Working Paper in Economics

202302

Intangible Cycles

Shalini Mitra
Gareth Liu-Evans

Intangible Cycles[*]

Shalini Mitra[†]
Gareth Liu-Evans[‡]

Current version: May 22, 2023
First version: January 8, 2021

Abstract

What is the role of an intangible investment technology shock in driving and propagating business cycles? We show that the ability of entrepreneurs to reallocate physical investment across (final goods and intangible investment) sectors within the firm, in response to such a shock, is key to quantitatively generating volatility and comovement among macroeconomic aggregates at business cycle frequencies. Such a channel is consistent with observed within-firm procyclical investment dispersion in the data and does not arise in the case of household ownership of intangible-capital-accumulating firms. Moreover, the unmeasured nature of intangible investments and the hump-shaped response of final goods implies that there is an almost zero correlation of the positive intangible investment shock to measured aggregate total factor productivity.

Keywords: Intangible investment shock, reallocation, intangible capital, business cycles, comovement, investment dispersion
JEL Classifications: E13, E22, E32, O33

---

[*] We would like to thank Oliver de Groot, Ricardo Reis and internal seminar participants at the University of Liverpool Management School.

[†] University of Liverpool

[‡] University of Liverpool Management School, Liverpool, L69 7ZH, email:shalini.mitra@liv.ac.uk

1 Introduction

Reallocation of investment spending is one of the main ways a firm mitigates the effects of frictions or constraints, whether internal or external, and the majority (68%) of firm level investment dispersion in a typical industry occurs within rather than across firms, according to Kehrig and Vincent (2019) (see also Bachmann and Bayer (2014)). In another line of research, Peters and Taylor (2017) (also Ewens et al. (2019)) find that a firm's intangible assets are largely internally generated and therefore not recorded as investment but rather expensed by the firm. Specifically, the mean (median) firm in their data, which includes the universe of Compustat firms, (barring regulated utilities and financial firms) purchases only 19% (3%) of its intangible capital externally, meaning the vast majority of a firms' intangible assets are unmeasured, or missing from their balance sheets. In this paper we bring these two facts together in an otherwise standard macroeconomic model, and show that an intangible investment technology shock alone may drive and propagate business cycles, when physical investment reallocation within the firm is the key channel of spending adjustment.

We assume firms are owned by borrowing constrained entrepreneurs and the intangible investment producing technology is internal to the firm, utilizing firm resources such as labor, tangible and intangible capital. An intangible investment shock raises the marginal productivity of capital (MPK) and labor (MPL) in intangibles (short for intangible investment) relative to final goods causing hours and physical investment to rise in this sector. An increase in physical capital investment in intangibles, raises the future productivity of final goods by relaxing the future intangible capital (IC) constraint of the firm. This generates a wealth effect, similar to that induced by news shocks, whereby households increase their consumption of final goods and *leisure* in anticipation of higher future output. Since hours rise in intangibles in response to the shock, the drop in aggregate hours comes mainly from a reduction in employment in final goods as the shock hits. This causes final goods to fall as well upon impact. The eventual increase in the firm's IC stock increases MPK and MPL in final goods relative to intangibles, in the shock's aftermath, causing employment and output in final goods to surge, driving up aggregate employment, aggregate consumption and aggregate investment with aggregate output.

Since an intangible investment shock does not raise aggregate output upon impact, any increase in physical investment in intangibles must be financed by either drawing down aggregate consumption, or by reallocating investment spending between sectors initially. As long as entrepreneurs do not exhibit too little risk aversion (i.e. in case of risk neutrality), reallocation of investment spending from final goods to intangibles is the key channel of spending adjustment in response to the shock. This causes aggregate investment to not change much upon impact, and it only increases over time with final goods output as explained earlier.

Note that one unit of physical investment in intangibles, produces final goods for consumption with a greater delay and higher uncertainty (due to the intangible investment shock) than the same unit invested in final goods (which is not subject to shocks). Thus less (more) risk averse entrepreneurs increase physical investment in intangibles more (less) in response to the shock. Moreover, given constant relative risk aversion (or CRRA) utility of entrepreneurs, a lower entrepreneurial risk aversion is associated with a greater tolerance for more intertemporal consumption substitution. Consequently, when entrepreneurs are too little risk averse, as in the case of risk neutrality, physical investment rises in both intangibles and final goods (in anticipation of the higher IC stock next period), causing aggregate investment to rise at the cost of aggregate consumption which is driven down by a large drop in entrepreneurial profits. There is no physical capital reallocation across sectors and the model generates negative aggregate comovement between consumption and investment in this case.

Reallocation of investment spending as a channel is also weak or absent when entrepreneurs are 'too' risk averse. In this case, physical investment in final goods registers a large drop even though investment rises little in intangibles, as too risk averse entrepreneurs reduce aggregate investment spending in order to stem the fall in profits due to higher wage bills. Note that the wage bill rises even though aggregate hours fall upon the shock's impact due to a large increase in labor's marginal product. Household consumption increases as a result, largely offsetting the fall in entrepreneurial profits causing aggregate consumption to change little upon impact. However, when entrepreneurial risk aversion is too high, as in the case of log utility - similar to households in the model - profits do not fall enough given the strong preference of entrepreneurs for lower intertemporal substitution in consumption which causes them to cut aggregate investment spending. Aggregate consumption thus rises, driven by the increase in household consumption, as aggregate investment falls, generating a negative consumption-investment comovement once again.

The wealth effect and the investment reallocation effect are both key to generating business cycle fluctuations and comovement in response to an intangible investment shock. The wealth effect ensures comovement in aggregate consumption and hours and arises when entrepreneurs, and not households, own the firms. We show that while borrowing constraints of entrepreneurs increase the range of entrepreneurial risk aversion for which comovement arises, it is not the main factor generating comovement in the model. The investment reallocation effect, on the other hand, ensures that consumption and investment comove in the aggregate, and arises when entrepreneurial risk aversion is neither too high (log-utility) nor too low (risk neutrality).

The model is consistent with procyclical within-firm investment dispersion as in the data (Kehrig and Vincent (2019), Bachmann and Bayer (2014)). Dispersion is simply the absolute value of the difference between the two sector's physical investment. Steady state physical investment spending in intangibles is only a fraction of physical investment in final goods. While the shock raises physical investment in intangibles relative to final goods immediately, it is the adjustment back to steady state, of physical investment in both sectors after the initial impact of the shock, that causes firm-level investment dispersion to be strongly procyclical in the model.

Lastly, the benchmark model generates a correlation of -0.11, between measured aggregate TFP and the intangible investment shock. This almost zero correlation arises because of the hump-shaped response of final goods to a positive, one-time shock to intangibles. Angeletos et al. (2020) motivate the search for a "main business cylce shock" that causes strong co-movements among macro variables but is disconnected from TFP. They rule out aggregate productivity and investment specific shocks as candidate business cycle drivers as a result. We show that an intangible investment specific technology shock alone *can* generate cycles while appearing disconnected to measured aggregate TFP. The unmeasured nature of intangibles implies, the immediate rise in intangible investment in response to the shock is not recorded, while measured output of final goods only rises over time as a larger IC stock becomes available. Thus measured TFP rises over time before peaking and returning to steady state, which generates a near-zero correlation of the positive intangible investment shock with measured aggregate TFP.

2 Literature

Our paper contributes to two main strands of literature. One relates to the question of what causes business cycles. The other is the relatively new, but fast advancing literature on the role of intangible capital in the economy.

An intangible investment technology shock, is related to but distinct from both aggregate technology shocks and investment specific technology (IST) or marginal efficiency of investment (MEI) shocks, both widely studied in the literature (see Mitra (2019) for a discussion on the nature of the difference between the shocks). The seminal paper of Justiniano et al. (2010) find that investment specific shocks explain a large chunk of the cycle, excluding consumption. However, Christiano et al. (2014) show that once they add a financial accelerator to this setup and estimate it with financial series, the influence of the investment shock nearly vanishes. Instead, shocks to the dispersion of firms' productivity, or risk shocks, become the main driver of the economy. The intangible investment shock in this paper produces comovement in the key macroeconomic aggregates, including consumption, as long as entrepreneurial risk aversion is neither too high nor too low. While the shock generates procyclical within-firm investment dispersion in this paper, we show in a related paper (Liu-Evans and Mitra (2022)) that the intangible investment shock is also associated with countercylical dispersion in firm level TFP, output and employment, in addition to generating procyclical investment dispersion.

McGrattan (2020) incorporates intangible investments into a multi-sector general equilibrium model and uses data from an updated U.S. input-output table to parameterize income and cost shares, with intangible investments reassigned from intermediate to final uses. They however, focus on shocks to industry and aggregate TFP. Employing maximum-likelihood methods and quarterly observations on sectoral gross outputs for the U.S. to estimate processes for latent sectoral TFPs that have common and sector-specific components she finds that sector-specific shocks and industry linkages play an important role in accounting for fluctuations and comovements in aggregate and industry-level U.S. data.

Chahrour et al. (2021) suggest that production networks do not account for all of the observed cyclical variation in output. In their model sectoral news coverage is the main driver of business cycle fluctuations. Their main premise is that - firms relying on media for news on sectoral or economic developments, hire too little or too much relative to actual changes in output since such news, although accurate is unrepresentative of the whole economy. This creates the appearance of a shock that is orthogonal to aggregate total factor productivity (TFP), even in the absence of any non-TFP shocks, in keeping with Angeletos et al. (2020). We show, similar to Chahrour et al. (2021), that business cycles are generated in our model by what is essentially a technology shock to the intangible investment producing sector, although the shock appears largely uncorrelated to aggregate measured TFP due to the unmeasured nature of intangibles.

vom Lehn and Winberry (2021) focus on the role of investment network and show that sector-specific shocks to a few key investment hubs and their key suppliers account for an increasing share of aggregate fluctuations, generating declines in cyclicality of labor productivity and other business cycle changes since the 1980s. Mitra (2019), focusing on the same period, also explains jointly the rising volatility of hours and wages and the declining cyclicality of labor productivity during a period of declining output volatility. She shows that these changes can be attributed to the growing importance of shocks to the intangible investment producing sector, relative to aggregate TFP shocks, as intangibles become more important in production.

The intangible investment shock in this paper is similar to the one in Mitra (2019), however, unlike that paper we do not rely on an aggregate TFP shock to generate business cycles. Nor does the model feature any other technology or non-technology shocks. To our knowledge, the role of an intangible investment shock in driving business cycle fluctuations and aggregate comovement has not been studied in the literature. We highlight the conditions in this paper, under which aggregate joint dynamics arise quantitatively in response to the shock, and the main mechanism behind the results.

In this vein, our paper contributes to the literature on the rising importance of intangible capital in the economy. Seminal works in this area include McGrattan and Prescott (2010) and Prescott and McGrattan (2012) who show that incorporating intangible capital into the standard neoclassical framework can help explain the loss of procyciclicality in labor productivity as well as the puzzling boom of the 1990's. More recent work by Crouzet and Eberly (2021) and Crouzet et al. (2022) have more generally emphasized the role of intangible capital in driving key macroeconomic trends since the 1990's, such as the relatively low TFP growth, the declining labor share, weak tangible investment, rising valuations of firms, increasing market concentration and rising inequality.

The rest of the paper proceeds as follows. Section 3 sets out the model, Section 4 discusses the calibration strategy, in Section 5 we present the quantitative results of the benchmark model, the model with non-binding financial constraints and the model with households as firm owners. Section 6 concludes.

3 Model

The economy is composed of firms, owned by borrowing constrained entrepreneurs, and households who lend to entrepreneurs and supply labor to the firms. Firms combine labor, physical and intangible capital to produce final goods and spend additional resources (in the form of labor, physical and intangible capital), to produce intangible investment. Thus firms accumulate intangible capital in addition to tangible (or physical) capital.

A **representative entrepreneur** maximizes profits, which is their deferred consumption, every period,

$$\text{Max} \sum_{t=0}^{\infty} \beta_e^t \frac{c_{e,t}^{1-\iota_e}}{1-\iota_e}, \tag{1}$$

where β_e is the discount factor of the entrepreneur while ι_e captures their risk aversion. c_e is entrepreneurial consumption or profits.

Entrepreneurs are subject to the following flow of funds constraint,

$$c_{e,t} + w_t l_t + x_{k,t} + R_t b_{t-1} = y_t + b_t, \tag{2}$$

where total spending on consumption, wage bill, physical capital investment and loan repayments on the left is financed by the production of final goods plus new loans on the right.

The production function is given by:

$$y_t = A_y \left(k_{y,t}^{\alpha} z_t^{\gamma} l_{y,t}^{1-\alpha-\gamma} \right)^{\mu}, \tag{3}$$

where A_y is a parameter measuring the productivity of final goods. k_y, l_y and z are firm level inputs of capital, labor and intangible capital respectively, while $\alpha\mu$ and $\gamma\mu$ are the firm level elasticities of tangible and intangible capital input in production. Note that unlike physical capital and labor, IC is not sector-specific within the firm. This is because, the firm's entire IC stock is available for both final goods production and IC investments by the firm. In other words, knowledge can be used to produce both automobiles and ideas. This feature of IC is well documented in the literature and commonly described as the scaleability of IC (McGrattan and Prescott (2010); Haskel and Westlake (2018); Crouzet et al. (2022)).

Intangible capital accumulates according to,

$$z_{t+1} = (1 - \delta_z) z_t + x_{z,t}, \tag{4}$$

where δ_z is the IC depreciation rate and x_z is new investment in IC.

x_z requires labor, tangible and intangible capital and is produced according to the following production function:

$$x_{z,t} = A_z T_t \left(k_{z,t}^{\alpha} z_t^{\gamma} l_{z,t}^{1-\alpha-\gamma} \right), \tag{5}$$

where A_z is a parameter measuring the productivity of the IC investment producing technology and T_t is a productivity shock affecting this sector, which follows an $AR(1)$

process,

$$\log T_t = \rho \log T_{t-1} + \epsilon_t, \tag{6}$$

with $\epsilon_t \sim [0, \sigma^2]$.

Tangible capital accumulates according to,

$$k_{t+1} = (1 - \delta_k) k_t + x_{k,t} - s\left(\frac{x_{k,t}}{k_t}\right) k_t. \tag{7}$$

δ_k is the depreciation rate of physical capital and $s(.)$ is a capital adjustment cost function defined as in Hayashi (1985):

$$s\left(\frac{x_{k,t}}{k_t}\right) = \frac{\phi}{2}\left(\frac{x_{k,t}}{k_t} - \delta_k\right)^2. \tag{8}$$

The main role of equation 8 is to ensure the benchmark model does not generate too much physical investment volatility relative to the data.

Following Buera and Moll (2015), entrepreneurs face the following borrowing constraint,

$$b_t \leq \theta k_t, \tag{9}$$

which implies, they can borrow at most a fraction θ of their physical capital stock, due to limited commitment for example (among other underlying frictions).

Households supply labour and lend to entrepreneurs every period. They solve the following problem,

$$\underset{c_{h,t}, l_t, b_t}{\text{Max}} \quad E_t \sum_t \beta^t \left(\frac{c_{h,t}^{1-\iota_h}}{1-\iota_h} - \frac{\psi l_t^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}}\right), \tag{10}$$

subject to,

$$c_{h,t} + b_t = w_t l_t + R_t b_{t-1}, \tag{11}$$

where β is the household discount factor, c_h is household consumption and l is total hours worked. ι_h denotes the household's risk aversion, while η is the elasticity of labor supply. From their budget constraint given by equation (10), households equate their wage income $w_t l_t$ and interest payments on last period's loans $R_t b_{t-1}$, on the right, to consumption $c_{h,t}$ and new loans b_t on the left, every period.

3.1 First order conditions

The firms' first order conditions with respect to physical capital in final goods and intangibles respectively are:

$$E_t\Big[\beta_e u'(c_{e,t+1})\left(1-\delta_k - s\left(\frac{x_{ky,t+1}}{k_{y,t+1}}\right) + s'\left(\frac{x_{ky,t+1}}{k_{y,t+1}}\right)\frac{x_{ky,t+1}}{k_{y,t+1}} + \alpha\mu\frac{y_{t+1}}{k_{y,t+1}}\right) + \lambda_{t+1}\theta\Big] \\ = u'(c_{e,t})\left(1 + s'\left(\frac{x_{ky,t}}{k_{y,t}}\right)\right), \tag{12}$$

and,

$$E_t\Big[\beta_e u'(c_{e,t+1})\left(1-\delta_k - s\left(\frac{x_{kz,t+1}}{k_{z,t+1}}\right) + s'\left(\frac{x_{kz,t+1}}{k_{z,t+1}}\right)\frac{x_{kz,t+1}}{k_{z,t+1}}\right) + \zeta_{t+1}\alpha\frac{x_{z,t+1}}{k_{z,t+1}} + \lambda_{t+1}\theta\Big] \\ = u'(c_{e,t})\left(1 + s'\left(\frac{x_{kz,t}}{k_{z,t}}\right)\right) \tag{13}$$

where λ_t and ζ_t are the Lagrange multipliers associated with the borrowing constraint and the IC accumulation equation respectively. ζ_t can be thought of as the "shadow" value of the firm's IC constraint, given by equation 4. Both equations 12 and 13 equate the marginal cost of acquiring an additional unit of physical capital on the left to its marginal benefit on the right. Note that the marginal cost is the same in both cases, a unit of current consumption which is given up plus the associated adjustment cost of the added unit of investment, $s'(.)$. Also same in both equations is the financial constraint term $\lambda_{t+1}\theta$ on the left, denoting that an additional unit of capital accumulated, irrespective of the sector, helps relax the financial constraint going forward.

In equation 12 the marginal benefit of an additional unit of $k_{y,t+1}$ on the left is composed of the discounted marginal product of physical capital, the value to the firm of undepreciated future capital and the contribution of the new unit of capital to the marginal decline in installation costs in the future.

In equation 13, the marginal product term on the left is weighted by ζ_{t+1}, the future shadow value of the IC constraint. That is, the contribution to marginal revenue generated from an additional unit of $k_{z,t+1}$ depends on the expected value to the firm of its future IC constraint. This highlights the key difference between physical capital investment in the two sectors. The marginal product of new physical capital investment in final goods is realized the period after in the form of new final goods for consumption, while the marginal product of additional physical capital investment in intangibles relaxes the future IC constraint of the firm and is therefore weighted by the value to the firm next period (in utility terms) of a change in this constraint. The rest of the left hand side terms in equation 13, have analogous interpretations to the corresponding terms in equation 12 above.

The first order condition with respect to next period's IC is given by,

$$E_t\left[\beta_e u'(c_{e,t+1})\left(\gamma\mu\frac{y_{t+1}}{z_{t+1}}\right)+\zeta_{t+1}\left(\gamma\tfrac{x_{z,t+1}}{z_{t+1}}+(1-\delta_z)\right)\right]=\zeta_t. \tag{14}$$

Equation 14 states that the current period shadow value of the IC constraint, ζ_t, on the right, equals the expected value of the marginal benefit from having an extra unit of z_{t+1}, on the left, which in turn has two components. First is the contribution of the additional unit of IC to an increase in future output of final goods by the amount of its marginal productivity in this sector. This is multiplied by the discounted marginal utility of a change in future consumption. Second is the contribution of the additional IC to an increase in new intangible investment by the amount of its marginal product in the intangibles sector, multiplied by ζ_{t+1} - the value in utility terms to the firm of a change in the IC constraint, along with the un-depreciated amount of IC.

Optimality conditions for labor in final goods,

$$\mu\left(1-\alpha-\gamma\right)\frac{y_t}{l_{y,t}} = w_t, \tag{15}$$

and intangibles,

$$\zeta_t\left(1-\alpha-\gamma\right)\frac{x_{z,t}}{l_{z,t}} = w_t u'(c_{e,t}). \tag{16}$$

The firm equates the marginal cost of employing an additional unit of labor, on the right to its marginal benefit on the left, in equations 15 and 16. The marginal benefit of an additional unit of labor in final goods is simply the marginal product of labor in this sector while its marginal cost is the real wage. The marginal benefit of an additional unit of labor in intangibles, however, internalizes the effect of this unit of labor on the shadow value of the IC constraint to the firm, ζ_t. The entrepreneur sacrifices current profits to hire additional labor in intangibles which does not generate additional final goods during the period, but relaxes the IC constraint, from which the entrepreneur derives value (in utility terms).

The first order conditions with respect to entrepreneurial and household borrowing are:

$$E_t\left[\lambda_{t+1} + R_t\left(\beta_{e,t+1} u'(c_{e,t+1})\right)\right] = u'(c_{e,t}) \tag{17}$$

$$\beta u'(c_{h,t+1}) R_t = u'(c_{h,t}), \tag{18}$$

where equation 18 is the standard Euler equation of households. Finally, the labor supply decision of the household is given by,

$$u'(c_{h,t}) w_t = \psi l_t^{\frac{1}{\eta}}.$$

3.2 Definition of equilibrium

An equilibrium in this economy is defined as a sequence of wages, $\{w_t\}_{t=0}^{\infty}$, and interest rates $\{R_t\}_{t=0}^{\infty}$ and corresponding labor inputs in the two sectors $\{l_{y,t}, l_{z,t}\}_{t=0}^{\infty}$ and loans $\{b_t\}_{t=0}^{\infty}$ such that (i) firms maximize profits subject to Eqs.(2) –(8) and households maximize utility subject to Eq.(10) taking as given the exogenous and endogenous states $\{T_t\}$, $\{k_{y,t}, k_{z,t}, z_t\}$ and the price sequences $\{w_t\}_{t=0}^{\infty}$ and $\{R_t\}_{t=0}^{\infty}$, and (ii) capital, labor, goods and bonds markets clear:

$$k_{y,t} + k_{z,t} = k_t \tag{19}$$

$$l_{y,t} + l_{z,t} = l_t \tag{20}$$

$$c_t + x_{k,t} = y_t, \tag{21}$$

where $c_t = c_{e,t} + c_{h,t}$ and $x_{k,t} = x_{ky,t} + x_{kz,t}$

3.3 Sectoral physical capital share

From the Euler equation of households (equation 18) at steady state, $R = 1/\beta$. Substituting R into the entrepreneur's first order condition for borrowing in equation 17, gives $\lambda = \beta - \beta_e$. This standard condition for the Lagrange multiplier associated with the borrowing constraint simply implies that the constraint is binding when $\beta \geq \beta_e$ and not when $\beta = \beta_e$. While we assume binding financial constraints in the benchmark model, the results for which are presented in Section 5.1, we present and discuss the case with non-binding financial constraints in Section 5.2.

From equation (4) at steady state, $x_z = \delta_z z$ which, plugged into equation (11) gives ζ, the Lagrange multiplier associated with intangible capital accumulation. Once we have both ζ and λ, they can be substituted into the entrepreneur's first order conditions with respect to physical capital in the two sectors - equations 12 and 13 - can be used to arrive at the sectoral physical capital ratio:

$$\frac{k_z}{k_y} = \frac{\gamma}{1-\gamma}. \tag{22}$$

The ratio depends only on γ - the income share of intangible capital in aggregate output and is less than 1. The literature estimates a value for γ around 0.1. For instance, Corrado et al. (2009) find that including intangible assets lowers the labor share of income from about 70% to 60% in the U.S. data, and raises capital's share from about 30% to 40%. Thus the ratio of physical capital (investment) in intangibles to final goods in steady state, from equation 22, is a small one.

An intangible investment shock, which raises physical investment in intangibles causes investment dispersion to fall as the shock hits. However, this is followed by steadily rising physical investment in final goods in the model, while physical investment in intangibles returns to steady state, for several periods, in the aftermath of the shock. This ensures investment dispersion rises with final goods over time in the model.

4 Calibration

We calibrate the model to the US economy at an annual frequency. Table 1 presents the calibrated parameters. The household discount factor is set equal to 0.96 reflecting a quarterly interest rate target of 1%. The depreciation rate of physical capital, δ_k, is set equal to its standard annual value of 0.1. The depreciation rate of intangible capital has been the source of much debate in the literature with the main difficulty being, little knowledge about, and the widely varying rates of, intangible capital depreciation depending on its type and industry. Corrado et al. (2009) for instance use an annual rate of 33% for computerized information, 15% for R&D, 60% for advertising and 40% percent for firm specific resources. Furthermore, when focusing on organizational capital, Ewens et al. (2019) find huge cross-sectional variation in its depreciation rate across industries, varying from 19% in consumer industries to 49% in healthcare. Given the literature, we assume a benchmark annual depreciation rate for intangible capital of $\delta_z = 0.4$ and present results for lower and higher values of δ_z in Section 5.4. We show that the model's results are not very sensitive to the depreciation rate of IC.

Following Corrado et al. (2009), we set the income share of physical capital α to 0.3 and the income share of intangible capital to $\gamma = 0.1$, in order that the total share of capital in income is 40% of aggregate returns with labor garnering the remaining 60%. While intangibles have a constant returns to scale technology, final goods in the model exhibit decreasing returns with the returns to scale parameter, μ, set to a standard value of 0.85.

Both households and entrepreneurs have CRRA utility. While household's risk aversion parameter, ι_h, is set equal to 1, such that the household displays log-utility throughout, we allow the value of the risk aversion parameter of the entrepreneurs, ι_e, to vary from 0 - the case of risk neutrality, to 0.75 in the next section. The Frisch elasticity of labor supply is set to $\eta = 5$ following Aoki and Nikolov (2015) and is within the range used in macroeconomic studies (see for instance Gertler and Kiyotaki (2010)). A lower η does not alter the model's results but generates between 5-10% lower employment volatility. Given η, the dis-utility parameter for labor supply, ψ, is chosen to target a total steady state hours worked of 0.35.

Using the productivity of the final goods sector A_y as the numeraire, we use the productivity of intangible investment A_z, to target the average firm-level intangible intensity in the data, defined as the share of intangible capital in the total capital ($tangible + intangible$) stock. We use the estimates of Peters and Taylor (2017) for IC intensity of Compustat firms (barring regulated utilities, financial firms and firms categorized as public service, international affairs, or non-operating establishments) between 1975-2011. Their IC estimates include balance sheet intangibles - those that are externally acquired (purchased) by firms and non-balance sheet or internally generated intangibles such as knowledge and organization capital. Their sample has a mean intangible intensity of 43% in a typical firm-year. However, an average of 19% of intangibles is acquired externally by their mean firm. Given our focus on internally generated intangibles, we use the productivity parameter for IC investments, A_z to target a slightly lower IC intensity (of 37%) in the benchmark model, than that obtained by the authors. However, higher IC intensities do not alter the main results of the model.

To allow for binding financial frictions, the discount factor of the entrepreneur is set at a value lower than the household's, such that $\beta_f = 0.90$. We alternatively set $\beta_f = \beta = 0.96$ in Section 5.2 to study the effect of non-binding financial frictions. Given β_f, the financial constraint parameter, θ is chosen to target an average non-financial business debt-to-income ratio of 0.76 between 1990-2020 from the Financial Accounts of the United States - Z.1 (obtainable at: $https : //www.federalreserve.gov/releases/z1/dataviz/z1/nonfinancial_debt/char$ $growth; series : business$).

The standard deviation of innovations to the IC shock, σ_t, is set to 0.1 in the benchmark model, to generate the post-1980's output volatility in US data - an average standard deviation of output of 1.4. This was a period of markedly lower aggregate volatility in the US as documented by a large literature (see for instance, Stock and Watson (2002) and Galí and Gambetti (2009) among others). We assign the persistence parameter, ρ_t a value of 0.85 in keeping with a quarterly persistence rate of 0.96 used for productivity shocks in the literature.

Table 1: Parameter Values

Parameter	Value	Target
Household discount factor,β	0.96	Annual interest rate of 4%
Entrepreneur discount factor,β_f	0.90	Literature
Phys. Capital income share,α	0.30	Corrado et al. (2009)
IC income share of l-type,γ_l	0.1	Corrado et al. (2009)
Phys. Capital depreciation rate,δ_k	0.075	Literature
Phys. capital adj. cost param, κ	0.5	Phy. inv. vol. 3.5 times output vol.
Financial constraint parameter, θ	0.5	Non-financial business debt-to-income
IC depreciation rate,δ_z	0.4	Ewens et al. (2019) and Corrado et al. (2009)
Household risk aversion,ι_h	1	Log-utility
Entrepreneur risk aversion,ι_e	0.25	-
Frisch elasticity,η	5	Aoki and Nikolov (2015)
Labor disutility,ψ	3.5	Hours worked=0.35
Returns to scale parameter,μ	0.85	Literature
Final goods productivity,A_y	1	Numeraire
IC investment productivity,A_z	3	IC to total capital ratio of 37%
S.D. of shock,σ	0.1	Post-1980's US output volatility of 1.4
Persistence of shock,ρ	0.85	Equivalent to quarterly persistence of 0.96 for tech. shocks

5 Results

In this section we present the results of a one standard deviation shock to the intangible investment production technology and discuss the mechanisms behind the results. We first look at the benchmark case with binding financial constraints on the entrepreneurs who own the firms. Next we allow financial constraints to be non-binding, and finally, we present and discuss the case when households and not entrepreneurs own the firms. We preserve the macroeconomic targets of the previous section, wherever possible.

5.1 Effect of an intangible investment shock

Table 2 presents the main results of the model for varying degrees of entrepreneurial risk aversion ι_e, where $\iota_e \in [0, 0.1, 0.25, 0.5, 0.75]$. Focusing first on the benchmark case of $\iota_e = 0.25$, the model generates a volatility of consumption about 65% that of output, investment volatility, 3.5 times so, while aggregate hours is about 60% as volatile as output. While the relative volatilities of consumption and investment are largely in line with aggregate business cycle data, the model clearly generates too little volatility of hours in the benchmark case[1]. This last is a well known weakness of the standard real business cycle paradigm, and our model does not seem to improve upon it. Note however, that both hours and investment volatility rise with entrepreneurial risk aversion while consumption volatility falls. This is because more risk averse entrepreneurs adjust investment more in order to smooth profit (and hence consumption) volatility. Greater investment adjustment, in the model is associated with higher adjustment of hours as we explain below, leading to more volatile hours. This is also seen in the strengthening correlation of aggregate hours with aggregate investment as entrepreneurial risk aversion rises, in Table 2.

Table 2: Business cycle moments

	(1)	(2)	**(3)**	(4)	(5)
ι_e	0	0.1	**0.25**	0.5	0.75
$vol(y)$	1.38	1.39	**1.44**	1.49	1.52
$vol(x_k)$	4.19	3.17	**5.07**	7.94	9.71
$vol(c)$	1.85	1.38	**0.93**	0.58	0.49
$vol(l)$	0.28	0.55	**0.87**	1.18	1.36
$corr(x_k, y)$	-0.09	0.60	**0.92**	0.95	0.96
$corr(c, y)$	0.91	0.94	**0.92**	0.67	0.22
$corr(l, y)$	0.85	0.95	**0.95**	0.96	0.96
$corr(c, x_k)$	-0.49	0.3	**0.68**	0.41	-0.06
$corr(c, l)$	0.61	0.87	**0.8**	0.44	-0.05
$corr(l, x_k)$	0.33	0.63	**0.95**	0.99	1
$corr(x_k disp, Y)$	0.59	0.86	**0.91**	0.92	0.93
$corr(T, TFP)$	-0.2	-0.15	**-0.11**	-0.06	-0.03

Model-implied moments in response to a one standard deviation shock to the intangible investment. ι_e is risk aversion of entrepreneurs. All variables are as defined in the model. $x_{kdisp} = |x_{k,y} - x_{k,z}|$. All series are HP-filtered and expressed as percentage deviations from the HP-trend before computing the moments.

Barring the extreme cases of $\iota_e = 0$ and $\iota_e = 0.75$, the model successfully generates co-movement among the key macroeconomic aggregates of output, investment, consumption and hours, as seen in Table 2. The impulse responses in the first two rows of Figure 1 further highlight the hump-shaped nature of the responses of these aggregates to the

---

[1] From Table 1 of King and Rebelo (2000), the volatility relative of output, of aggregate consumption, investment and hours are 0.74, 2.93 and 0.99 respectively

shock. In what follows, we highlight the model's mechanisms for the benchmark case of $\iota_e = 0.25$, following which we discuss the important role of entrepreneurial risk aversion and investment reallocation in particular in driving business cyles.

The shock initially causes i) MPK_z to rise relative to MPK_y and ii) MPL_z to rise relative to MPL_y. The effect of i) in the model is to raise $x_{k,z}$ relative to $x_{k,y}$. From the first order condition of the entrepreneurs relative to sectoral capital we know that an increase in $x_{kz,t}$ works to relax the future IC constraint of the firm, allowing it to produce higher final goods further into the future than the same unit invested in final goods. This increase in $x_{k,z}$ relative to $x_{k,y}$ has a wealth effect in the model, which causes households to increase their consumption of final goods and *leisure*, in anticipation of higher future income. Thus the model generates a fall in aggregate hours upon impact of the shock. From ii) above (and from panel (h)) hours in intangibles rises, which means the fall in aggregate hours manifests mainly as a drop in hours worked in final goods, as seen in panel (i). This causes aggregate (final goods) output to fall upon impact and so does aggregate consumption and investment in the benchmark case.

After the initial impact, however, MPK_y and MPL_y rise relative to MPK_z and MPL_z, with the higher availability of IC, which raises employment and physical investment in final goods. This causes aggregate output and consumption to increase in the aftermath of the shock allowing the model to generate hump-shaped responses and comovement among the key macroeconomic aggregates. From Table 2, however, we see that comovement, particularly between aggregate consumption and investment is not achieved in the two extreme cases of ι_e we consider - that is, when entrepreneurs exhibit too little or too much risk aversion.

Risk aversion of entrepreneurs matter because, as mentioned above, one unit of physical investment in intangibles, produces final goods for consumption with a greater delay and higher uncertainty than the same unit invested in final goods. Thus less (more) risk averse entrepreneurs invest more (less) in intangibles. Moreover, given the entrepreneur's CRRA utility, lower (higher) risk aversion is also associated with higher (lower) tolerance of larger intertemporal substitution in consumption. Note, from Figure 1 that for the intermediary values of entrepreneurial risk aversion considered, an increase in $x_{k,z}$ is accompanied by a fall in $x_{k,y}$. In other words, there is a reallocation of investment spending from final goods to intangibles upon the shock's impact. However, when entrepreneurs are too little risk averse, as in the case of $\iota_e = 0$, there is no reallocation of investment spending in response to the shock as a large increase in $x_{k,z}$ is accompanied by an increase in $x_{k,y}$ as well (in anticipation of a higher stock of IC given its higher current productivity of IC investments). Risk neutral entrepreneurs care little about large intertemporal substitutions in consumption and choose to increase investment in both sectors at the cost of a large drop in profits. This latter causes aggregate consumption to fall significantly as aggregate investment jumps upon the impact of the shock in panels (b) and (c) for $\iota_e = 0$, generating a negative correlation (-0.49) between the two.

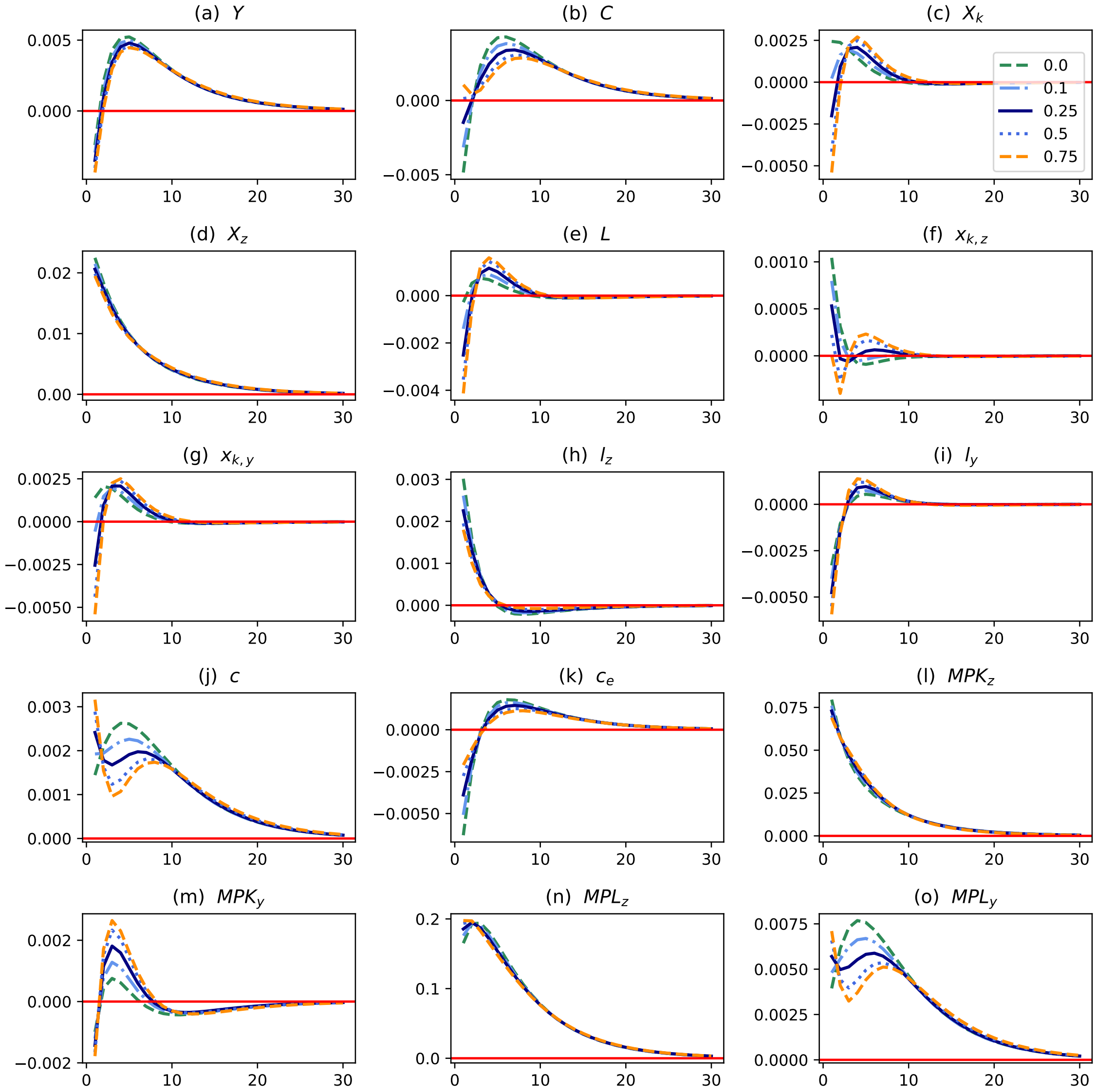

Figure 1: Impulse responses to a one standard deviation shock to intangible investment, for different values of ι_e. We denote aggregate output, aggregate consumption, aggregate hours and aggregate tangible and intangible investment, Y, C, L, X_k and X_z respectively. The rest of the variables are as defined in the paper. Responses are percent deviations from steady state.

When their risk aversion is too high, as in the case of $\iota_e = 0.75$, entrepreneurs lower total investment spending in order to stem the fall in profits (consumption) due to the rise in wage bill. The rise in MPL, mainly in intangibles, in response to the shock drives a large increase in the real wage as the shock hits, which pushes the wage bill up even as aggregate hours decline upon impact. This causes entrepreneurial profits to fall. When entrepreneurial risk aversion is high, such as at $\iota_e = 0.75$, physical investment in intangibles rise little, but investment in final goods falls significantly. Once again, in this case, there is no reallocation of physical investment spending across sectors, and the large drop in investment spending by entrepreneurs allows profits to decline little, causing the increase in household consumption to play a larger role in aggregate consumption's response to the shock. This once again negates the comovement between aggregate consumption and investment as seen in Table 2.

Thus reallocation of investment spending between intangibles and final goods within the firm, is key to generating aggregate comovement in response to an intangible investment shock. When this channel is absent or weak, as in the cases of too low or too high an entrepreneurial risk aversion, there is no aggregate comovement, particularly between consumption and investment.

5.1.1 Cyclicality of investment dispersion

Procyclicality of firm level investment dispersion is a key feature of business cycle data documented for instance by Bachmann and Bayer (2014) while Kehrig and Vincent (2019) shows that 68% of firm level investment dispersion in a typical industry occurs within rather than across firms. In our set-up, which only has a within-firm investment adjustment margin, investment dispersion is simply the absolute value of the range, of physical investment spending in the two sectors [2]. In other words, $x_{kdisp} = |x_{k,y} - x_{k,z}|$.

Given the calibrated value of $\gamma = 0.1$, the ratio of physical capital investment in intangibles to final goods from equation 22 is quite small in steady state. The increase in $x_{k,z}$ relative to $x_{k,y}$ due to the shock causes investment dispersion to fall upon impact. However, the increase in final goods in the aftermath of the shock causes physical investment in final goods to rise over time, alongside a decline in physical investment in intangibles, causing investment dispersion to also rise over time generating the strong procyclicality of investment dispersion in the model.

From Table 2, the benchmark model generates a correlation coefficient of 0.91 between x_{kdisp} and aggregate output. This is almost double the correlation of 0.45 reported by Bachmann and Bayer (2014) in the data. The much stronger procyclicality of investment dispersion arises in our model due to the dominance of the within-firm channel of investment dispersion. We show in Liu-Evans and Mitra (2022), that the correlation falls to the levels observed in the data when heterogenous firms are present. In other words,

---

[2] Alternatively we have also defined it as the absolute value of the difference in log of investment in the two sectors.

within-firm investment dispersion is weaker, although still dominant, in the presence of a between-firm reallocation channel of investment.

5.1.2 Total factor productivity and the intangible investment shock

The final row of Table 2 shows that in the benchmark case of the model, aggregate TFP has a correlation of -0.11 with the intangible investment shock T. The main reason for this near lack of correlation of TFP with what is essentially a TFP shock to intangible investments, is a lack of measurement of these investments in firm balance sheets. In other words, while the shock causes an immediate jump in intangible investment, such an increase is not recorded as part of measured output. Final goods which is measured on the other hand, only rises over time in response to the shock. As aggregate (final goods) output initially falls upon the shock's impact before rising above and eventually returning to, its steady state value, so does measured TFP.

Thus a positive intangible investment shock appears largely uncorrelated to aggregate TFP, due to the unmeasured nature of intangible investments. We further show in Section 5.4, that this correlation varies from mildly negative to mildly positive depending on the assumed depreciation rate of intangibles, while not altering the key results of the model.

5.2 Role of financial frictions

In this section we present the results of non-binding financial frictions. Table 3 presents the moments for the same set of entrepreneurial risk aversion ι_e as in Section 5.1, while Figure 2 presents the impulse responses.

Table 3: Business cycle moments: non-binding financial constraints

	(1)	(2)	**(3)**	(4)	(5)
ι_e	0	0.1	**0.25**	0.5	0.75
$vol(y)$	1.4	1.43	**1.48**	1.52	1.54
$vol(x_k)$	4.07	3.19	**5.27**	7.48	8.63
$vol(c)$	2.03	1.28	**0.75**	0.48	0.49
$vol(l)$	0.32	0.64	**0.96**	1.21	1.33
$corr(x_k, y)$	-0.06	0.76	**0.95**	0.97	0.97
$corr(c, y)$	0.88	0.93	**0.92**	0.36	-0.15
$corr(l, y)$	0.79	0.95	**0.96**	0.96	0.97
$corr(c, x_k)$	-0.52	0.46	**0.65**	0.12	-0.38
$corr(c, l)$	0.44	0.83	**0.7**	0.1	-0.04
$corr(l, x_k)$	0.49	0.82	**0.98**	0.99	1
$corr(x_k disp, y)$	0.56	0.9	**0.93**	0.93	0.94
$corr(T, TFP)$	-0.2	-0.16	**-0.12**	-0.08	-0.05

Model-implied moments in response to a one standard deviation shock to the intangible investment production technology. ι_e is risk aversion of entrepreneurs. All variables are as defined in the model. $x_{kdisp} = |x_{k,y} - x_{k,z}|$. All series are HP-filtered and expressed as percentage deviations from the HP-trend before computing the moments.

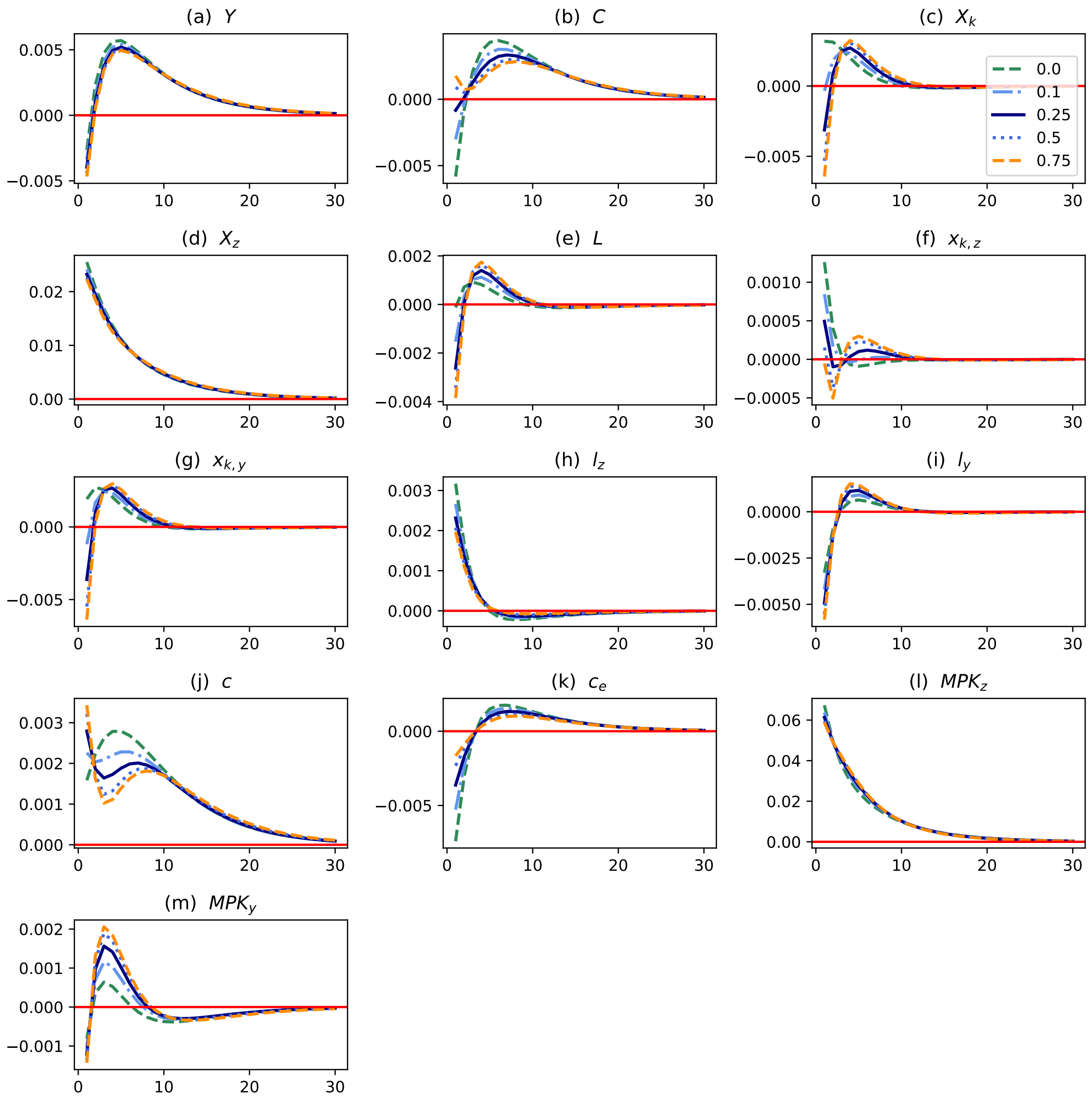

Figure 2: Figures show impulse responses to a one standard deviation shock to intangible investment when financial frictions are not binding. Results are for different values of entrepreneurial risk aversion, ι_e. The responses are percent deviations from steady state.

Note first of all that the dynamics of the model are largely similar whether or not financial frictions are binding. That is, depending on the entrepreneur's risk aversion, the intangible investment shock causes reallocation of investment spending and hours from final goods to intangible investment. Unless entrepreneurs are risk neutral, physical investment in final goods rises with a delay (just as more intangible investment becomes available as a result of the previous period's physical investment in intangible investments). As the stock of the firm's intangible capital in general and physical capital in final goods rises so does final goods production.

The strong focus on investment and hours reallocation as the shock hits causes aggregate investment and hours to move little on impact, with both rising smoothly over time driven by the rise in investment and hours in the final goods sector. Entrepreneurial profits fall while household consumption rises due to the increase in the wage bill due to the higher marginal product of labor, particularly in intangibles. Therefore, aggregate consumption, like other aggregates, changes little upon impact, rising over time with aggregate output. Thus the model generates the joint quantitative business cycle dynamics even when financial constraints are not binding.

In the absence of binding financial constraints however, risk averse entrepreneurs, who prefer a smoother profit (consumption) stream, can do so more effectively. Thus the correlation between aggregate consumption and investment starts to fall at a lower level of entrepreneurial risk aversion when financial frictions are not binding compared to when the are. At $\iota_e = 0.5$, greater consumption smoothing leads to a consumpion-investment correlation of 0.12 in this section compared to 0.41 in the previous section when frictions were binding. While $\iota_e = 0.75$ now generates a correlation of -0.38 between the two aggregates, compared to only -0.06 in the case with binding frictions.

Thus reallocation of investment spending between sectors, with or without binding financial frictions, is key to generating business cycle dynamics in the model. The instances when the reallocation channel is not the main mechanism behind the model's dynamics, that is when the entrepreneur is either too little or too much risk averse, are also the instances when the model fails to generate aggregate comovement particularly between aggregate consumption and investment. Reallocation of investment spending as a channel is absent when firms are owned by households and not entrepreneurs. We show next that such a model also fails to generate aggregate comovement, in this case between consumption and hours, in addition to generating too little volatility of consumption and investment.

5.3 Household ownership of firms

The model in this section is the same as the model in Section 3 with the only difference being, households own the firms and rent physical capital to them in addition to supplying labor. Firms allocate physical capital optimally between the two sectors of production - final goods and intangible investment.

Firm's problem:

$$\text{Max} \sum_{t=0}^{\infty} M^t d_t, \tag{23}$$

subject to,

$$\begin{aligned} d_t = y_t - w_t l_t - (r_t + \delta_k) K_t, && (24) \\ y_t = A_y \left(k_{y,t}^{\alpha} z_t^{\gamma} l_{y,t}^{1-\alpha-\gamma} \right), && (25) \\ z_{t+1} = (1-\delta_z) z_t + x_{z,t}, && (26) \\ x_{z,t} = s_t A_z \left(k_{z,t}^{\alpha} z_t^{\gamma} l_{z,t}^{1-\alpha-\gamma} \right), && (27) \\ \log T_t = \rho \log T_{t-1} + \epsilon_t, && (28) \end{aligned}$$

where $\epsilon_t \sim [0, \sigma^2]$.

First order conditions w.r.t physical capital in final goods and intangibles respectively are:

$$\begin{aligned} \alpha\mu \frac{y_t}{k_{y,t}} - (r_t + \delta_k) &= 0, && (29) \\ \zeta(\alpha \frac{x_{z,t}}{k_{z,t}}) - (r_t + \delta_k) &= 0, && (30) \\ & && (31) \end{aligned}$$

First order condition with respect to next period's IC and hours in the two sectors are,

$$\begin{aligned} \beta_{e,t+1} \left(\gamma\mu \frac{y_{t+1}}{z_{t+1}} \right) + \zeta_{t+1} \left(\gamma \tfrac{x_{z,t+1}}{z_{t+1}} + (1-\delta_z) \right) &= \zeta && (32) \\ \mu \left(1-\alpha-\gamma\right) \frac{y_t}{w_t} &= l_{y,t}, && (33) \\ \zeta_t \left(1-\alpha-\gamma\right) \frac{x_{z,t}}{w_t} &= l_{z,t}. && (34) \end{aligned}$$

Household's problem:

$$\underset{c_t, l_t}{\text{Max}} \quad E_t \sum_t \beta^t \left(\frac{c_t^{1-\iota_h}}{1-\iota_h} - \frac{\psi l_t^{1+\frac{1}{\eta}}}{1+\frac{1}{\eta}} \right), \tag{35}$$

subject to,

$$c_t = w_t l_t + (r_t + \delta_k) K_t + d_t. \tag{36}$$

The household's optimality conditions now become,

$$u'(c_t) w_t = \psi l_t^{\frac{1}{\eta}}, \tag{37}$$

$$\beta u'(c_{t+1})(r_{t+1} + 1) = u'(c_t). \tag{38}$$

The market clearing conditions are given by equations (19), (20) and (21) as before.

Households earn wages from labor and the rental rate r on physical capital. The rental rate equalizes the marginal product of capital across sectors. Physical capital demand in final goods plays a much larger direct role in the determination of r, compared to its demand in the production of intangible investments. Therefore, the rental rate reflects the dynamics of MPK_y more closely - rising over time before returning to steady state, although being larger than MPK_y due to the higher relative value of MPK_z in response to the shock.

We calibrate the model to target the same variables as in Section 5.1 where possible. The risk aversion parameter, ι, of households, who are the investors in this case, is allowed to take three values, such that, $\iota = [0.5, 0.75, 1]$. Figure 3 presents the impulse responses.

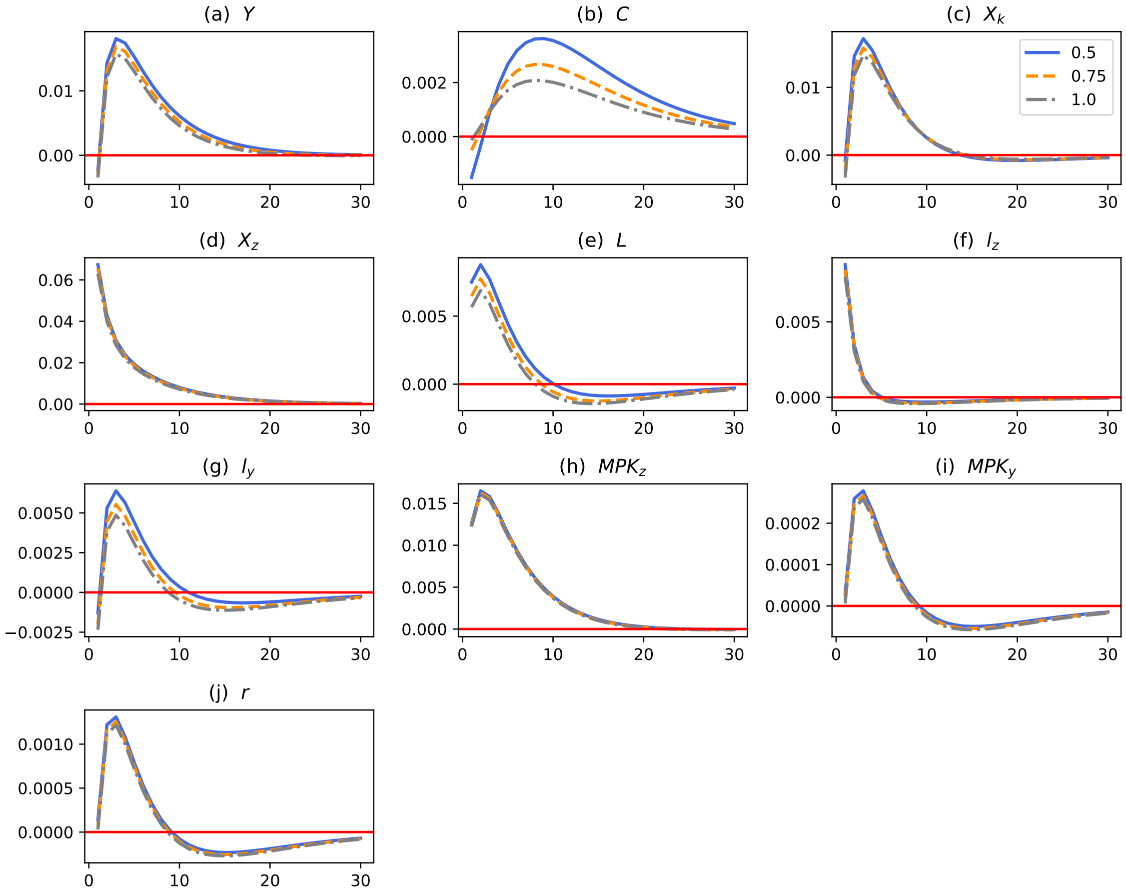

Figure 3: Impulse responses to a one standard deviation shock to intangible investment, for different values of ι, the household/investor risk aversion parameter. We denote aggregate output, aggregate consumption, aggregate hours and aggregate tangible and intangible investment, Y, C, L, X_k and X_z respectively. The rest of the variables are as defined in the paper. Responses are percent deviations from steady state.

The main difference when Figure 3 is compared to Figure 1 is that aggregate hours, L, jump upon impact of the shock. This is because instead of investment spending, firms reallocate physical capital between sectors as the shock hits. In other words, there is an immediate, one-time increase in physical capital in intangibles as firms reallocate capital across sectors in response to the shock. The rental rate, which equates the marginal product of capital across sectors, rises over time as demand for physical capital in final goods rise. Aggregate investment also rises over time in response to the rental rate. A one time increase in physical capital in intangibles, implies, hours jump in intangibles but does not fall in final goods, as households do not have an incentive to increase leisure or consumption in the current period. In other words the shock does not generate a wealth effect. Thus aggregate hours rise upon impact of the shock while aggregate consumption does not, generating a negative comovement between the two aggregates.

From Table 4, the model generates strong negative correlation between consumption and hours in the aggregate especially in the case where there are no capital adjustment costs (CACs). Table 4, further shows that the model's results improve with *strong* capital adjustment costs. However, notice that the case for CACs is ill justified in this case, given the model without CACs already generates too little volatility of investment (1.45 times that of output) while aggregate consumption volatility varies between 30-60% of aggregate output volatility. Needless to say, consumption and investment volatility worsens once CAC's are included. With a CAC parameter $\kappa = 2$, the correlation between aggregate consumption and hours improves especially for the non-zero values of ι, but not by much. The correlation rises from a low of 0.16 at $\iota = 0.5$, to a mild value of 0.27 at $\iota = 0.75$, before going back down to -0.27 at $\iota = 1$.

As expected the CACs slow down the response of aggregate investment, causing aggregate consumption to rise sooner. The slower response of investment implies, physical capital rises more slowly in final goods in the aftermath of the shock thus slowing down the rise in hours in this sector after the initial jump in hours in intangibles. Thus hours exhibit a little more of a hump shape after the initial jump while aggregate consumption rises sooner. Both effects culminate in a slightly improved correlation of aggregate hours and consumption in the presence of CACs in this set-up.

Table 4: Business cycle moments with households as firm-owners

	no CACs				with CACs			
ι	0	0.5	0.75	1	0.75	0	0.5	0.75
$vol(y)$	1.74	1.28	1.24	1.22	1.46	1.2	1.28	1.17
$vol(x_k)$	6.9	1.86	1.79	1.76	2.07	1.66	1.66	1.71
$vol(c)$	14.3	0.62	0.38	0.27	3.12	0.4	0.29	0.27
$vol(l)$	1.28	1.19	1.08	1.01	1.1	0.9	0.84	0.96
$corr(x_k, y)$	0.09	0.99	1	1	0.73	1	1	1
$corr(c, y)$	0.27	0.28	0.41	0.5	0.43	0.75	0.74	0.5
$corr(l, y)$	0.76	0.53	0.5	0.49	0.65	0.49	0.47	0.45
$corr(c, x_k)$	-0.93	0.12	0.32	0.44	-0.3	0.69	0.7	0.44
$corr(c, l)$	-0.37	-0.57	-0.41	-0.27	-0.37	0.16	0.27	-0.27
$corr(l, x_k)$	0.69	0.64	0.57	0.53	0.96	0.52	0.48	0.49

Model-implied moments in response to a one standard deviation shock to the intangible investment production technology. ι is the CRRA risk aversion parameter of households/investors. All series are HP-filtered and expressed as percentage deviations from the HP-trend before computing the moments.

5.4 Depreciation of intangibles

In this section, we allow the depreciation rate of intangibles to vary. Recall, from Section 4, that IC depreciation rates vary from 20-60% depending on the type of IC and for the same type of IC across industries. Brands, for example, have a depreciation rate of 60% according to Corrado et al. (2009), while the depreciation rate of organization capital varies dramatically across industries going from 19% in consumer industries to 49% in healthcare according to Ewens et al. (2019). They also find that knowledge capital (or R&D) goes from a low of 0.30 in "other" industries to a high of 0.46 in high-tech firms, much higher numbers than traditionally used for R&D in the literature. In this section we study the effect of these widely different depreciation rates on the model's results. We particularly allow the depreciation rate to vary from 0.3 to 0.6. The benchmark case of Section 5.1 is highlighted in bold.

Table 5: Business cycle moments: varying IC depreciation

δ_z	0.3	0.4	0.5	0.6
$vol(y)$	1.3	**1.44**	1.56	1.67
$vol(x_k)$	3.69	**5.07**	6.29	7.41
$vol(c)$	0.99	**0.93**	0.88	0.84
$vol(l)$	0.76	**0.87**	1.14	1.4
$corr(x_k, y)$	0.8	**0.92**	0.93	0.94
$corr(c, Y)$	0.95	**0.92**	0.88	0.84
$corr(l, y)$	0.95	**0.95**	0.94	0.94
$corr(c, x_k)$	0.7	**0.68**	0.65	0.61
$corr(c, l)$	0.84	**0.8**	0.75	0.69
$corr(l, x_k)$	0.95	**0.95**	0.95	0.95
$corr(x_k disp, y)$	0.9	**0.91**	0.92	0.93
$corr(T, TFP)$	-0.35	**-0.11**	0.05	0.18

Model-implied moments in response to a one standard deviation shock to the intangible investment production technology. IC is intangible capital; All series are HP-filtered and expressed as percentage deviations from the HP-trend before computing the moments.

From Table 5 we see that the model's results are largely unaffected by the rate of IC depreciation. The only difference, if any, is in the correlation of the shock with measured TFP in the final row. At a relatively lower depreciation rate of 0.3, the correlation is quite negative, falling to near zero and then becoming mildly positive as the depreciation rate rises to the highest values considered in the literature.

A higher rate of IC depreciation causes intangibles to take longer to accumulate. Recall that IC investments are risky not only because it is subject to productivity shocks unlike final goods, but also because a unit of physical capital invested in intangibles takes longer to produce final goods than the same unit invested in final goods itself, thus delaying consumption. For any given level of risk aversion of entrepreneurs, a higher depreciation rate of IC raises its 'riskiness' by delaying even further its accumulation and therefore the generation of final goods. This causes a smaller reallocation of hours and investment spending to intangibles as the shock hits. Intangible investment rises less with the shock while final goods rise and peak sooner after the shock's impact as seen in Figure 4. The smaller initial increase in intangible investment also causes final goods to fall more quickly back to steady state after the shock. The faster general rise and fall in output in response to the shock, contributes to the rising volatility of output with IC depreciation, in Table 5.

When IC displays lower rates of depreciation, the peak in TFP occurs later as entrepreneurs reallocate more spending (in investment and hours) to the unmeasured IC investment sector. Thus final goods fall more upon impact and rise more slowly over time contributing to a slightly more negative skew in the TFP response in Figure 4, and pushing the TFP-to-shock correlation towards to negative.

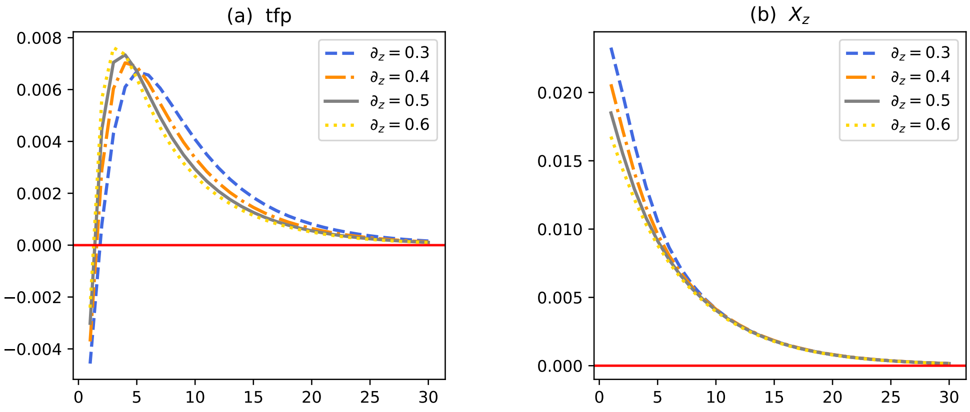

Figure 4: The figures show impulse responses to a one standard deviation shock to intangible investment. Results are for different values of entrepreneurial risk aversion, ι_e. The responses are percent deviations from steady state.

6 Conclusion

This paper highlights the channels through which an intangible investment technology shock may work to generate quantitatively important business cycles. The model with intangible capital successfully reproduces the joint dynamics of output, consumption, investment and employment when borrowing constrained entrepreneurs and not households own the physical capital. While financial constraints, extend the parameter space for which aggregate business cycle dynamics arise in the model, it is not the main force behind the model's results. What is key is the reallocation of investment spending, from final goods to investment within firms. Such a channel arises when entrepreneurs, and not households, act as firm owners, and entrepreneurs exhibit low to moderately high risk aversion.

The model generates strong within-firm procyclicality in investment dispersion as physical investment across sectors adjust back to their respective steady states in the aftermath of the shock. Finally, we show that, although a technology shock by definition, the intangible investment shock has an almost zero correlation with measured aggregate TFP. This is in keeping with Angeletos et al (2020) and arises in our model because intangible investments are unmeasured.

References

Angeletos, G.-M., F. Collard, and H. Dellas (2020): "Business-Cycle Anatomy," *American Economic Review*, 110, 3030–70.

Aoki, K. and K. Nikolov (2015): "Bubbles, banks and financial stability," *Journal of Monetary Economics*, 74, 33–51.

Bachmann, R. and C. Bayer (2014): "Investment Dispersion and the Business Cycle," *American Economic Review*, 104, 1392–1416.

Buera, F. J. and B. Moll (2015): "Aggregate Implications of a Credit Crunch: The Importance of Heterogeneity," *American Economic Journal: Macroeconomics*, 7, 1–42.

Chahrour, R., K. Nimark, and S. Pitschner (2021): "Sectoral Media Focus and Aggregate Fluctuations," *American Economic Review*, 111, 3872–3922.

Christiano, L. J., R. Motto, and M. Rostagno (2014): "Risk Shocks," *American Economic Review*, 104, 27–65.

Corrado, C., C. Hulten, and D. Sichel (2009): "INTANGIBLE CAPITAL AND U.S. ECONOMIC GROWTH," *Review of Income and Wealth*, 55, 661–685.

Crouzet, N. and J. Eberly (2021): "Intangibles, markups, and the measurement of productivity growth," *Journal of Monetary Economics*, 124, S92–S109, the Real Interest Rate and the MarginalProduct of Capital in the XXIst CenturyOctober 15-16, 2020.

Crouzet, N., J. C. Eberly, A. L. Eisfeldt, and D. Papanikolaou (2022): "The Economics of Intangible Capital," *Journal of Economic Perspectives*, 36, 29–52.

Ewens, M., R. H. Peters, and S. Wang (2019): "Measuring Intangible Capital with Market Prices," Working Paper 25960, National Bureau of Economic Research.

Galí, J. and L. Gambetti (2009): "On the Sources of the Great Moderation," *American Economic Journal: Macroeconomics*, 1, 26–57.

Gertler, M. and N. Kiyotaki (2010): "Chapter 11 - Financial Intermediation and Credit Policy in Business Cycle Analysis," Elsevier, vol. 3 of *Handbook of Monetary Economics*, 547–599.

Haskel, J. and S. Westlake (2018): *Capitalism without Capital: The Rise of the Intangible Economy*, Princeton University Press.

Hayashi, F. (1985): "Corporate finance side of the Q theory of investment," *Journal of Public Economics*, 27, 261–280.

Justiniano, A., G. E. Primiceri, and A. Tambalotti (2010): “Investment shocks and business cycles,” *Journal of Monetary Economics*, 57, 132–145.

Kehrig, M. and N. Vincent (2019): “Good Dispersion, Bad Dispersion,” Working Paper 25923, National Bureau of Economic Research.

McGrattan, E. R. (2020): “Intangible capital and measured productivity,” *Review of Economic Dynamics*, 37, S147–S166, the twenty-fifth anniversary of “Frontiers of Business Cycle Research“.

McGrattan, E. R. and E. C. Prescott (2010): ‘Unmeasured Investment and the Puzzling US Boom in the 1990s,’ *American Economic Journal: Macroeconomics*, 2, 88–123.

Mitra, S. (2019): ‘Intangible capital and the rise in wage and hours volatility,’ *Journal of Economic Dynamics and Control*, 100, 70–85.

Peters, R. H. and L. A. Taylor (2017): ‘Intangible capital and the investment-q relation,’ *Journal of Financial Economics*, 123, 251–272.

Prescott, E. and E. McGrattan (2012): ‘The Labor Productivity Puzzle,’ 2012 Meeting Papers 644, Society for Economic Dynamics.

Stock, J. H. and M. W. Watson (2002): ‘Has the Business Cycle Changed and Why?’ Working Paper 9127, National Bureau of Economic Research.

vom Lehn, C. and T. Winberry (2021): ‘The Investment Network, Sectoral Comovement, and the Changing U.S. Business Cycle*,’ *The Quarterly Journal of Economics*, 137, 387–433.