux_t - \frac{1}{2}x_t^2 - p_t x_t,$$

The ensuing optimal quantities are given by: $x_d^* = (1+\alpha)u - p_d - \alpha p_t, \quad x_t^* = u - p_t$.

The incumbent operator fails to recognise the positive externality and does not internalise it into their optimal pricing decisions. As a consequence, equilibrium prices follow the case $\alpha = 0$:

$$p_d^{M,nI'} = \frac{u + c_d}{2}, \quad p_t^{M,nI'} = \frac{u + c_t}{2}.$$

The ensuing demand quantities, where senders internalise the externality, are:

$$x_d^{M,nI'} = \frac{(1+\alpha)u - c_d - ac_t}{2}, \quad x_t^{M,nI'} = \frac{u - c_t}{2}.$$

To see that profit is higher than Result 2 (where all parties neglect α) but lower than Result 1 (where all parties internalise α), we employ the same line of reasoning as in Result 1. We established that if the postal operator acts as if $\alpha = 0$ but the senders recognise $\alpha > 0$, postal operator profit increases with α. Therefore, profit is lowest when all parties neglect α. However, the observation that the postal operator adjusts prices with α must, by the property of Nash equilibrium, indicate that profit is highest in Result 1 where α is fully internalised in prices.

Proof of Result 4a. The result follows immediately from the expressions for price and quantity in the main paper.

Proof of Result 4b. The resulting expressions for the optimal price and quantity are below. The result follows immediately from these expressions.

$$p_d^M = \frac{2u_d + \alpha u_t + (2-\alpha^2)c_d - \alpha c_t}{4 - \alpha^2}, \quad p_t^M = \frac{(2-\alpha^2)u_t - \alpha u_d + \alpha c_d + 2c_t}{4 - \alpha^2}$$

$$x_d^M = \frac{2u_d + \alpha u_t - \alpha c_t - 2c_d}{4 - \alpha^2}, \quad x_t^M = \frac{2u_t + \alpha u_t - \alpha c_d - 2c_t}{4 - \alpha^2}$$

Proof of Result 4c. The result follows immediately from the expressions for price and quantity above.

Proof of Result 5. In the case of a single decision maker sending both types of mail, the single mail sender will maximise:

$$\max_{x_t, x_d} \pi = [V(A_t) - p_t]\, x_t + [V(A_d) - p_d]\, x_d \tag{18}$$

Inserting the linear attention function:

$$\max_{x_t, x_d} \pi = (u + \alpha x_t)x_d - \frac{1}{2}x_d^2 - p_d x_d + ux_t - \frac{1}{2}x_t^2 - p_t x_t.$$

Taking the FOC's and solving yield:

$$x_d = u - p_d + \alpha x_t$$

$$x_t = u - p_t + \alpha x_d$$

Remark. In the case with independent decision makers for the senders, the initial FOC's for the advertising mail sender is unchanged but the FOC for the non-advertising mail sender would be $x_t = u - p_t$. Therefore, for given prices, the best response indicates that the quantity demanded of non-advertising mail is higher when there is a single decision maker sending the mail. This does not in itself demonstrate that the final quantity will be higher with only one single decision maker, as the postal operator may adjust prices in response. However, it does indicate that postal operator profit will be higher when there is a single decision maker sending both types of mail, rather than independent decision makers.

Solving the First Order Conditions above yields:

$$x_t = \frac{u - p_t + \alpha u - \alpha p_d}{1 - \alpha^2}$$

$$x_d = \frac{u - p_d + \alpha u - \alpha p_t}{1 - \alpha^2}$$

We now solve the postal operator's maximisation problem to solve for the optimal prices, conditional on the demand functions above.

$$\pi_{PO} = (p_t - c_t)x_t + (p_d - c_d)x_d$$

Inserting the best response functions for x_t and x_d yields:

$$\pi_{PO} = (p_t - c_t)\left(\frac{u - p_t + \alpha u - \alpha p_d}{1 - \alpha^2}\right) + (p_d - c_d)\left(\frac{u - p_d + \alpha u - \alpha p_t}{1 - \alpha^2}\right)$$

Taking the FOC's yields:

$$p_t = \frac{u + \alpha u + c_t + \alpha c_d}{2} - p_d, \quad p_d = \frac{u + \alpha u + \alpha c_t + c_d}{2} - \alpha p_t$$

Solving the FOC's yields:

$$p_t = \frac{u + c_t}{2}, \quad p_t = \frac{u + c_d}{2}$$

Note that these are the same as the main model when $\alpha = 0$. Inserting the equilibrium prices into the quantity expressions above yields:

$$x_t = \frac{u + \alpha u - c_t - \alpha c_d}{2(1 - \alpha^2)}, \quad x_d = \frac{u + \alpha u - c_d - \alpha c_t}{2(1 - \alpha^2)}$$

Comparisons with standard case of independent mail senders.

Claim: The price of advertising mail is lower and the price of non-advertising mail is higher, when there is a single decision maker choosing both quantities of mail than when there are independent decision makers for the two mail types.

Proof: Prices when there is a single decision maker are equivalent to the case of $\alpha = 0$. With independent decision makers, p_t decreases with α and p_d increases with α. With a single decision maker this is no longer true.

Claim: Non-advertising mail equilibrium quantity is higher when there is a single decision maker.

Proof: In the case of independent decision makers: $x_t^M = \frac{(2+\alpha)u-2c_t-\alpha c_d}{4-\alpha^2}$. With a single decision maker: $x_t = \frac{u+\alpha u-c_t-\alpha c_d}{2(1-\alpha^2)}$. It can be shown that the case with a single decision maker is higher.

Claim: Advertising mail equilibrium quantity is higher when there is a single decision maker.

Proof: In the case of independent decision makers: $x_d^M = \frac{(2+\alpha)u-2c_d-\alpha c_t}{4-\alpha^2}$. With a single decision maker: $x_t = \frac{u+\alpha u-c_t-\alpha c_d}{2(1-\alpha^2)}$. It can be shown that the case with a single decision maker is higher.

Claim: Postal Operator profit is higher when there is a single decision maker than independent decision makers.

Proof: To see this most simply, observe from the initial first order conditions for x_t, x_d that when prices p_d, p_t are fixed, the quantity of non-advertising mail would be higher when there is a single decision maker, whilst the quantity of advertising mail would be the same. Therefore, if prices were to remain unchanged from the case with independent decision makers, when there is a single decision maker, profit would be higher with a single decision maker. In the Nash Equilibrium, the postal operator finds it optimal to adjust prices to take account of the single decision maker. This must yield an even higher profit than could be obtained by pricing as if there are independent decision makers.

Proof of Result 6. Final Expressions for the Two Sender Model: This section solves the expressions for closed form solutions. To begin, sub $p_{2,t}$ into $p_{1,t}$ to yield:

$$p_{1,t} = \frac{u + c_t}{2} - \frac{2\alpha\gamma_1(p_d - c_d)}{3}$$

By symmetry of the optimisation process:

$$p_{2,t} = \frac{u + c_t}{2} - \frac{2\alpha\gamma_2(p_d - c_d)}{3}$$

Substituting these values into the expression for p_d and simplifying:

$$p_d^* = \frac{9}{9 - 4\alpha^2(\gamma_1^2 + \gamma_2^2 - \gamma_1\gamma_2)} \left[\frac{u + c_d}{2} + \frac{\alpha}{3}\left[\frac{(u - c_t)(\gamma_1 + \gamma_2)}{2} - \frac{4\alpha c_d(\gamma_1^2 + \gamma_2^2 - \gamma_1\gamma_2)}{3}\right]\right]$$

$$p_{1,t}^* = \frac{u + c_t}{2}$$
$$+\frac{2\alpha\gamma_1}{3}\left(c_d - \frac{9}{9 - 4\alpha^2(\gamma_1^2 + \gamma_2^2 - \gamma_1\gamma_2)}\left[\frac{u + c_d}{2} + \frac{\alpha}{3}\left[\frac{(u - c_t)(\gamma_1 + \gamma_2)}{2} - \frac{4\alpha c_d(\gamma_1^2 + \gamma_2^2 - \gamma_1\gamma_2)}{3}\right]\right]\right)$$

$$p^*_{2,t} = \frac{u + c_t}{2}$$
$$+\frac{2\alpha\gamma_2}{3}\left(c_d - \frac{9}{9 - 4\alpha^2(\gamma_1^2 + \gamma_2^2 - \gamma_1\gamma_2)}\left[\frac{u + c_d}{2} + \frac{\alpha}{3}\left[\frac{(u - c_t)(\gamma_1 + \gamma_2)}{2} - \frac{4\alpha c_d(\gamma_1^2 + \gamma_2^2 - \gamma_1\gamma_2)}{3}\right]\right]\right)$$

Proof of Result 7. From Result 6, when there are two senders of the same mail type, the first order conditions for the non-advertising mail senders' optimisations are:

$$p_{1,t} = \frac{u + c_t}{2} - \frac{2\alpha\gamma_1(p_d - c_d)}{3} \tag{19}$$

$$p_{2,t} = \frac{u + c_t}{2} - \frac{2\alpha\gamma_2(p_d - c_d)}{3} \tag{20}$$

Therefore, using (19) and (20) , if $\gamma_1 > \gamma_2$, then $p_{1,t} < p_{2,t}$.

Proof of Result 8. This result is established using numerical simulations and a numerical example is given in the main text.

Proof of Result 9a.

Step 1: We first establish marginal cost pricing for advertising mail. It is well-known that in Bertrand price competition with constant marginal costs, the unique pure strategy equilibrium involves marginal cost pricing. Therefore, $p_d = \hat{p_d} = c_d = \hat{c_d}$.

The equilibrium must involve symmetric prices: $p_d = \hat{p_d}$. For any $p_d > \hat{p_d} > c_d$, the incumbent obtains zero demand for advertising mail. There exists a profitable deviation to $p_d = \hat{p_d} - \epsilon > c_d,\ \epsilon > 0$. (Similarly for $\hat{p_d} > p_d > c_d$). Any pair of prices involving at least one firm choosing a price below marginal cost also cannot be an equilibrium.

For any $p_d = \hat{p_d} > c_d$, there exists a profitable deviation for either firm to marginally undercut to obtain the entire market demand for advertising mail.

Step 2: Fix the price of advertising mail at c_d. The optimisation of the advertising mail sender becomes:

$$\pi_{dms} = \left(u + \alpha x_t - \frac{1}{2}x_d\right)x_d - c_d x_d$$

where x_d refers to the total quantity of advertising mail sent by the single advertising mail sender. This quantity may be allocated to one postal operator of split between both operators if prices are tied (which we will show is the case). Also note in the expression for the advertising mail sender: $p_d = c_d$ in the final term from Step 1.

The best response yields: $x_d = u + \alpha x_t - c_d$.

Step 3: We now solve for the non-advertising mail sender's optimal quantity: $x_t = u - p_t$.

Step 4: We now determine the optimal price set by the incumbent for non-advertising mail, which follows the no-interdependence case. The intuition is clear when writing the profit function of the incumbent:

$$\pi = (p_d - c_d)x_t + (p_d - c_d)x_d$$

where the second term is zero as $p_d = c_d$ due to entry. Therefore, $p_t^* = \frac{u+c_t}{2}$. Using the best response in Step 4; $x_t^* = \frac{u-c_t}{2}$.

Step 5: The total quantity of advertising mail is: $x_d = u - c_d + \alpha(\frac{u-c_t}{2})$. Note that this is the total quantity of advertising mail sent by the advertising mail sender and each postal operator receives their share.

Step 6: The entrant earns zero profit and the incumbent only derives profit from advertising mail, which is equivalent to the profit the firm would earn without entry if there were no-interdependence between the main types. Profit for the incumbent is: $(p_t - c_t)x_t = (\frac{u-c_t}{2})^2$.

Claim: x_d is higher following entry by a competitive entrant into the advertising mail market than with only the monopoly incumbent operating in the market.

Step 1: x_d without entry is $\frac{(1+\alpha)u-2c_d-\alpha c_t}{4-\alpha^2}$ and with competitive entry is $\frac{4u-4c_d+2\alpha u-2\alpha c_t}{4}$.

Step 2: Therefore, we need to show:

$$\frac{(1+\alpha)u - 2c_d - \alpha c_t}{4-\alpha^2} < \frac{4u - 4c_d + 2\alpha u - 2\alpha c_t}{4}$$

Proceeding as follows:

$$\frac{4(u + \alpha u - 2c_d - \alpha c_t)}{1} < \frac{(4-\alpha^2)(4u - 4c_d + 2\alpha u - 2\alpha c_t)}{1}$$

This simplifies to:

$$0 < (4 - 2\alpha^2)(u - c_d) + (u - c_t)(2\alpha - \alpha^3)$$

This condition is satisfied.

Proof of Result 9b. Firstly, we can write the profit for the incumbent prior to entry, using the optimal prices and quantities taken from the fully recognised interdependency case of the single sender model. The required optimal values are:

$$p_d^M = \frac{(2+\alpha)u + (2-\alpha^2)c_d - \alpha c_t}{4-\alpha^2}, \quad p_t^M = \frac{(2-\alpha-\alpha^2)u + \alpha c_d + 2c_t}{4-\alpha^2}$$

$$x_d^M = \frac{(2+\alpha)u - 2c_d - \alpha c_t}{4-\alpha^2}, \quad x_t^M = \frac{(2+\alpha)u - 2c_t - \alpha c_d}{4-\alpha^2}$$

Therefore;

$$\pi_{PO}^{M,I} = (p_t^M - c_t)x_t^M + (p_d^M - c_d)x_d^M$$

$$\pi_{PO}^M = \left(\frac{(2-\alpha-\alpha^2)u + \alpha c_d + 2c_t}{4-\alpha^2} - c_t\right)\left[\frac{(2+\alpha)u - 2c_t - \alpha c_d}{4-\alpha^2}\right]$$

$$+ \left(\frac{(2+\alpha)u + (2-\alpha^2)c_d - \alpha c_t}{4-\alpha^2} - c_d\right)\left[\frac{(2+\alpha)u - 2c_d - \alpha c_t}{4-\alpha^2}\right]$$

Secondly, write the incumbent's profit post-entry by an entrant who takes away some of the advertising mail demand. The incumbent retains share g. This is on the assumption of price coordination.

$$\pi_{PO} = \left(\frac{(2-\alpha-\alpha^2)u + \alpha c_d + 2c_t}{4-\alpha^2} - c_t \right) \left[\frac{(2+\alpha)u - 2c_t - \alpha c_d}{4-\alpha^2} \right]$$
$$+g \cdot \left(\frac{(2+\alpha)u + (2-\alpha^2)c_d - \alpha c_t}{4-\alpha^2} - c_d \right) \left[\frac{(2+\alpha)u - 2c_d - \alpha c_t}{4-\alpha^2} \right]$$

Thirdly, we can write the profit for the entrant when engaging in coordination. This is simply the share $(1-g)$ of the total non-advertising mail profit from the incumbent prior to entry.

$$\pi = (1-g) \cdot \left(\frac{(2+\alpha)u + (2-\alpha^2)c_d - \alpha c_t}{4-\alpha^2} - c_d \right) \left[\frac{(2+\alpha)u - 2c_d - \alpha c_t}{4-\alpha^2} \right]$$

Fourthly if we have a case of competition, the incumbent drives the price and profit of advertising mail to zero. The profit for the incumbent follows the profit from non-advertising mail only in the no-interdependency case from the model with one sender of each type:

$$p_d^{M,nI} = \frac{u + c_d}{2},\ x_d^{M,nI} = \frac{u - c_d}{2}$$

Therefore, in that case $\pi = (\frac{(u-c_d)}{2})^2$ for the incumbent. We can use these expressions to demonstrate the claims in Result 9b.

Proof of Result 9c. The postal operator will maximise:

$$\max_{p_d, p_t} \pi = (p_t - c_t)x_t^* + (p_d - c_d)\frac{x_d^*}{2}$$

The final term captures the fact that the incumbent only receives have of the advertising mail demand. The best response functions for the quantities remain the same as in the single sender model with full inter-dependence:

$$x_d^* = (1+\alpha)u - p_d - \alpha p_t, \quad x_t^* = u - p_t.$$

Substituting these expressions and taking the FOC yields the best response for prices:

$$p_t = \frac{u + c_t}{2} - \frac{\alpha}{\mathbf{4}}(p_d - c_d), \quad p_d = \frac{(1+\alpha)u - \alpha p_t + c_d}{2}$$

The only difference from the standard case without anticipating entry in the SS model is the 4 in bold (which was 2). Therefore, when $\alpha = 0$, anticipating entry has no effect on prices.

Solving simultaneously:

$$p_t = \frac{1}{8-\alpha^2}\left(u(4-\alpha-\alpha^2) + 4c_t + \alpha c_d\right)$$

We now explore the comparative statics.

Claim: The price of non-advertising mail when the incumbent anticipates entry is lower than when there is no adjustment for entry.

Proof: In the case without entry (from the single sender model);

$$p_t^{M,I} = \frac{(2-\alpha-\alpha^2)u + \alpha c_d + 2c_t}{4-\alpha^2}$$

In the case with anticipating entry:

$$p_t = \frac{1}{8-\alpha^2}\left(u(4-\alpha-\alpha^2) + 4c_t + \alpha c_d\right)$$

It can be demonstrated that the second is larger than the firs:

$$\frac{1}{8-\alpha^2}\left(u(4-\alpha-\alpha^2) + 4c_t + \alpha c_d\right) \geq \frac{(2-\alpha-\alpha^2)u + \alpha c_d + 2c_t}{4-\alpha^2}$$

Cross multiply:

$$(4-\alpha^2)\left(u(4-\alpha-\alpha^2) + 4c_t + \alpha c_d\right) \geq (8-\alpha^2)((2-\alpha-\alpha^2)u + \alpha c_d + 2c_t)$$

Simplify

$$2\alpha^2 u - 4\alpha c_d - \alpha^2 2c_t + 4\alpha u \geq 0$$

Rearrange:

$$4\alpha(u-c_d) + 2\alpha^2(u-c_t) \geq 0$$

Clearly the price of non-advertising mail is lower when the incumbent anticipates entry providing $\alpha > 0$.

Claim: The price of advertising mail is lower when the incumbent anticipates entry.

Proof: This is evident from the best response for advertising mail; the best response itself is identical but it is decreasing in p_t. Therefore, as p_t is now larger, p_d is now lower.

We can also show this formally by comparing the price of advertising mail in the standard case and in the case anticipating entry.

The price of advertising mail in the standard case without entry is:

$$p_d^{M,I} = \frac{(2+\alpha)u + (2-\alpha^2)c_d - \alpha c_t}{4-\alpha^2}$$

and anticipating entry by a price matching entrant the price of advertising mail is:[32]

$$p_d = \frac{4u + 2\alpha u + 4c_d - 2\alpha c_t - \alpha^2 c_d}{8-\alpha^2}$$

We can now show that the price of advertising mail is lower when the incumbent anticipates a price-matching entrant.

The condition becomes:

[32]This is obtained by solving the maximisation of the postal operator when they receive only half of the advertising mail demand.

$$4\alpha(u - c_d) + 2\alpha^2(u - c_t) > 0$$

This is satisfied for any $\alpha > 0$. Again if $\alpha = 0$, the possibility of price matching entry has no impact on these prices.

Proof of Result 9d.

$$\max_{x_d} \pi_{dms} = (u + \alpha x_t)x_d - \frac{1}{2}x_d^2 - \hat{p_d}x_d$$

The first order condition yields the optimal quantity:

$$\hat{x_d} = u - \hat{p_d} + \alpha x_t = u - \hat{p_d} + \frac{\alpha(u - c_t)}{2}$$

The optimal price for advertising mail set by the entrant becomes:

$$p_d^* = \frac{u + \hat{c_d}}{2} + \frac{\alpha(u - c_t)}{4}$$

The closed form quantity chosen by the advertising mail sender is therefore:

$$x_d^* = \frac{(u - \hat{c_d})}{2} + \frac{\alpha(u - c_t)}{4}$$

The closed form profit for the entrant postal operator is:

$$\hat{\pi} = \left(\frac{u - \hat{c_d}}{2} + \frac{\alpha(u - c_t)}{4}\right)^2$$