Displacing Random Sensors: Coverage and Interference Problems

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Outline

- Motivation
- Problem and Model
- Interference
- Coverage
- 2D

Motivation

What are Sensors?

- used in cameras and smartphones,. . .

- . . .in smart-watches, hubs, and for gas-detection

- . . .and it's only the beginning!

Sensors in Mobile Robots

- Robots equipped with sensors ...

What are Sensors Used for?

- ... to enable/enhance communicion.

“Edson Arantes do Nascimento, 1940”

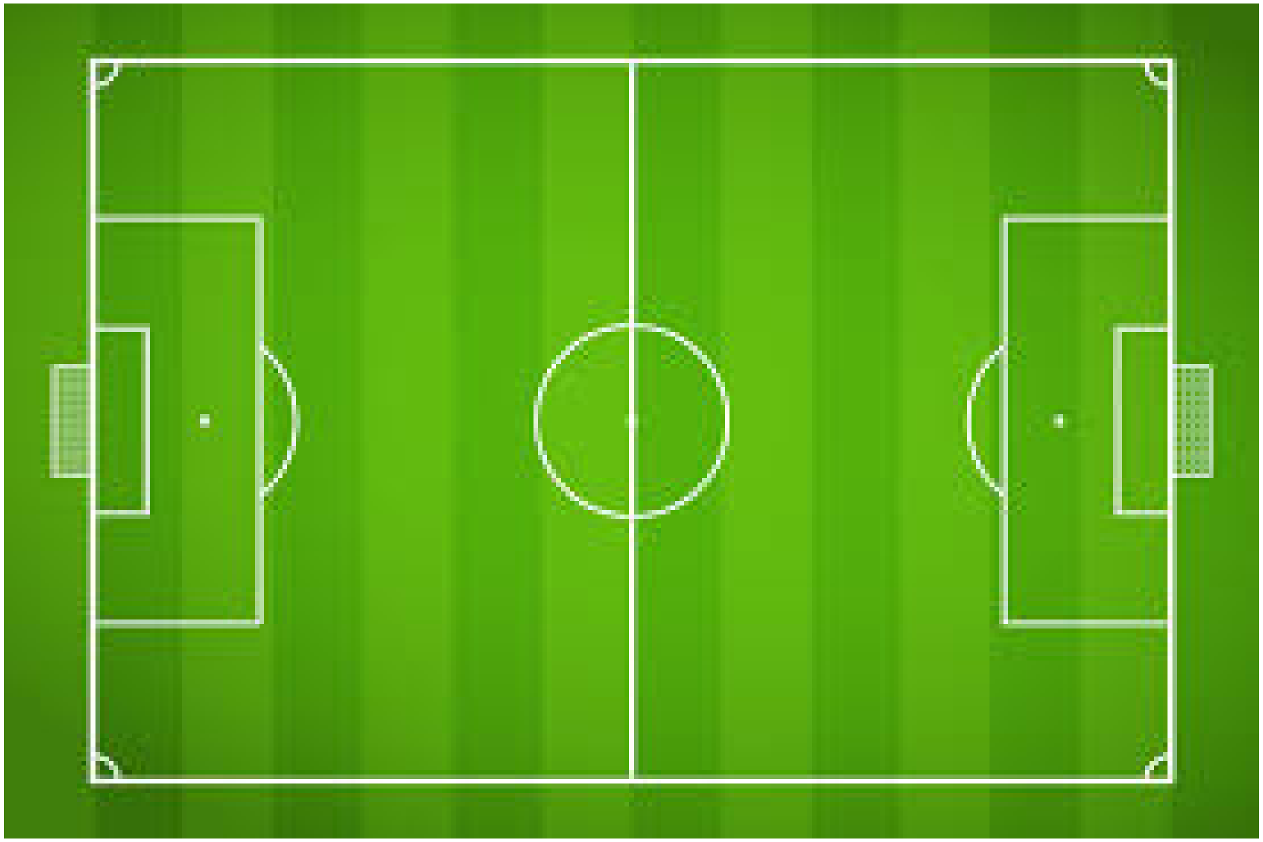
- From grapefruits. . .to rugs and socks...



- ...to modern technology!

The Most Important Questions in ihe Beautiful Game!

- **Did the Ball Cross the Line? Which Line? When?**



- The correct answer can cause goodwill or hostility?

The Beautiful ... ...

- Christmas Day 1914: soccer match between British and German troops. [a]
  - "A German looked over the trench—-no shots—-our men did the same, and then a few of our men went out and brought the dead in and buried them and the next thing a football kicked out of our trenches...and Germans and English played football."
- To be commemorated in 2014.

[a] British Mirror, December, 1914:

... and not so Beautiful Game!

- La guerra del fútbol (the Soccer War or 100 Hour War): brief war fought by El Salvador and Honduras in 1969.[a]
  - Began on 14 July 1969, (during a World Cup Qualifier) when the Salvadoran military launched an attack against Honduras: left thousands of civilians dead

[a]Wikipedia

On Measuring the Beautiful Game
"Why Soccer Matters", Pele, 2014

- Hang a ball with a rope on a tree[a]...
- ...and practice more ...

- ...to win!

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[a]Invented by Pele's father.

On Measuring the Beautiful Game

- Many Questions:
  - Put sensors on all players' feet, hands, heads,. . .!
  - Measure the total distance covered by the players of a team!
  - How many passes did a team make during the game?
  - What was the average length of a pass during a game?
  - . . .
- What should a winning team do?

Sensors in a Vineyard

- Making Canadian (in Ontario) “Ice Wine”.
- Very sensitive to temperature changes.

- … harvest late in the season and wait for the temperature to drop to $-7C$!

Problem & Model

Not too Close to Each Other
(Proximity and Sensor Interference)

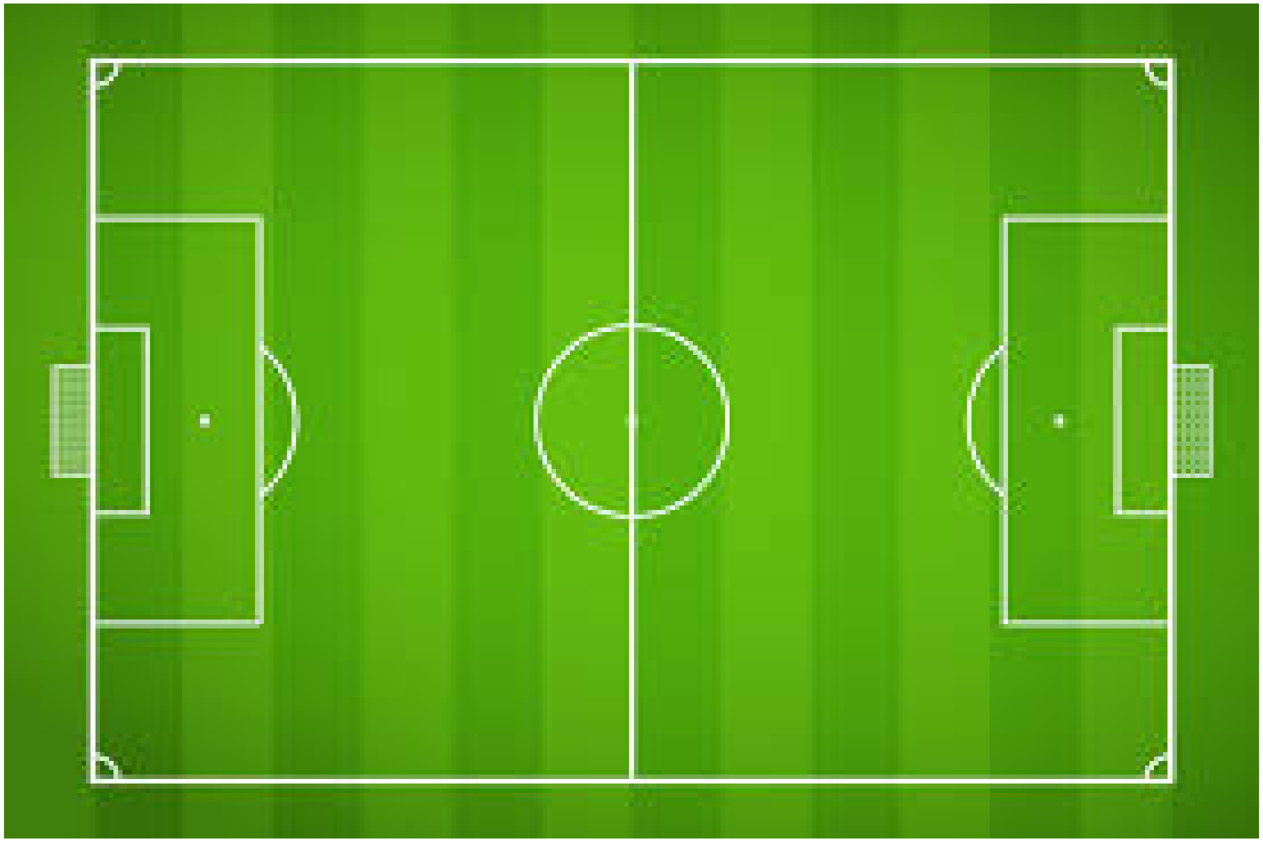
- Proximity affects transmission and reception signals and degrades performance:*the closer the distance the higher the resulting interference and hence performance degradation.*

  –

- In theoretical models, a critical value, say $s > 0$, is established and sensors must be kept a distance of at least s apart:
  - Two sensors' signals interfere with each other during communication if their distance is $< s$.

Not too Far from Each Other (Sensor Coverage)

- You want to cover a line (or any geometric domain) in such a way that every point on the line is within the range of a sensor.

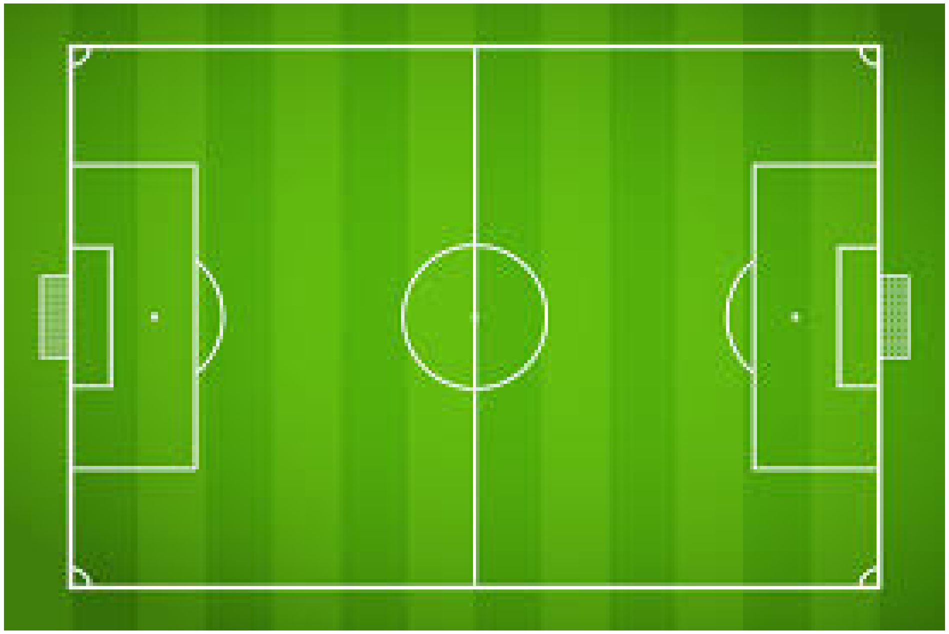


(Total, Max, etc) Movement

- Sensors' initial placement does not necessarily satisfy the coverage and/or interference requirements.
- An algorithm is required to specify how sensors should move.
- The cost is specified by
  - Sum of movements (or Total Movement) of sensors,
  - Max movement of a sensor,
  - etc

Problem Statement

- Sensors are placed on a specific domain, e.g.,
  - line, plane, etc



- Move the sensors along the domain so as to
  - satisfy the coverage and/or interference constraints, and
  - minimize the cost of sensor movement.

Communication and Movement Algorithms (1/2)

- Deterministic Input
  - How efficiently can you move the sensors?
    - ∗ Minimize the energy
    - ∗ Minimize the time
    - ∗ Minimize the number of sensors moved
  - How do sensors communicate?
    - ∗ Global
    - ∗ Local
- Some Recent Research
  - COCOA 08 (TCS 09), ADHOCNOW 09 & 10, PODC 13, ...

Communication and Movement Algorithms (2/2)

- Random Input
  - Type of distribution
  - Relationship of sensor range and movement
- Some Recent Research
  - SPAA 13, COCOON 14, ...
- Key references for Random Placement:
  - Kranakis et al. [2013][*Coverage*]
  - Kranakis and Shaikhet [2014a][*Interference*] M/D/1 Queues
  - Kranakis and Shaikhet [2014b][*Interference & Coverage*]: Queues G/G/1 (Coverage), G/D/1 (Interference)
  - Talagrand [2005][*The Generic Chaining*]

References

E. Kranakis and G. Shaikhet. Displacing sensors to avoid interference. In *Proceedings of 20th COCOON*, 2014a.

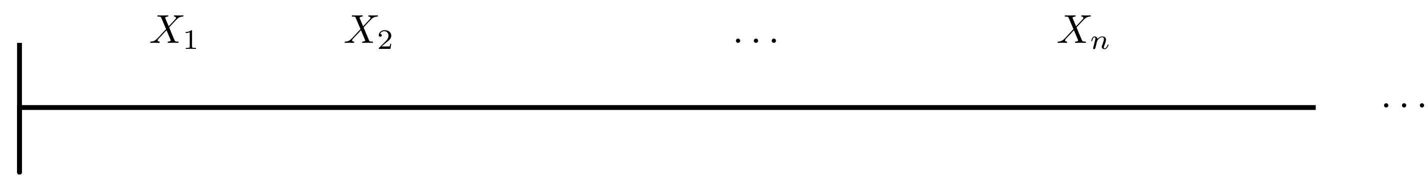
E. Kranakis and G. Shaikhet. Critical regimes for sensor coverage and interference. In *preparation*, 2014b.

E. Kranakis, D. Krizanc, O. Morales-Ponce, L. Narayanan, J. Opatrny, and S. Shende. Expected sum and maximum of displacement of random sensors for coverage of a domain. In *Proceedings of the 25th SPAA*, pages 73–82. ACM, 2013.

M. Talagrand. *The generic chaining*, volume 154 of *Springer Monographs in Mathematics*. Springer, 2005.

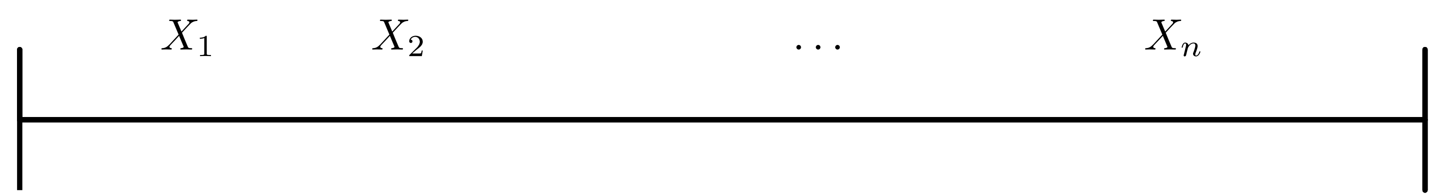
Model

- Random Variables $X_1, X_2, \ldots, X_n$ represent sensor positions.
- *Interference/Coverage* Problems in the half-line $[0, +\infty)$:



X_i is the i-th arrival in a Poisson (or General) process.

- *Interference/Coverage* Problems in the unit interval $[0, 1]$:



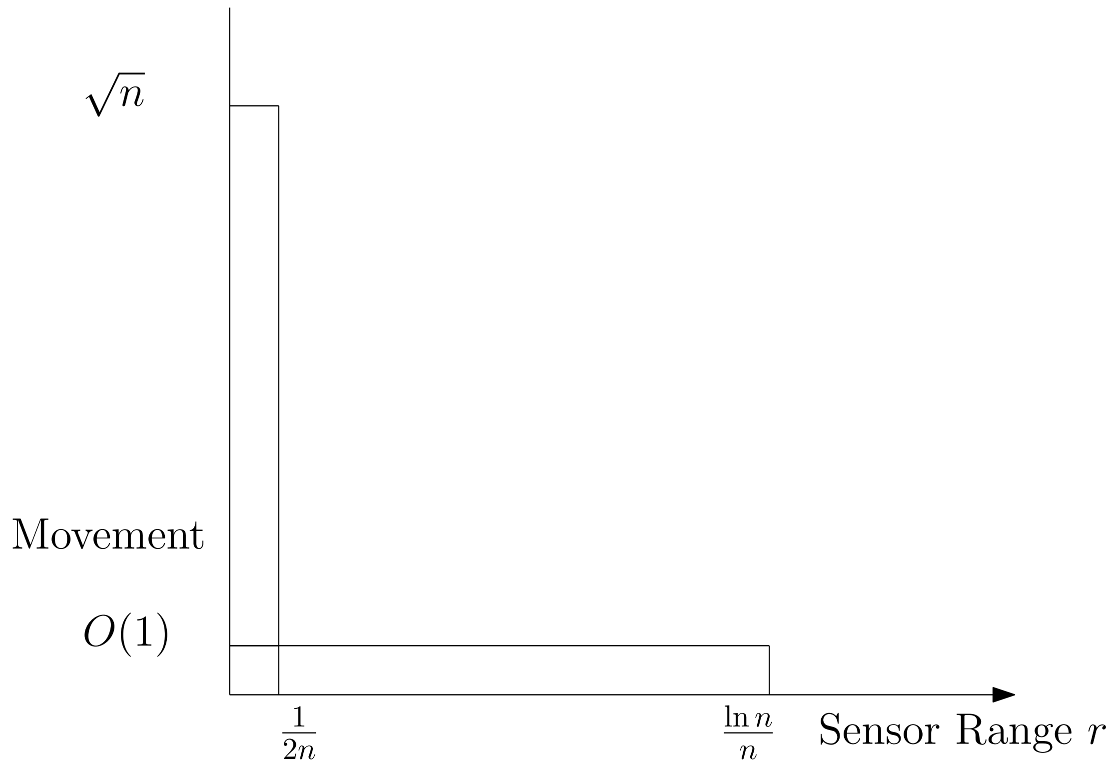
Sensors are thrown randomly and independently with the uniform distribution in the unit interval.

Coverage: Mathematical Motivation (1/3)

- Throw n sensors of radius $r := \frac{1}{2n}$ at random in a unit interval.
  - To ensure coverage of the interval they must be moved to anchors $a_i = \frac{i}{n} + \frac{1}{2n}$, for $i = 0, 1, \ldots, n-1$.
  - This is the worst-case total movement!
  - Why?
- Keep increasing the sensor radius.
  - The bigger the radius the less the movement! Why?
- When the radius reaches $\Theta(\frac{\ln n}{n})$ w.h.p. no sensor needs to move!
  - Why?

Coverage: Mathematical Motivation (2/3)

- Sensor movement as a function of the sensor range.



- The bigger the radius (range) the smaller the movement.

Interference: Mathematical Motivation (3/3)

- Throw n sensors at random in a unit interval. We we want to ensure no two sensors are at distance $< s$.
  - To ensure no two sensors are at distance $< \frac{1}{2n}$ they must all be placed to anchors $a_i = \frac{i}{n} + \frac{1}{2n}$, for $i = 0, 1, \ldots, n-1$. This is the worst-case total movement! Why?
- Keep decreasing the interference distance s.
  - The smaller the interference distance s the less the movement! Why?
- In general,

$$\text{Arrival Time of } i+1\text{st sensor} - \text{Arrival Time of } i\text{th sensor}$$

are the interarrival times of the Poisson process.

Interference

Displacement and Interference on a Line

- Consider sensors on a line. We are allowed to move the sensors (on the line), if needed, so as to avoid interference.
- We call *total movement* the sum of displacements that the sensors have to move so that the distance between any two sensors is $\geq s$.
- Assume that n sensors arrive according to a Poisson process having arrival rate $\lambda = n$ in the interval $[0, +\infty)$.
  - What is the expected minimum total distance that the sensors have to move from their initial position to a new destination so that any two sensors are at a distance more than s apart?

Results on Interference

- In Kranakis and Shaikhet [2014a], we study tradeoffs between the interference distance s and the expected minimum total movement, denoted by $E(s)$.

Interference Distance s	Total Displacement $E(s)$
$s - \frac{1}{n} \in \Omega\left(n^{-\alpha}\right), 2 \geq \alpha \geq 0$	$\Omega(n^{2-\alpha})$
$\left\|s - \frac{1}{n}\right\| \in O\left(n^{-3/2}\right)$	$\Theta(\sqrt{n})$
$s \leq \frac{1}{tn}, t > 1$	$\leq \frac{t^2}{(t-1)^3}$

- **Critical Regime:** Critical threshold around $\frac{1}{n}$,
  1. for s below $\frac{1}{n} - \frac{1}{n^{3/2}}$, $E(s)$ is a constant $O(1)$,
  2. for $s \in \left[\frac{1}{n} - \frac{1}{n^{3/2}}, \frac{1}{n} + \frac{1}{n^{3/2}}\right]$, $E(s)$ is in $\Theta(\sqrt{n})$,
  3. for s above $\frac{1}{n} + \frac{1}{n^{3/2}}$, $E(s)$ is above $\Theta(\sqrt{n})$.

Interference and G/D/1 Queues

- Can extend the results to arbitrary random processes. In Kranakis and Shaikhet [2014b] we prove:

  Theorem 1 *Assume the sensors arrive according to a general distribution. Let the interference distance be* $s = \frac{1}{tn}$*. Then the expected minimum sum of displacements of the sensors to ensure that any two of them are at distance at least* s*, is at most* $\frac{1}{2t(t-1)}$.

  – This is a result about $G/D/1$ queues.

  – Proof uses Little's theorem and the Pollaczek-Khinchine formula.

Coverage

Displacing for Coverage in $[0, 1]$

- n sensors with identical range $r = \frac{f(n)}{2n}$, for some $f(n) \geq 1$, for all n, are thrown randomly and independently with the uniform distribution in the unit interval $[0, 1]$.
- They are required to move to new positions so as to cover the entire unit interval in the sense that every point in the interval is within the range of a sensor.
- We obtain tradeoffs between the range r of the sensors and
  - the expected min sum (denoted by $E(r)$)

  of displacements of the sensors required to accomplish this task.

Results for the Unit Interval

In Kranakis et al. [2013] we prove:

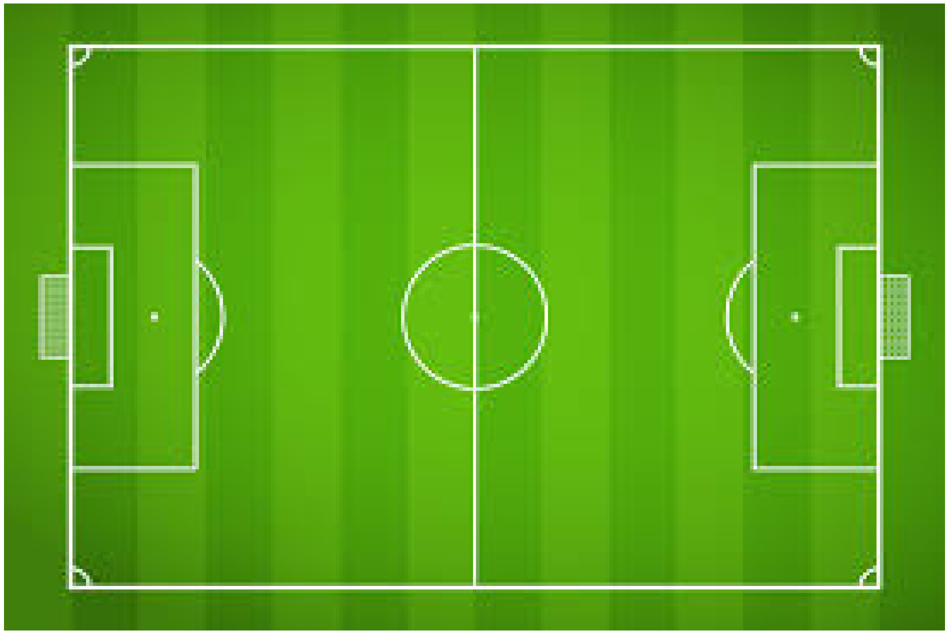
Sensor Range r	Total Displacement $E(r)$
$\frac{1}{2n}$	$\Theta(\sqrt{n})$
$\frac{f(n)}{2n}$ $(f(n) \geq 6)$	$O(\sqrt{\frac{\ln n}{f(n)}})$
$\frac{f(n)}{2n}$ $(12 \leq f(n) \leq \ln n - 2 \ln\ln n)$	$O(\frac{\ln n}{f(n)e^{f(n)/2}})$
$\frac{f(n)}{2n}$ $(1 < f(n) < \sqrt{n})$	$\Omega(\epsilon f(n) e^{-(1+\epsilon)f(n)}), \forall \epsilon > 0$

Interference and G/G/1 Queues

- Can we prove the existence of a critical regime for coverage on a line? (i.e., Can we prove tight bounds) for coverage on a line?)
- YES
- In Kranakis and Shaikhet [2014b], using Skorokhod maps (used in the theory of stochastic differential equations) we can show there is a critical regime for the coverage problem.
  - This is a result about $G/G/1$ queues.

2D

- Several deterministic/randomized results are known on



  - Covering a domain
  - Covering the perimeter of a domain
  - Preventing interference

Thank you