MONDAY 12TH MAY

9:45 Welcome from Prof Karl Coleman

Dean of the School of Physical Sciences
University of Liverpool

10:00 PLENARY
Chair: Prof Olivier Menoukeu Pamen

Prof Rachel Bearon
University of Liverpool

Insights from mathematical models of micro-tissues for drug uptake & cancer spread

Mathematical models can aid discovery in the life sciences, by providing predictive tools, and allowing efficient testing of 'what-if' scenarios. However, identifying the 'right' model, and suitably parameterizing it, is a challenging task which mathematicians are well-placed to contribute. I will discuss two projects based upon an experiments on 3D spheroid cell culture systems [1,2]. Cells cultured in such system have been shown to more closely resemble the functionality and morphology of cells *in-vivo*, and so there is increasing interest in using these systems for example in drug toxicity studies and for better understanding cancer metastasis. Time allowing, I will also present new results from a collaborative project with the University of Sheffield on drug delivery across the oral mucosa using a tissue engineering and mathematical modelling approach.

[1] Leedale, J. A., Kyffin, J., Harding, A., Colley, H., Murdoch, C., Sharma, P., Williams, D., Webb, S. & Bearon, R. (2020). Multiscale modelling of drug transport and metabolism in liver spheroids. Interface Focus, 10(2). doi:10.1098/rsfs.2019.0041

[2] Scott, M., Zychaluk, K. & Bearon, R. (2021) A mathematical framework for modelling 3D cell motility; applications to Glioblastoma cell migration. Mathematical Medicine and Biology, 38(3) doi.org/10.1093/imammb/dqab009

SESSION 1 11:30-13:30

Chair: Rockefeller

Dr Smitha Maretvadakethope
University of Liverpool

The interplay between bulk flow and boundary conditions on the distribution of micro-swimmers in channel flow

Biofilm formation impacts many fields, from medical technologies (e.g. catheter design) to infrastructure development (e.g. water supply pipes) due to contamination and infection risks. For the case of motile micro-swimmers, the early stages of biofilm behaviour are dependent on the physical properties of swimmers and their flow environments as these affect the likelihood of surface interactions and surface colonisation. In our work, we highlight the effect of boundary conditions on the bulk flow distributions, such as through the development of boundary layers or secondary peaks of cell accumulation in bulk-flow swimmer dynamics. For the case of a dilute swimmer suspension in 2D channel flow, we compare distributions (in physical and orientation space) obtained from individual-based stochastic models with those from continuum models, and identify mathematically sensible continuum boundary conditions for different physical scenarios (i.e. specular reflection, uniform random reflection and absorbing boundaries). We identify the dependence of the spread of preferred cell orientations on the interplay between rotation driven by sheared flows and rotational diffusion. We further highlight the effects of swimmer geometries, fluid shear, and the full history of bulk-flow dynamics on the orientation distributions of micro-swimmer wall incidence.

Pre-print: https://arxiv.org/abs/2209.05973

Camelle Kabiwa Kadje
African Institute for Mathematical Sciences (AIMS) Cameroon and University of Douala Cameroon

Exploring the role of water stress and soil on banana-plantain production: a mathematical modelling approach

The aim of this paper is to investigate the role of water stress and soil on banana plantain plants production. We propose a mathematical model for the dynamics growth of banana plantain-plants that takes into account the concentration of available water in the soil, water stress, banana- plantain plant production, plants compensation and delay. We present the theoretical analysis of the model with and without water stress. More precisely, we show that the full model is well-posedness. For each model, we compute the trivial equilibria and derive two thresholds parameters

that determine the outcome of water stress within a plantation. Further,
we perform numerical simulation on the case of banana-plantain simulations to support the theory. After, the role of the water stress on the plant production is numerically investigated. We found that the water stress can cause about 63.27% of loss of banana-plantain production within a plantation with 1600 rejets initially planted. This suggests that climate change plays a detrimental role on banana plantains production. We also numerically prove that Fine Sandy Loamy soil is
more resistant to water than Silty Clay soil and Loamy soil.
Keywords: Banana-plantain plant; Water stress; Delay; Mathematical models; Stability; Numerical simulation

Dr Cara Neal
University of Liverpool

The role of non-Newtonian fluids in microswimmer propulsion

The locomotion of microswimmers in non-Newtonian fluids is of crucial importance in many biologically processes including infection, fertilisation, and biofilm formation. The behaviour of microswimmers in these media remains an area with many conflicting results, with swimmers displaying varying responses depending on their precise morphology, propulsive mechanisms, and elastic properties, as well as the complex characteristics of the surrounding fluid. We present a computational technique which will be used to investigate the effect of non-Newtonian properties, including shear-thinning rheology and viscoelasticity, on both planar sperm-like swimmers and helical bacterial locomotion. This model utilises known Newtonian solution techniques to approximate the rapidly varying flow surrounding a swimmer, with a non-Newtonian correction term obtained through solving using the finite element method. For sperm-like swimmers, the inclusion of swimmer elasticity allows us to demonstrate that shear-thinning rheology can either enhance or hinder propulsion, depending on a balance of elastic and viscous forces as well as fluid properties. For bacteria, we present the framework for modelling cells in fluids exhibiting both shear-thinning and viscoelastic effects and discuss possible future developments for this model and their applications.

SESSION 2 14:00-16:00
Coffee break at 15:00
Chair: Dr Cara Neal

Rockefeller
Stellenbosch University and African Institute for Mathematical Sciences (AIMS)South Africa

An Approach to Modeling Dynamical Systems with Temporally Consistent State Spaces

One of the challenges of Recurrent Neural Networks (RNNs) is to build, store and incorporate information from previous time steps, and also learn a representation of the cumulative history using bounded storage, which must be updated online as more data is received. RNNs have been extensively used for learning non-linear dependencies in dynamical systems including financial and climate-related time series data. However, they suffer from many limitations including temporal inconsistency. In this talk, we will present the Historical Consistent Neural Networks (HCNNs) which allow the modeling of highly-interacting dynamical systems across multiple time scales in a temporally consistent manner. HCNNs do not draw any distinction between inputs and outputs, but model observables embedded in the dynamics of large state space. To extend the length of the forecast horizon, we will also present three different HCNNs' improvement architectures: Partial Teacher Forcing, Large Sparse State Transition Matrices, and an LSTM Formulation. After obtaining results on time-series datasets mathematically generated from chaotic systems, we will also present some early results on the forecasting of climate-related data across certain regions in the Western Cape, South Africa.

Dr Sean Edwards
University of Liverpool

Mathematical modelling of drug delivery through the oral mucosa

Systemic drug delivery through the oral mucosa (the tissue which lines the inside of the mouth) is an attractive alternative to traditional methods such as by injection or tablet, as treatments can be self-applied in a pain-free manner or by-pass first pass metabolism by the liver, respectively. Experiments to develop new oral mucosal drug treatments are often undertaken on animals or in vitro using 3D tissue-engineered human models. However, every time a new drug is proposed, experiments must be undertaken, which is both ethically and economically costly. To compliment these approaches, we are developing a mathematical model of drug permeation across the oral mucosa. Once developed this model will be used to screen new drugs based on their biophysical and biochemical properties, thus significantly reducing the need for animal testing.

We present the in silico model, which captures the histological structure of the epithelium: discrete cells separated by extracellular gaps. To ensure that the model is physically consistent the sizes of the cells and gaps are taken from experimental measurements of stained tissue sections and by transmission electron microscopy. Drug transport is modelled with diffusion, with diminished permeation rates across cell-membranes (due in part to a drugs lipophilicity).

The novelty of this project is that the development of the in silico model is directly informed by drug delivery data generated from in vitro experiments on tissue-engineered oral mucosa. A comparison with results from these in vitro experiments will be presented.

Anoumou Attiogbe (online)
African Institute for Mathematical Sciences (AIMS) Senegal

Short time existence and smoothness of the nonlocal mean curvature flow of graphs

In this talk, we consider the geometric evolution problem of entire graphs moving by fractional mean curvature. For this, we study the associated nonlocal quasilinear evolution equation satisfied by the family of graph functions. We establish, using an analytic semigroup approach, short time existence, uniqueness and optimal H\"older regularity in time and space of classical solutions of the nonlocal equation, depending on the regularity of the initial graph. The method also yields $C^\infty-$smoothness estimates of the evolving graphs for positive times. The results from this project have been obtained in a joint work with M. M. Fall and T. Weth.

TUESDAY 13TH MAY

10:00 PLENARY
Chair: Prof Olivier Menoukeu Pamen

Dr Emmanuel Coffie
University of Liverpool

On the analysis of a highly nonlinear two-factor stochastic volatility model under local Lipschitz condition

We establish theoretical properties of the true solution to a two- variance-driven asset price model with super-linear coefficient terms. Since this model is not analytically tractable, we construct an implementable numerical method to approximate it and prove the finite-time strong convergence theory under the local Lipschitz condition. Finally, we provide simulation examples to validate the theoretical results.

SESSION 1 11:30-13:30

Chair: Elizabeth Dadzie

Dr Kira Henshaw
University of Liverpool

A stochastic model of the effect of insurance on households susceptible to random proportional losses

In this talk, we adopt a risk process with deterministic investment and multiplicative jumps to model the capital of a low-income household. The household is assumed to be susceptible to capital shocks that are a random proportion of their accumulated capital. These shocks may be due to, for example, severe illness, the death of a household member or breadwinner and catastrophic events such as floods and earthquakes. For remaining proportions of capital after shock events that follow a special case of the beta distribution, closed-form expressions for the probability of a household falling below the poverty line (the trapping probability) are derived via analysis of the Laplace transform of the infinitesimal generator of the process. Exploring the impact of insurance on this probability, which mimics an insurer's ruin probability, an insurance product offering proportional coverage is considered and the corresponding probability studied by recursively solving the associated integro-differential equation.

Rhoss Beauneur Likibi Pellat
African Institute for Mathematical Sciences (AIMS) Ghana

Path-regularity of Path-dependent Quadratic Forward-Backward SDEs with Dini-continuous drift

We analyze the path regularity of solution for forward-backward SDEs(FBSDEs) with low regularity assumptions on the drift and path dependent functional terminal value. More precisely we are interested in the numerical approximation of solutions to FBSDEs with quadratic drivers and non Lipschitz continuous drift.

Cedric Koffi
University of Liverpool

Impact of social factors on microloans defaults

Micro-finance institutions (MFI) are major loan providers to low-income individuals in developing countries. Even though this endeavour is laudable, micro-finances put themselves at risk of going bankrupt since such customers carry a high risk of default. This is due to the fact that many MFI do not have adequate credit scoring and rating models for their customers. In this work, our objective is to build a model that can be used as benchmark to predict the future behaviour of customers throughout the repayment period of their loans. This would automatically enable the financial institution to take appropriate steps to circumvent defaults, allocate loan amounts to customers more confidently, and set up their portfolio more efficiently. To build this model, we have relied on techniques from Survival Analysis to model the heterogeneity between customers repayments, and used a logit link function to estimate time dependent transition probabilities.

SESSION 2 14:00-16:00
Coffee break at 15:00
Chair: Dr Kira Henshaw

Elizabeth Dadzie
African Institute for Mathematical Sciences (AIMS) Ghana and University of Ghana

Pricing Weather-linked Insurance Derivative for Agricultural Goods

Weather derivatives are simply futures contracts or options on those contracts where the underlying commodity is a weather index. Insurance derivatives that are dependent on the weather are fast rising as they have many applications and uses.

The recent phenomenon of climate change has led to many uncertainties (weather) for farmers. We examine commodity (agricultural goods) yields that depend on weather factors such as temperature, rainfall, etc. We consider a farmer who wants to purchase an insurance derivative to mitigate the weather risk from an agent (insurer).

This insurance derivative depends on some weather factors namely temperature and rainfall. The weather factors are assumed to be correlated to some tradable assets in the market. The agent invests in a derivative linked with the tradable assets. We assume the preference of the agent is given by the classical expected utility.

We present some numerics on the weather factors. We find the price at which the agent is indifferent, regarding his utility, whether or not to invest in the derivative and obtain closed-form indifference prices for the agent under certain conditions.

Keywords: weather factors, indifference pricing, exponential utility.

Ozan Hur
University of Liverpool

Multi-fractional Stochastic Dominance

We study a novel family of multi-fractional stochastic dominance (MFSD). MFSD generalizes the notion of fractional stochastic dominance (FSD) recently introduced by Müller et al. (2016) to interpolate between integer degree dominance relations. The family of MFSDs is parametrized by an arbitrary non-decreasing function, which has two-fold significance: (i) it economically controls decision maker's preferences in terms of risk aversion and greediness at a local level, and (ii) it mathematically provides a local interpolation phenomenon between first and second stochastic dominance on different portions of supports of distributions. Our generalization permits the ordering of distribution functions that cannot be ordered by FSD in non-trivial cases. We will discuss some mathematical as well as economical properties of MFSD and show how it provides a more comprehensive framework for decision analysis.

[1] Alfred Müller, Marco Scarsini, Ilia Tsetlin, Robert L. Winkler (2016) Between First- and Second-Order Stochastic Dominance. Management Science 63(9):2933-2947. https://doi.org/10.1287/mnsc.2016.2486

Maalvladedon Ganet Some
African Institute for Mathematical Sciences (AIMS) Ghana

Optimal Management of a Residential Heating System: The Prosumer's Problem

We formulate a model for a residential heating system of a representative prosumer connected to a central heating system and equipped with a solar thermal collector and an external storage. The objective of the prosumer is to minimize the cost of satisfying his/her heat and hot water demand. We formulate and solve a stochastic optimal control problem and present numerical results for the value function and the optimal control.

WEDNESDAY 14th MAY

10:00 PLENARY
Coffee break at 11am
Chair: Dr Kamila Zychaluk

Dr Susha Parameswaran
University of Liverpool

Dark Energy in String Theory

Abstract: The surprising discovery in 1998 that our Universe is undergoing an accelerated expansion presents arguably the biggest challenge in fundamental physics today. It implies that our Universe is dominated by a so-called Dark Energy, whose microscopic nature is unknown. I will discuss how ideas in string theory may help to answer the question of what Dark Energy is and how Dark Energy may lead to observable consequences for string theory.

11:30-12:00
Dr Benjamin Holmes
University of Liverpool

Using a Markov Chain Model to Identify Optimal Football Match Tactics

In this paper, we present a Markov chain model of football. The chain allows teams to move the ball around the pitch in various manners, whilst the probability of each transition is dependent on the abilities of the players on each team. We estimate two multinomial models that estimate the locations of players in attacking and defending roles, allowing us to build novel variables representing a team's abilities in the different areas of the pitch. We can simulate the chain numerous times using different tactical set-ups to determine what is the optimal strategy for a particular match. We see this model becoming a useful tool for football coaches and managers.

OUTREACH ACTIVITY 13:00-15:00
All registered participants

THURSDAY 15TH MAY

10:00 PLENARY
Chair: Dr David Marti-Pete

Prof Lasse Rempe
University of Liverpool

Pure mathematics and dynamical systems at the University of Liverpool

I will begin by giving a short overview of the three research areas within the pure mathematics research cluster at Liverpool: Algebraic Geometry, Geometry & Topology, and Dynamical Systems.

I will then give an introduction to the theory of Dynamical Systems - these are systems that evolve over time according to fixed rules. Dynamical systems often display surprisingly complex behaviour, even if the underlying rules are simple.

In Liverpool, many of us are particularly interested in the case where the state of the systems is described by a single complex variable. In the final part of the talk, I will discuss a problem known as Eremenko's conjecture, which poses what may at first appear to be a rather simple question about the behaviour of an analytic function defined in the complex plane. The conjecture was posed in 1989 and motivated much ground-breaking research in the intervening years. It was solved in 2022 in a collaboration between David Martí-Pete (Liverpool), James Waterman (Stony Brook, formerly a postdoc at Liverpool) and myself.

SESSION 1 11:30-13:30

Chair: Peguy Kem-Meka Tiotsop Kadzue

Andrew Brown
University of Liverpool

Counterexamples to the Strong Eremenko Conjecture: "How Slow Can You Grow?"

The strong Eremenko Conjecture was disproved in the paper of Rottenfußer, Rückert, Rempe and Schleicher with a counterexample function in the Eremenko—Lyubich Class B with infinite order growth. It was also shown in the same paper that the conjecture holds for functions of finite order growth (and finite compositions of them). This talk will discuss the work that has been done to reduce the order of growth of counterexamples.

Aissatou Seck
African Institute for Mathematical Sciences (AIMS) Rwanda

Projected Changes of Precipitation Extremes Over Africa

Climate change is expected to have significant impacts on the frequency and intensity of precipitation extremes, which can lead to more frequent and severe floods and droughts. Precipitation extremes refer to the occurrence of very high or very low levels of rainfall or snowfall in a given region, over a specific period of time. Understanding how these extremes may change in the future is important for developing effective adaptation strategies. The aim of this research has to anticipate alterations in the frequency, intensity, and timing of emergence of extreme precipitation events across Africa, based on future climate scenarios. To achieve this, we utilized a state-of-the-art global climate model from the Coupled Model Inter-comparison Project's Phase 6 (CMIP6) to simulate changes in precipitation extremes over Africa between 2071 and 2100, utilizing various Shared Socioeconomic Pathways (SSPs). The SSPs reflect potential combinations of social, economic, and environmental variables that may impact greenhouse gas emissions in the future. Our findings indicate a rise in the occurrence and severity of extreme precipitation events in numerous regions of Africa, particularly the Sahel East and eastern Africa, for consecutive wet days. These increases are predicted to persist throughout the 21st century, with the most significant rises occurring in the Sahel East and eastern Africa. However, certain areas such as western Africa and Central Africa have experienced a reduction in extreme precipitation events in recent years, which is predicted to continue. These changes can have significant consequences for Africa's population and its economy, potentially affecting food security, public health, water resources, and agriculture. On the other hand, increases in consecutive wet days can lead to flooding and landslides, causing significant damage to infrastructure and communities. These findings highlight the need for effective adaptation strategies to mitigate the impacts of changing precipitation patterns in Africa, particularly in vulnerable regions where the consequences can be most severe.

Keywords: climate change, Africa, precipitation extremes,CMIP6

Julia Münch
University of Liverpool

Extending rational maps on the Riemann sphere to $\boldsymbol{R}^3$

The main motivation for the talk the question whether we can use similar techniques and reasoning as in complex dynamics also in higher dimensions. For that we need to clarify which maps we would like to study and see whether the dynamics is interesting. Lattès maps are a family of holomorphic maps on the Riemann sphere with particular dynamical behaviour and as it was shown by Mayer one can extend them to R^3. I will explain the construction and indicate further projects.

SESSION 2 14:00-16:00
Coffee break at 15:00
Chair: Julia Munch

Peguy Kem-Meka Tiotsop Kadzue
Doctoral Training Program in Data Science in Quantum Leap Africa (QLA)
Topological Characterization of Climate Change in Selected African Countries using Persistent Homology

Climate change is a big challenge in all continents including Africa. Africa is particularly vulnerable to climate change impacts. Some of these impacts include floods, droughts, and storms which are the main sources of agricultural risk. This study examines the climate pattern in selected African countries (Kenya, Cameroon, Somalia, Tunisia, Burundi, Rwanda, Madagascar, and Zambia) for a period of 1905 to 2021. To detect climate change patterns, precipitation and temperature are analyzed using persistent homology, which is the main tool in topological data analysis (TDA) that focuses on qualitative information known as topological features of data. Results of the analyses show that there are changes in topological features during climate change. The study also found that the first persistence landscape (a functional topological summary of persistent homology) consistently detects climate change. This paper offers the possibility to develop an effective early detection system for climate change based on our approach.

James Dolan
University of Liverpool

Geometry of Continued Fractions

In this talk I will give an overview of a geometric approach to understanding continued fractions.

I will first go over what a continued fraction is and how we choose to classify them in this field, I will then give motivation for why it is appropriate to describe these objects by new geometric notions such as integer length and integer sine, and finally I will give a full description of Geometric Continued Fractions as the primary object of study in integer trigonometry.

Mamadou Diallo (online)
Doctoral Training Program in Data Science in Quantum Leap Africa (QLA)

Physics-Informed Neural Networks (PINNs)-Based Grain Drying Modeling

Grain drying is the process of drying grain to prevent spoilage during storage. Finding the appropriate storage and the optimal process have significant socio-economic benefits for Africa, in particular. To do this, a mathematical formulation in the form of high-dimensional partial differential equations (PDEs) is considered. In a few cases, analytical solutions to these PDEs are available, otherwise, numerical methods will be implemented. These methods use grids or mesh domains to compute the numerical solutions. However, they are computationally expensive for large PDEs. Thus, data-driven modeling such as physics-informed neural networks (PINN), a type of deep learning framework used to solve PDEs, is increasingly being used to circumvent the problems encountered in traditional numerical methods. This project will aim to develop PINN-based solutions for the problem of grain drying, as well as the associated storage.

FRIDAY 16TH MAY

10:00 PLENARY
Chair: Dr Kamila Zychaluk

Dr Linglong Yuan
University of Liverpool

Macroscopic patterns emerge from random individual behaviours

Probability theory has been widely used to model macroscopic phenomena in physics, biology, economics, and many fields. A natural and powerful approach is to build an individual-based model which describes justly how individuals interact with each other randomly. This talk will go through several examples to show how collective behaviours arise from random individual interactions in the long term.

SESSION 1 11:30-13:30

Chair: Aissatou Seck

Robert Mason
University of Liverpool

Neutrino mixing models from non-Hermitian quantum mechanics

Measured solar neutrino oscillation proved the first evidence for physics beyond the standard model requiring the neutrino to be described as massive particles. However, cosmological bounds have shown the mass to be incredibly small. Recent interest in quantum mechanics beyond the Hermitian paradigm has prompted interest in PT-symmetric theories and its application to various different areas including high energy physics. This talk discusses a non-Hermitian neutrino mass model and efforts to extend the theory to multiple generations in a consistent way.

James Roscoe
University of Liverpool

Linear Logarithmic Relaxation for Phase Transitions in Statistical Field Theory

Models with phase transitions are both interesting practically and theoretically. However, they are difficult to investigate using standard Markov-Chain Monte Carlo. There is need for more efficient and cost-effective methods to measure more complex models. This talk investigates using the linear logarithmic relaxation method to extract the density of states for the well known square lattice 2D Ising model and expands the method to two observables.

Marco Fava
University of Liverpool

Cohomology of a fine compactified universal Jacobian via topological stratification

A fine compactified universal Jacobian over the moduli space of pointed stable curves is a proper and smooth scheme. Therefore its cohomology can be obtained from its Hodge-Deligne polynomial, which in turn can be found by adding the polynomials of the Jacobians over the strata of the topological stratification of $\overline{\mathrm{M}_{\mathrm{g,n}}}$.

SESSION 2 14:00-15:00
Chair: Marco Fava

Shijie Xu

University of Liverpool

On the estimation of density and expected occupation density of Itô processes with irregular drift and diffusion coefficients

This is talk is a joint work with Dr. Paul Eisenberg and it is a quick overview of 2 papers. In the first part we give an upper bound for the density of Itô process with locally bounded drift coefficients and fixed diffusion coefficient. In the second part we give a sharp upper for the occupation density with bounded irregular drift and diffusion coefficients.

CLOSING REMARKS