The role of non-Newtonian fluids in microswimmer propulsion

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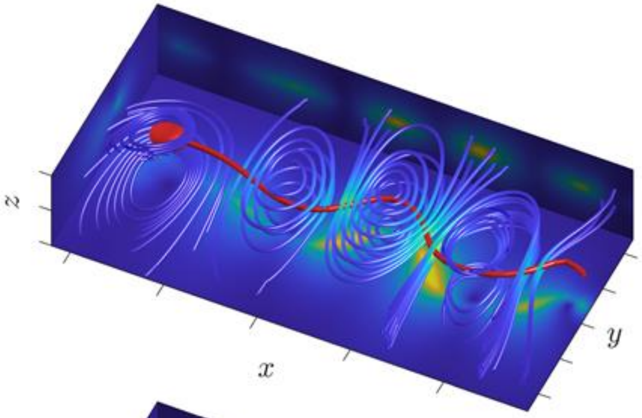

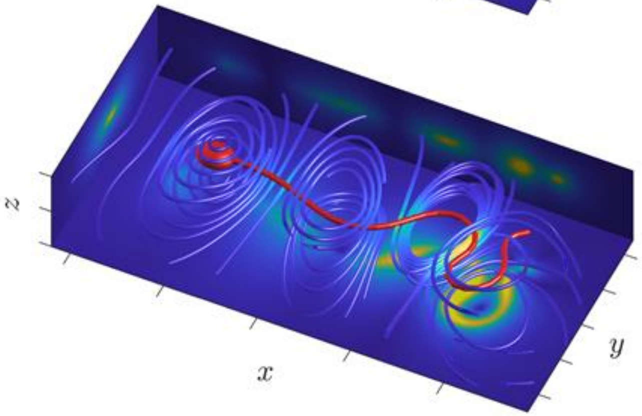

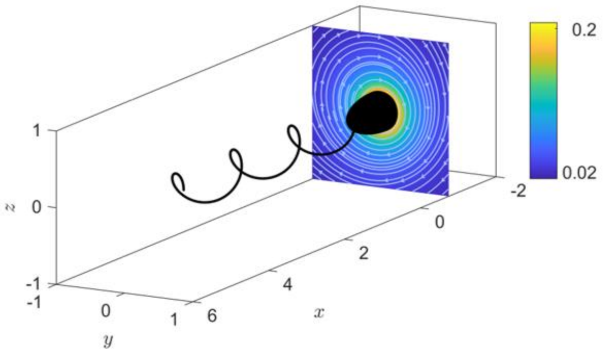

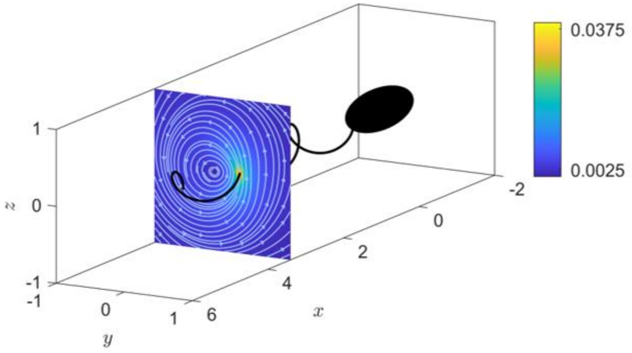

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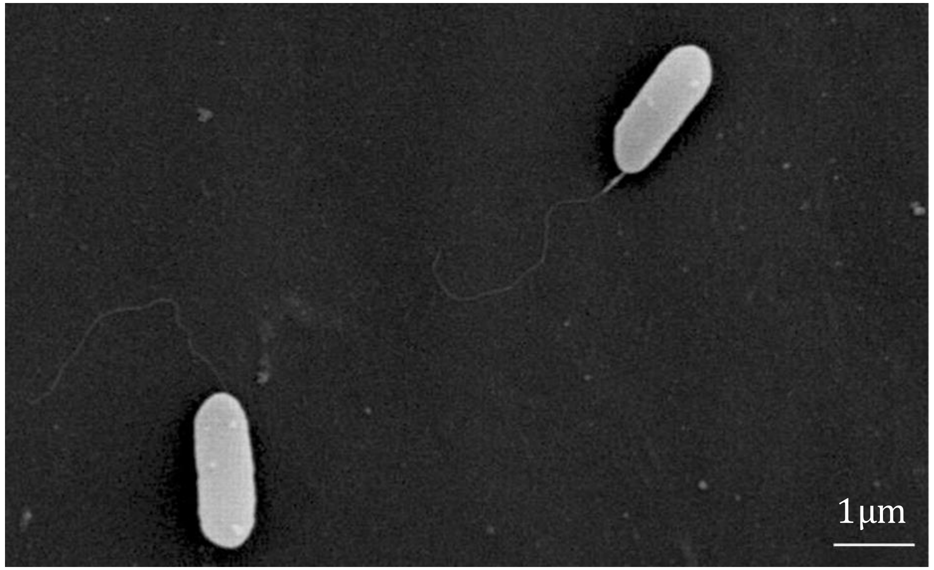
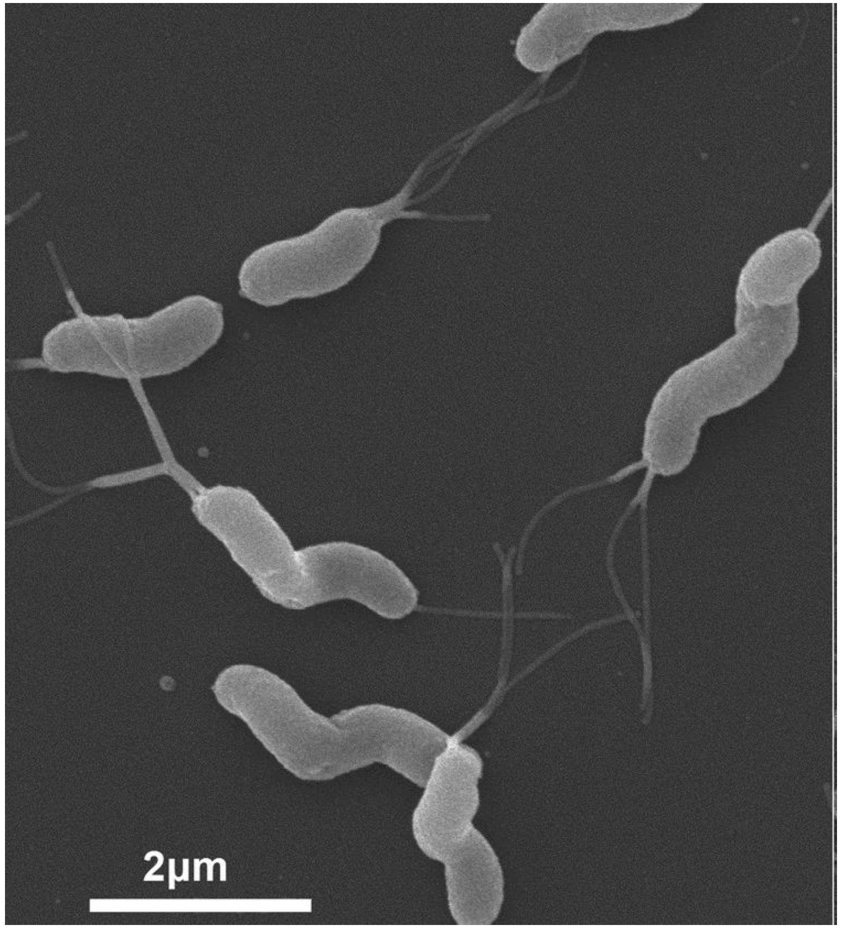
Swimming in non-Newtonian fluids

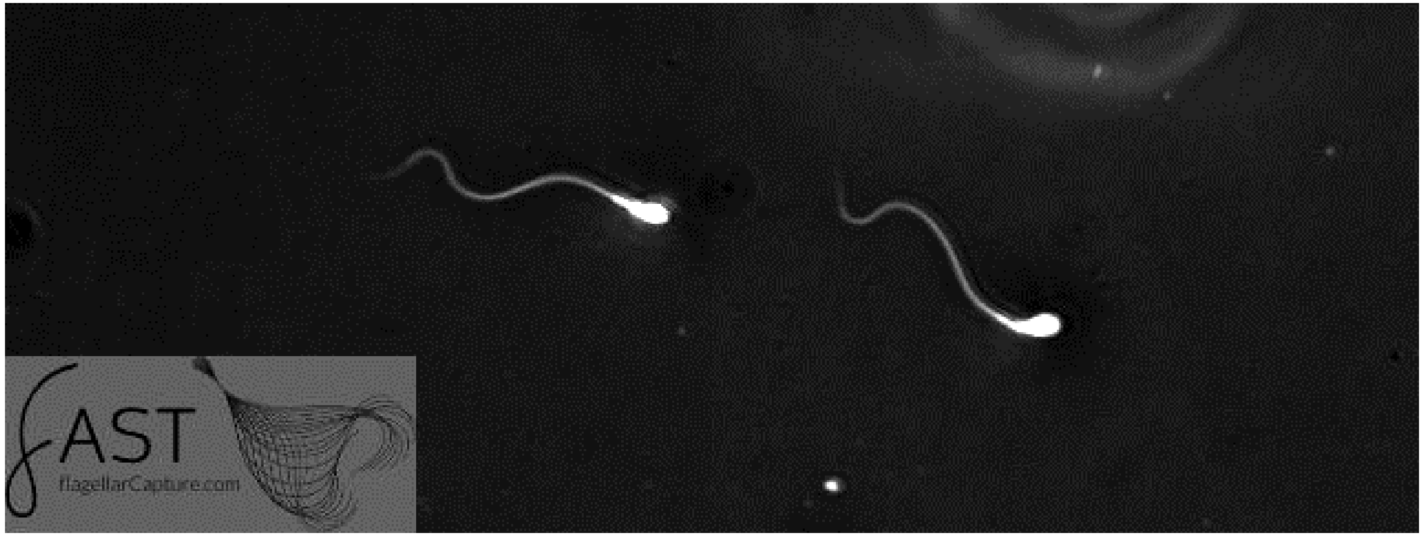


1. Pseudomonas aeruginosa progressing through the mucus-filled respiratory system.

2. Helicobacter pylori moving through the mucus layer covering the stomach.

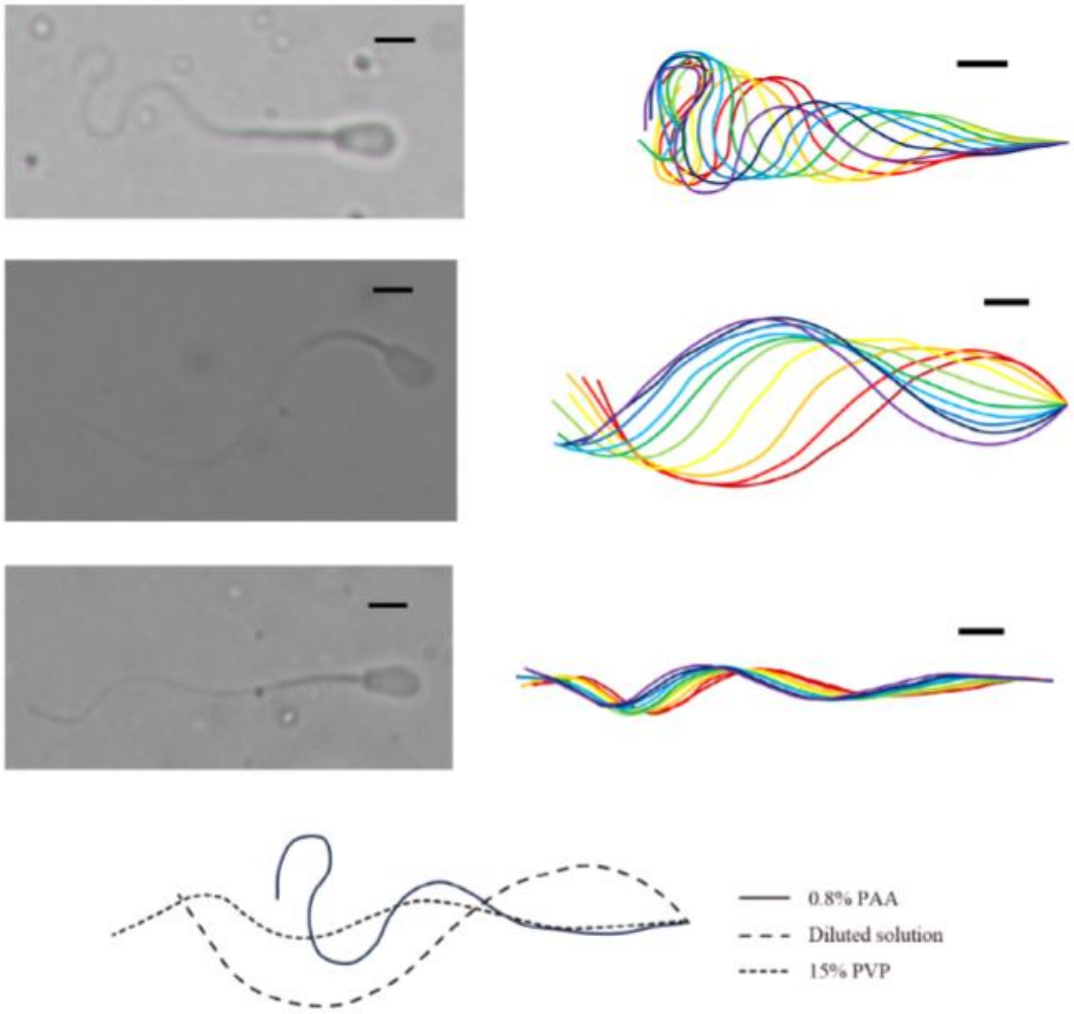


3. Human spermatozoa progressing through a cervical mucus analogue.



Image/video credits: (1) Tsang et al. *Eur. Respir. J.* 2003, (2) Dr. Nina Salama, (3) Gallagher et al. *Hum. Reprod*. 2019.

Defining the fluid-structure problem



Hyakutake, Sato & Sugita. *J. Biomech.* 2019.

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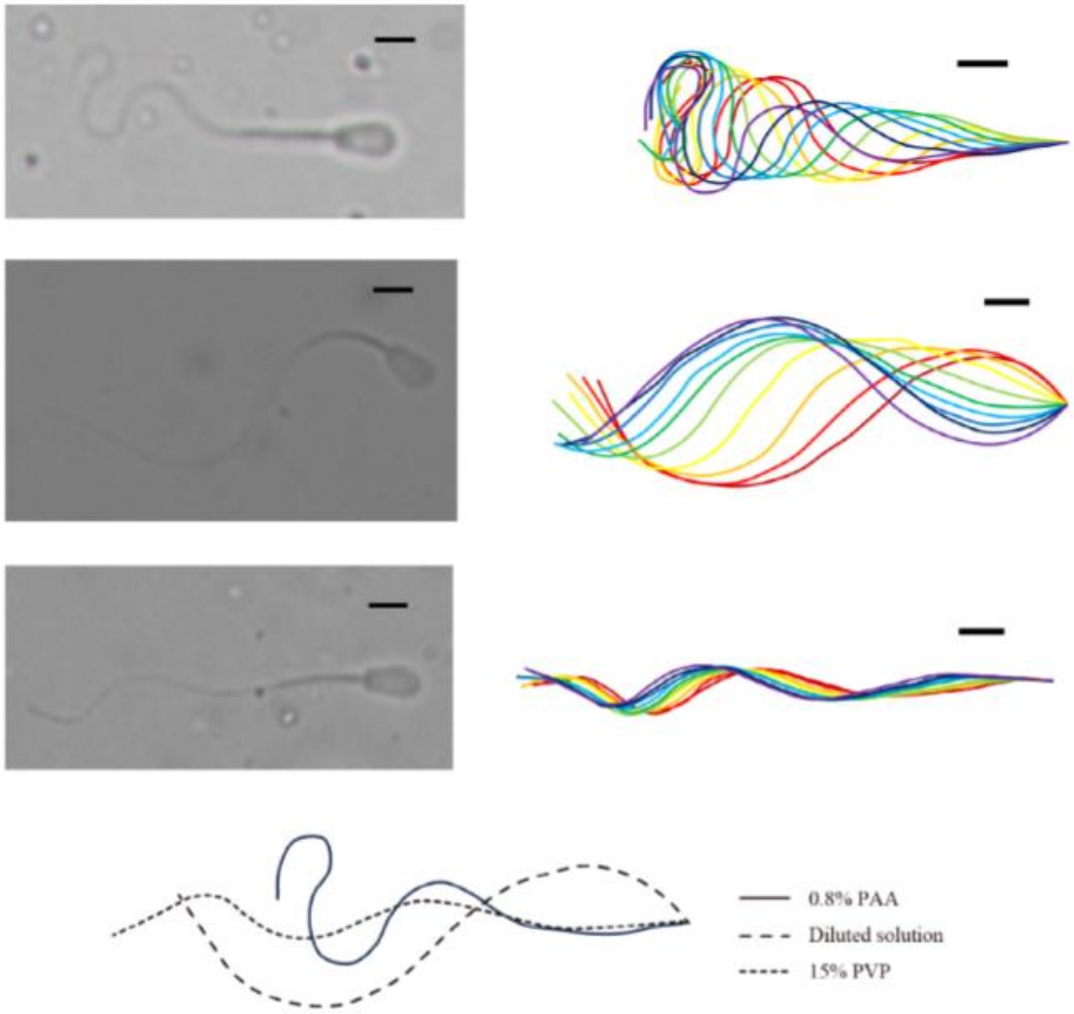

Hyakutake, Sato & Sugita. *J. Biomech.* 2019.

$$-\nabla p + \boldsymbol{\nabla} \cdot \boldsymbol{\tau} = \mathbf{0}$$
$$\boldsymbol{\nabla} \cdot \boldsymbol{u} = 0$$

$\mathrm{Re} = \frac{\rho U L}{\mu} \to 0$, steady, incompressible

Newtonian

$$\boldsymbol{\tau} = 2\mu \boldsymbol{D}(\boldsymbol{u})$$

Viscoelastic

$$\boldsymbol{\tau} + \lambda_1 \overset{\nabla}{\boldsymbol{\tau}} = 2\mu_0[\boldsymbol{D}(\boldsymbol{u}) + \lambda_2 \overset{\nabla}{\boldsymbol{D}}(\boldsymbol{u})]$$

Shear-dependent viscosity

$$\boldsymbol{\tau} = 2\mu(\dot{\gamma})\boldsymbol{D}(\boldsymbol{u})$$

The hybrid approach to non-linear swimming

The boundary value problem

$$\boldsymbol{\nabla} \cdot [2\mu(\dot{\gamma}(\boldsymbol{u}))\boldsymbol{D}(\boldsymbol{u})] - \nabla p + \boldsymbol{F} = \boldsymbol{0} \quad \text{in } \Omega$$

$$\boldsymbol{\nabla} \cdot \boldsymbol{u} = 0 \quad \text{in } \Omega$$

$$\boldsymbol{u} = \boldsymbol{0} \quad \text{on } \partial\Omega$$

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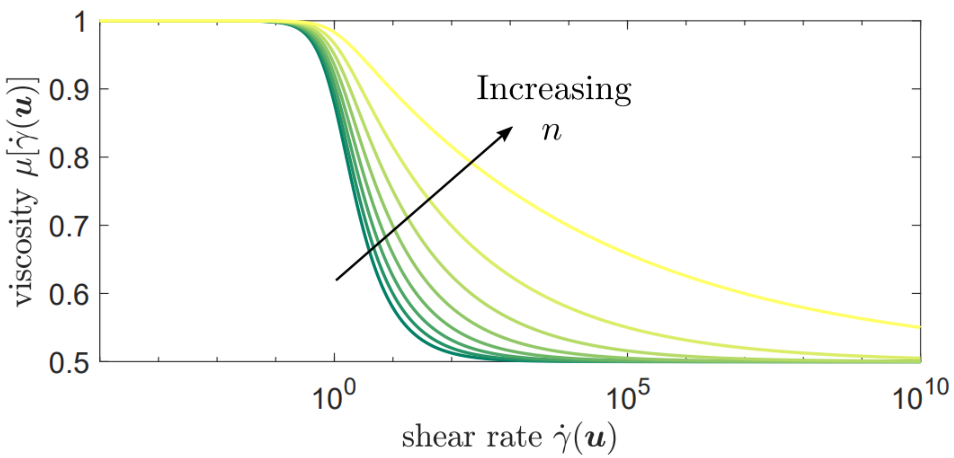
Constitutive law (Bird-Carreau-Cross)

$$\mu(\dot{\gamma}) = \mu_\infty + [\mu_0 - \mu_\infty][1 + [\lambda\dot{\gamma}]^2]^{\frac{n-1}{2}}$$



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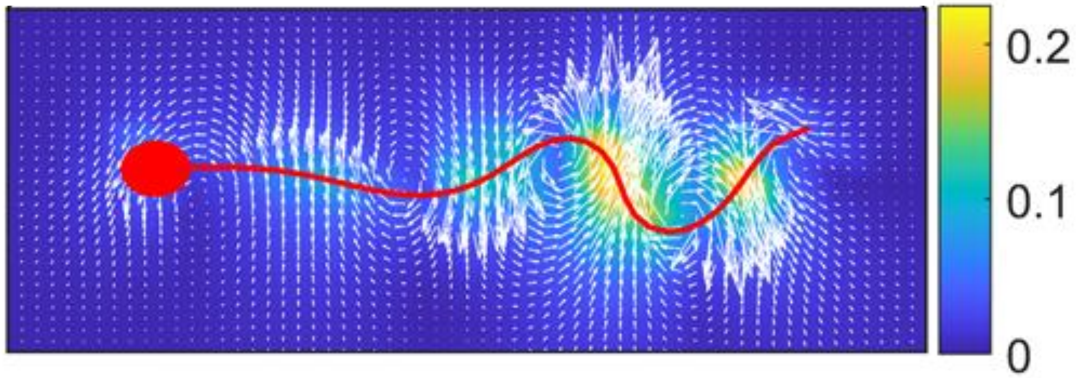
Fluid flow around a swimmer varies rapidly



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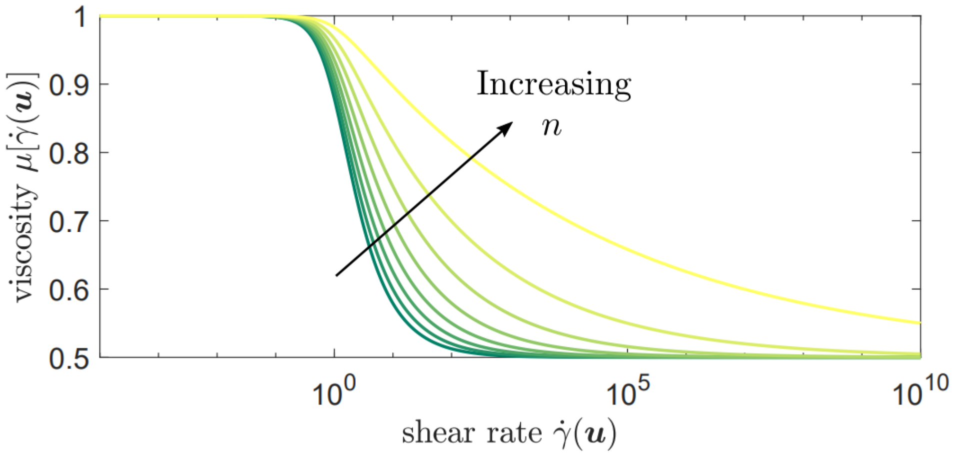

The hybrid method

$$\boldsymbol{u} = \boldsymbol{U} + \boldsymbol{u}_s$$

$$p = P + p_s$$

Non-Newtonian 'correction'

Newtonian 'approximation'

The hybrid approach to non-linear swimming: example

The boundary value problem

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$$\boldsymbol{\nabla} \cdot \boldsymbol{u} = 0 \quad \text{in } \Omega$$

$$\boldsymbol{u} = \boldsymbol{0} \quad \text{on } \partial\Omega$$

Newtonian flow approximation

If $\boldsymbol{F} = \boldsymbol{f}\delta(\boldsymbol{x} - \boldsymbol{y})$, (when $\mu(\dot{\gamma}) := \mu_N$)

$$\boldsymbol{u}_s = S_{ij}(\boldsymbol{x}, \boldsymbol{y})\boldsymbol{f}_j = \frac{1}{8\pi\mu_N}\left[\frac{\delta_{ij}}{r} + \frac{r_i r_j}{r^3}\right] f_j,$$

$$p_s = P_j(\boldsymbol{x}, \boldsymbol{y})\boldsymbol{f}_j = \frac{1}{4\pi}\left[\frac{r_j}{r^3}\right] f_j,$$

where $r_i = x_i - y_i,\ r = |\boldsymbol{x} - \boldsymbol{y}|$.

The hybrid approach to non-linear swimming: example

The boundary value problem

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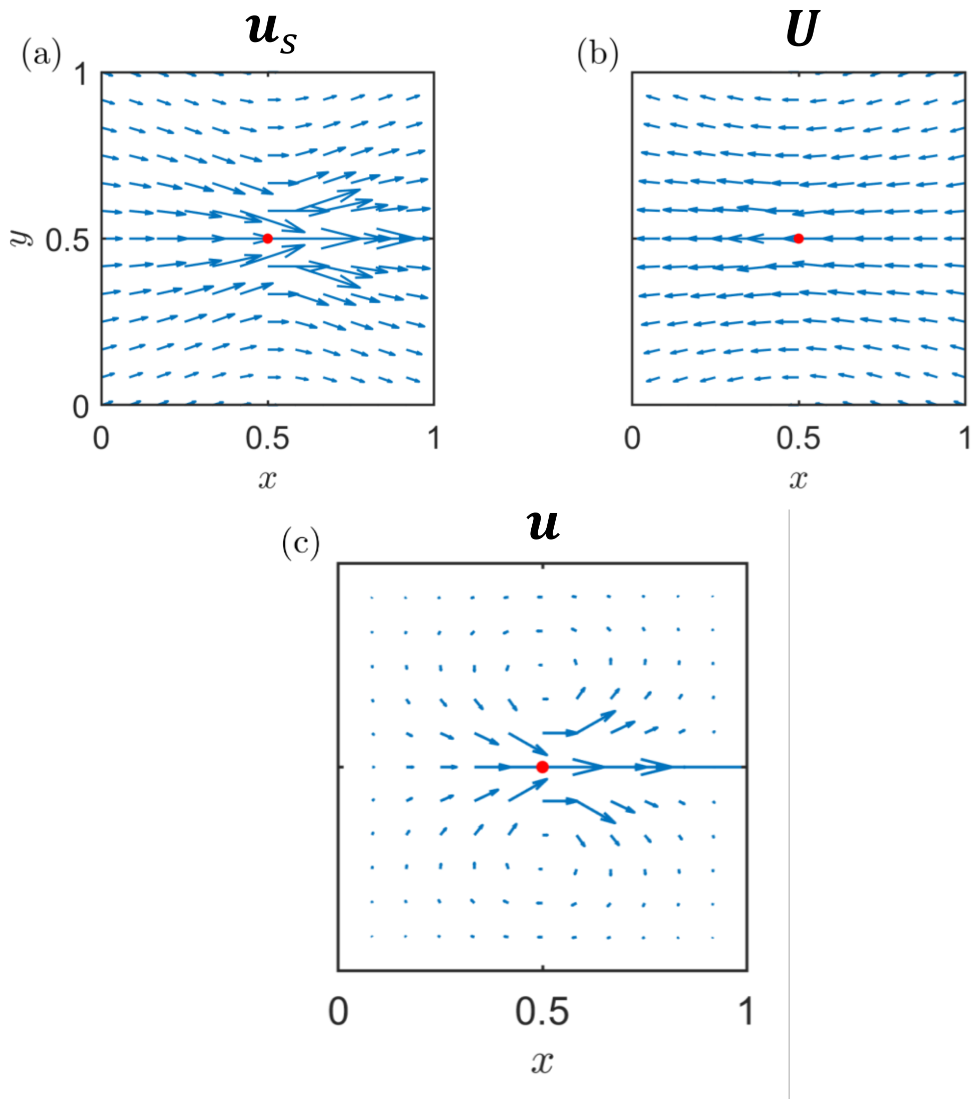
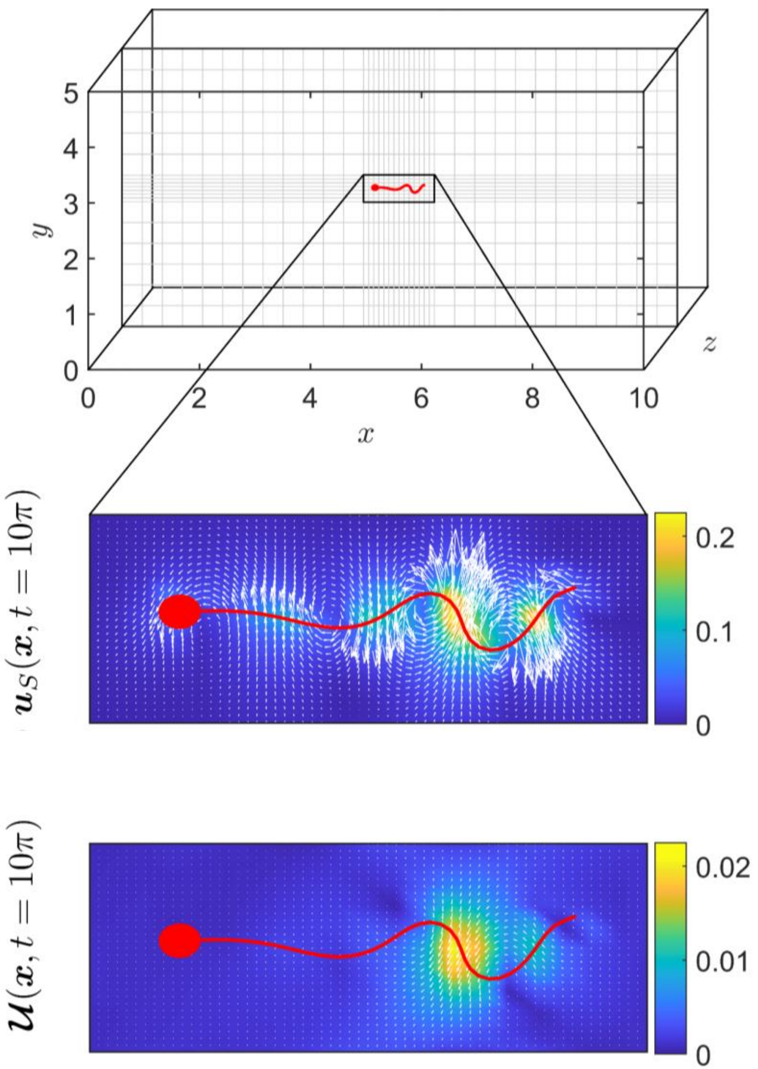


$n = 0.5, \lambda = 1, \mu_0 = 1, \mu_\infty = 0.5$

The effect of shear-thinning rheology on sperm propulsion

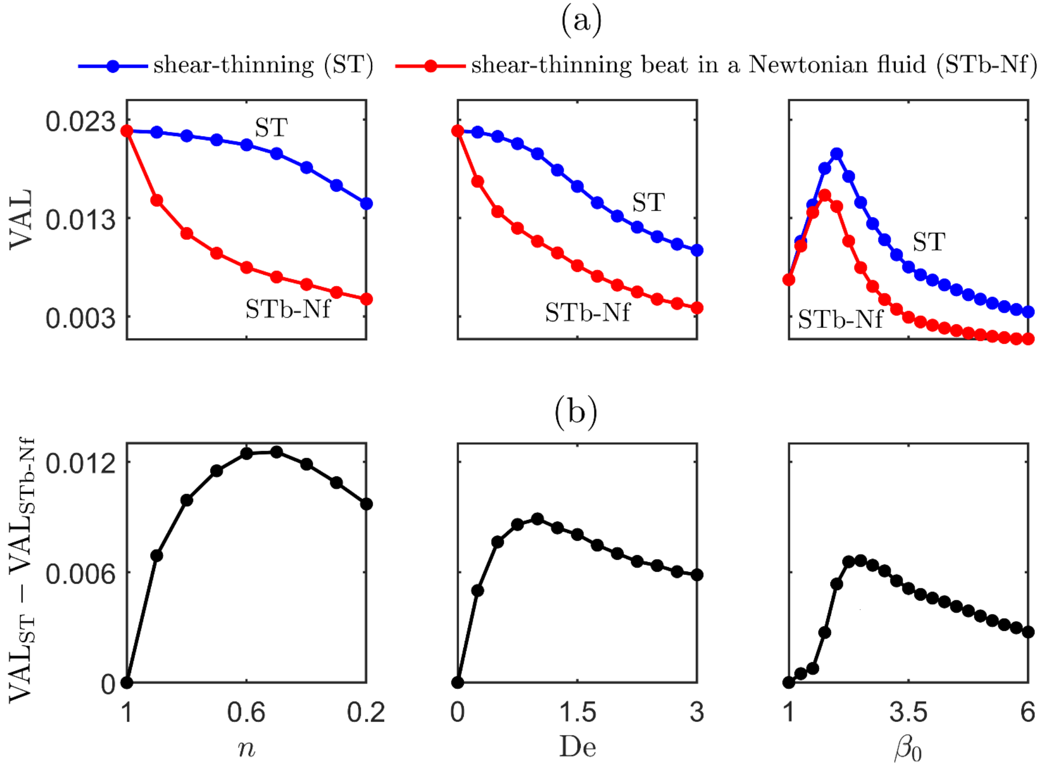


$n = 0.5, \lambda = 1, \mu_0 = 1, \mu_\infty = 0.5$

$$\mu(\dot{\gamma}) = 1 + [\beta_0 - 1][1 + [\mathrm{De}\,\dot{\gamma}]^2]^{\frac{n-1}{2}}$$

$$\mathrm{De} = \lambda\omega$$

$$\beta_0 = \frac{\mu_0}{\mu_\infty}$$



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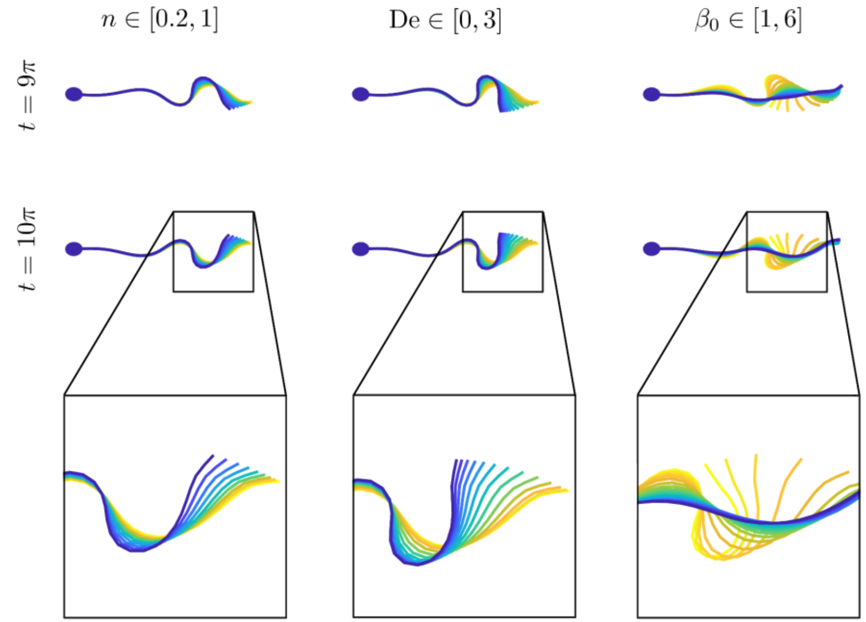
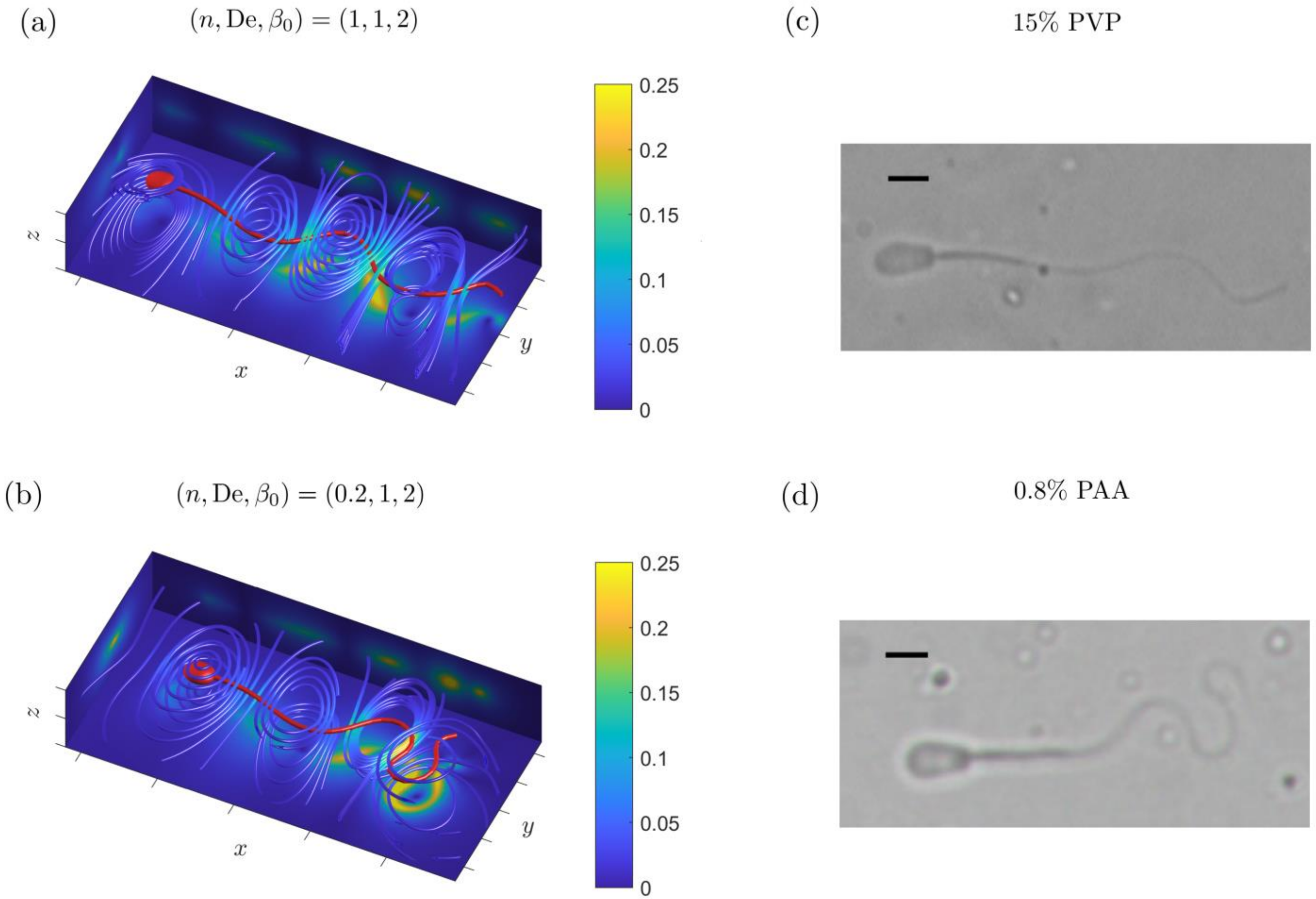
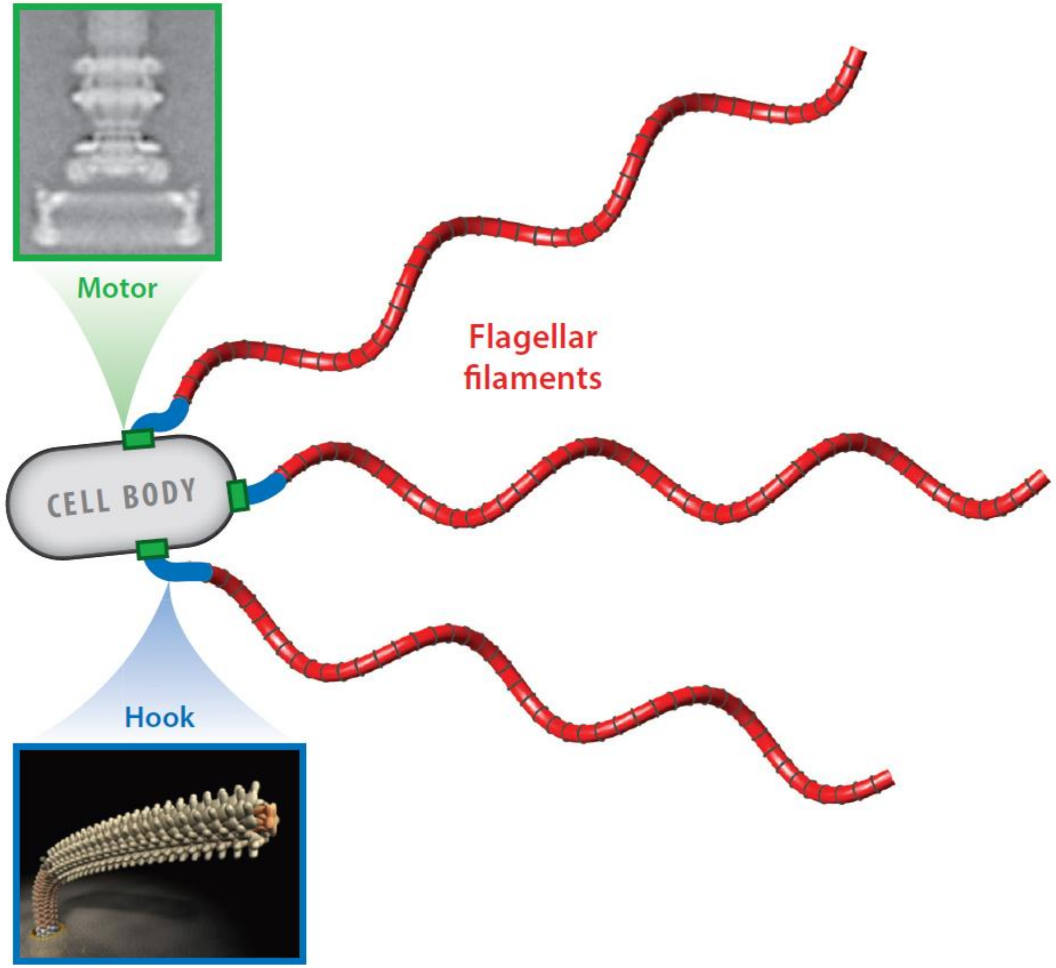
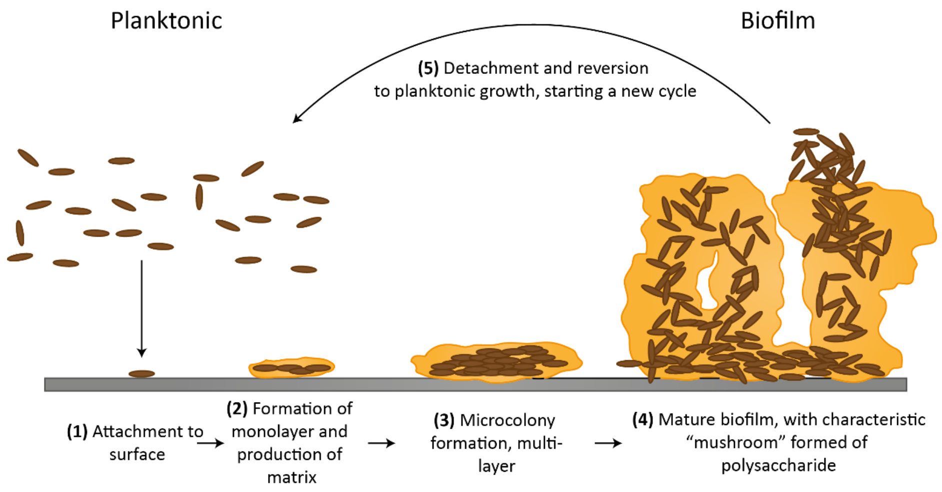




Bacterial locomotion and biofilm formation in non-Newtonian fluids



E. Lauga. *Annu. Rev. Fluid Mech.* 2016.



Vasudevan. *J. Microbiol. Exp.* 2014.

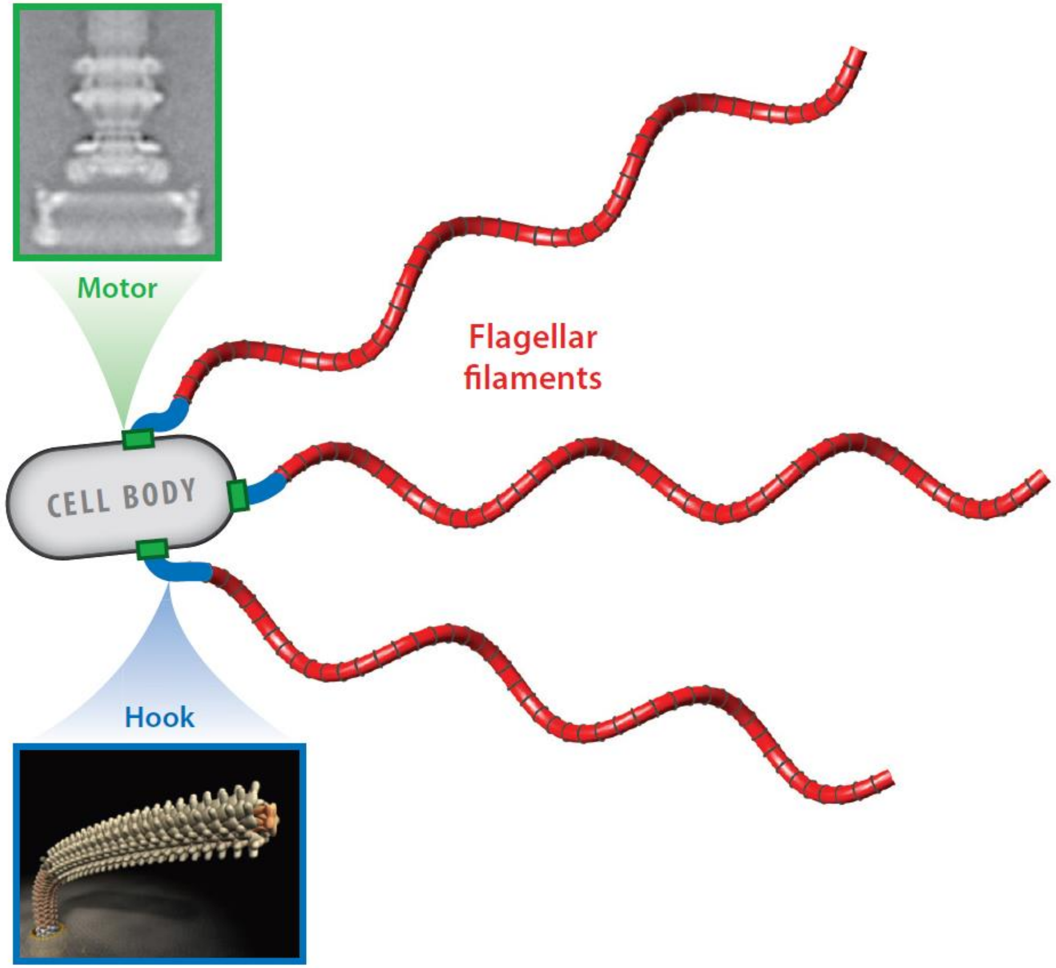
$$\boldsymbol{u} = \boldsymbol{U} + \boldsymbol{u}_s$$

$$p = P + p_s$$

Alter: viscoelastic + shear-thinning effects

Alter: bacterium model with rigid flagellum

The bacterium model



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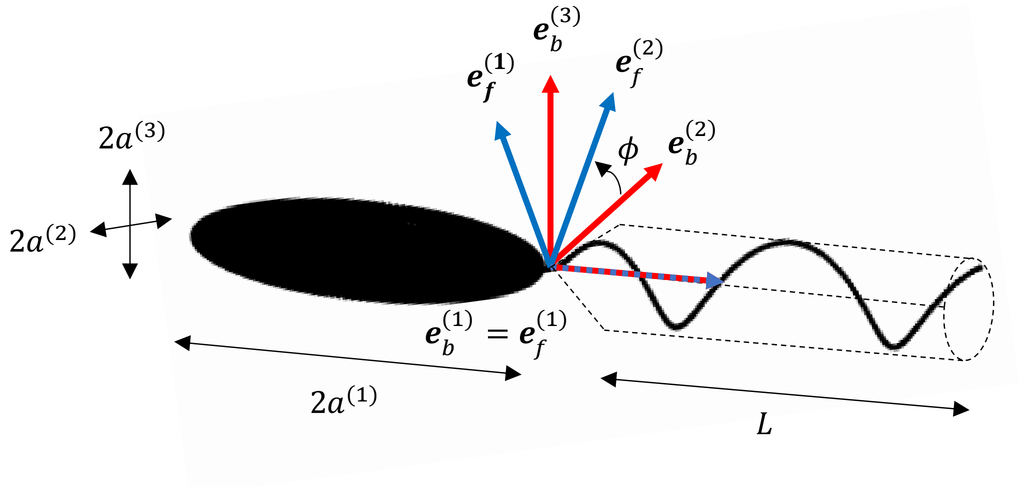


Motion is driven through a prescribed relative rotation rate between the body and flagellum

$$\omega = \frac{d\phi}{dt}.$$

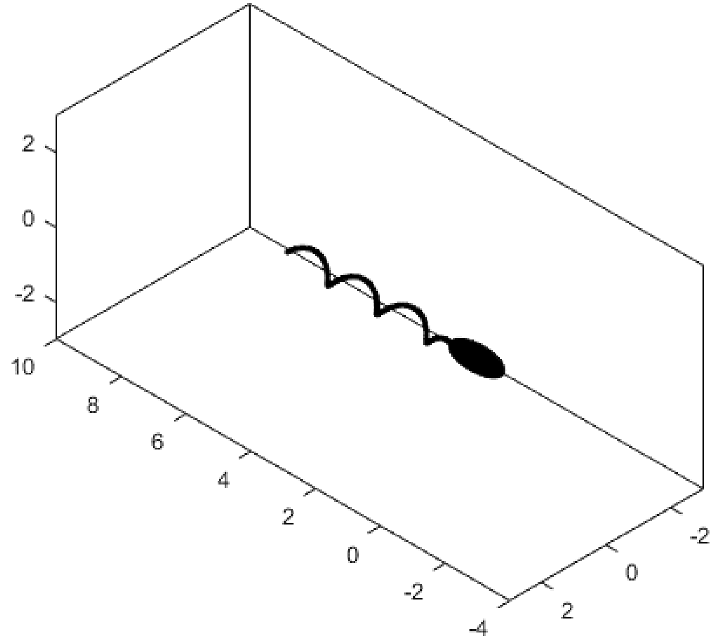
The surface velocity of points on the body and flagellum are described by

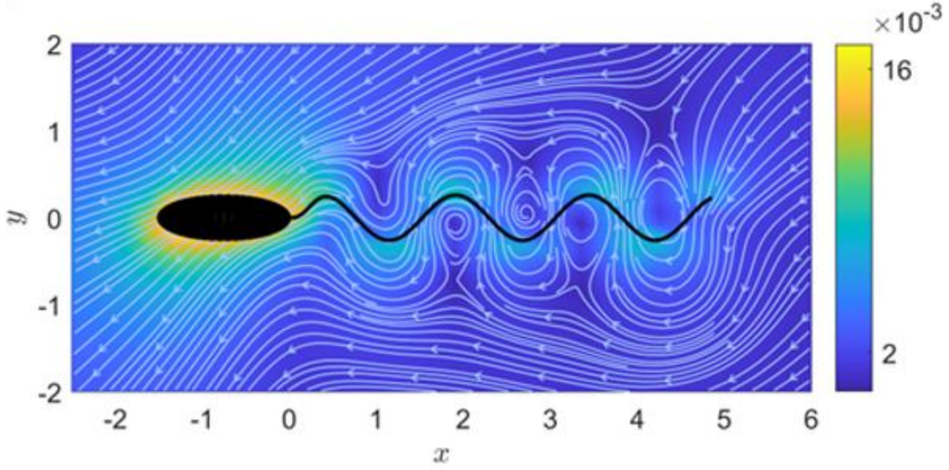
$$u(\boldsymbol{Y}) = \boldsymbol{V} + \boldsymbol{\Omega}_b \times [\boldsymbol{Y} - \boldsymbol{X}_0],$$

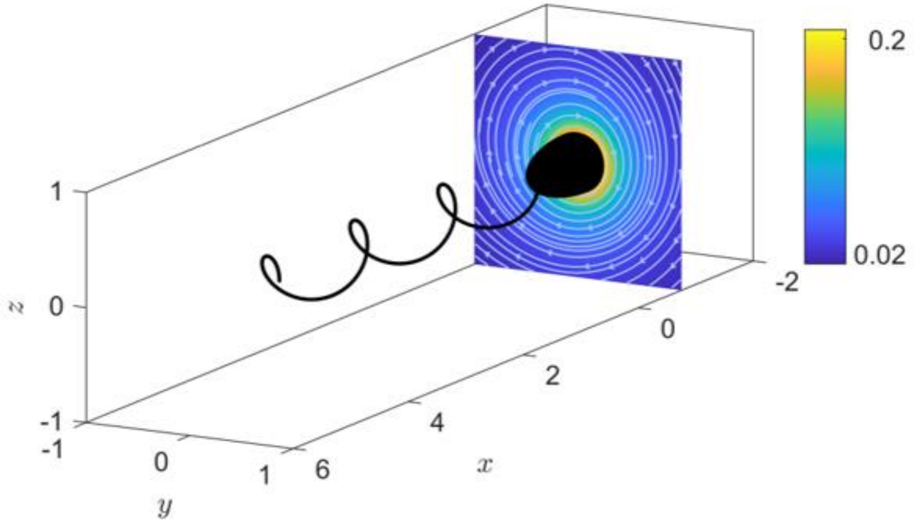
$$u(\boldsymbol{X}) = \boldsymbol{V} + [\boldsymbol{\Omega}_b + \boldsymbol{\Omega}_m] \times [\boldsymbol{X} - \boldsymbol{X}_0].$$

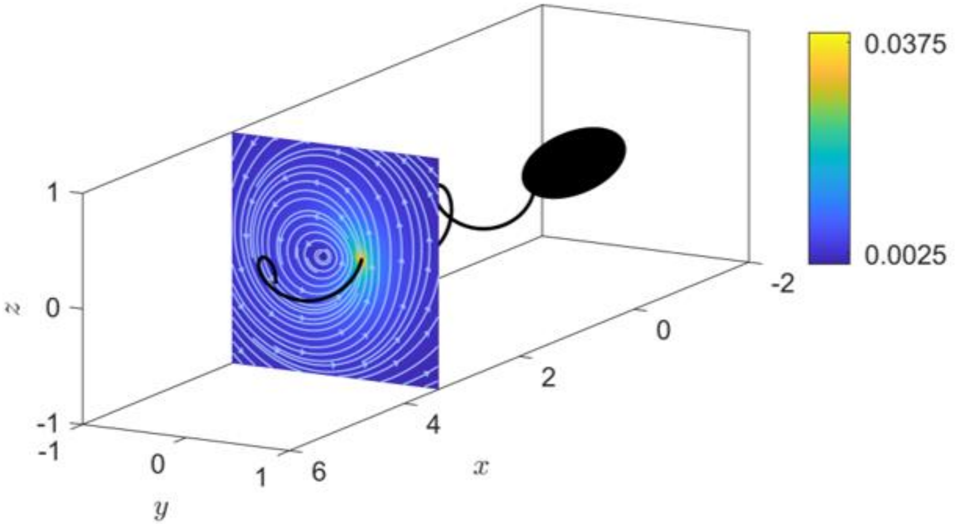
$(\boldsymbol{\Omega}_m = [\omega, 0,0]^T)$

The bacterium model







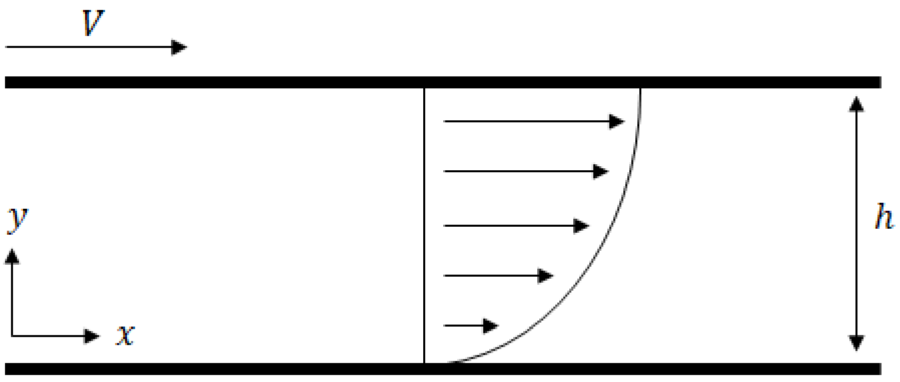


Couette-Poiseuille flow of a Giesekus fluid

Giesekus model

$$\boldsymbol{\tau} + \frac{\alpha\lambda}{\eta}(\boldsymbol{\tau}\cdot\boldsymbol{\tau}) + \lambda\overset{\triangledown}{\boldsymbol{\tau}} = 2\eta\boldsymbol{D}$$

The parameters λ and η are the fluid relaxation time and viscosity respectively. The parameter $0 < \alpha < 1$ is the dimensionless mobility.



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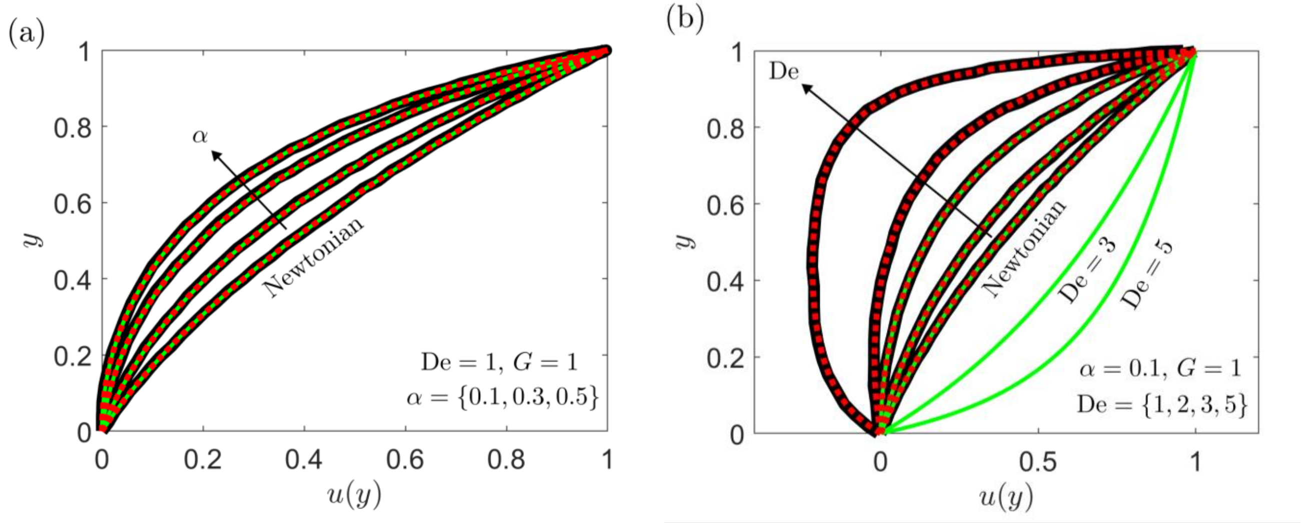
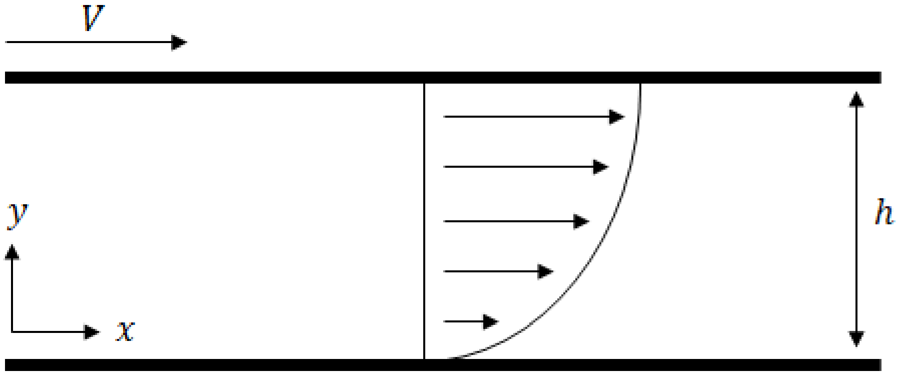


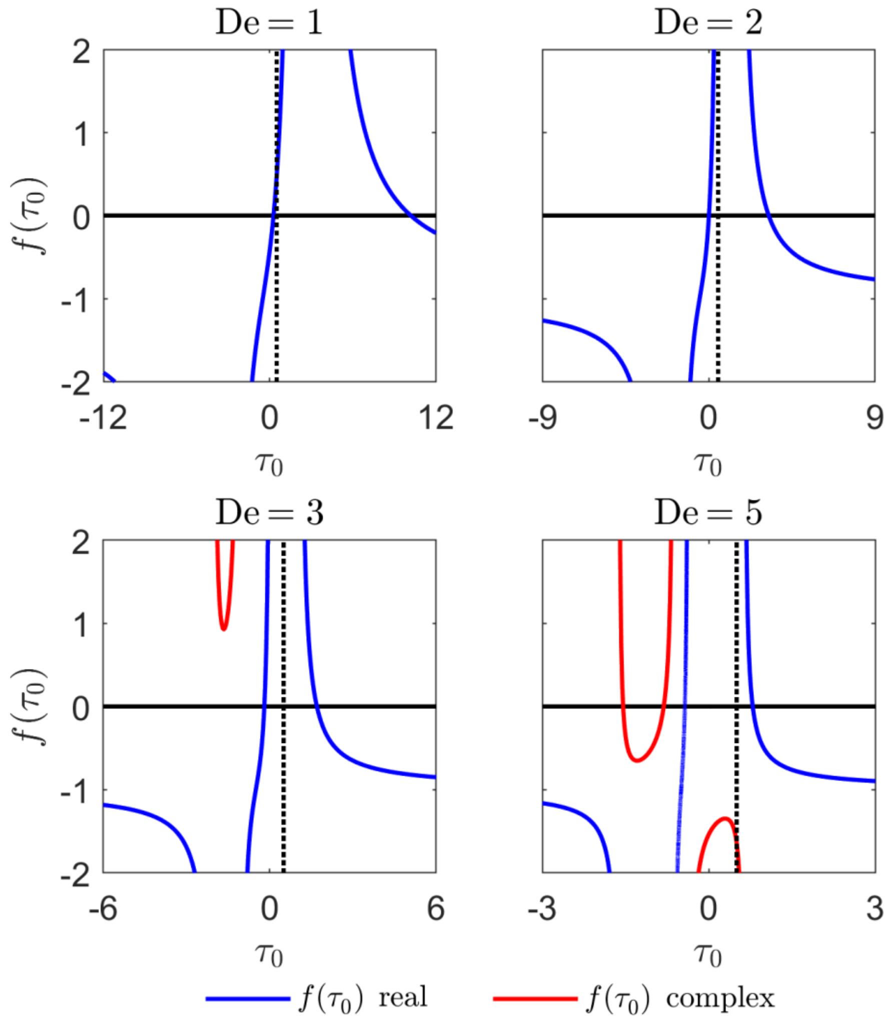
$$u(y) = -\frac{1}{2\alpha G \mathrm{De}^2}\left[\frac{2(\alpha-1)}{1-\alpha\mathrm{De}^2(\tau_0+Gy)^2} + (2\alpha-1)\ln\left(1-\alpha\mathrm{De}^2(\tau_0+Gy)^2\right)\right] + C$$

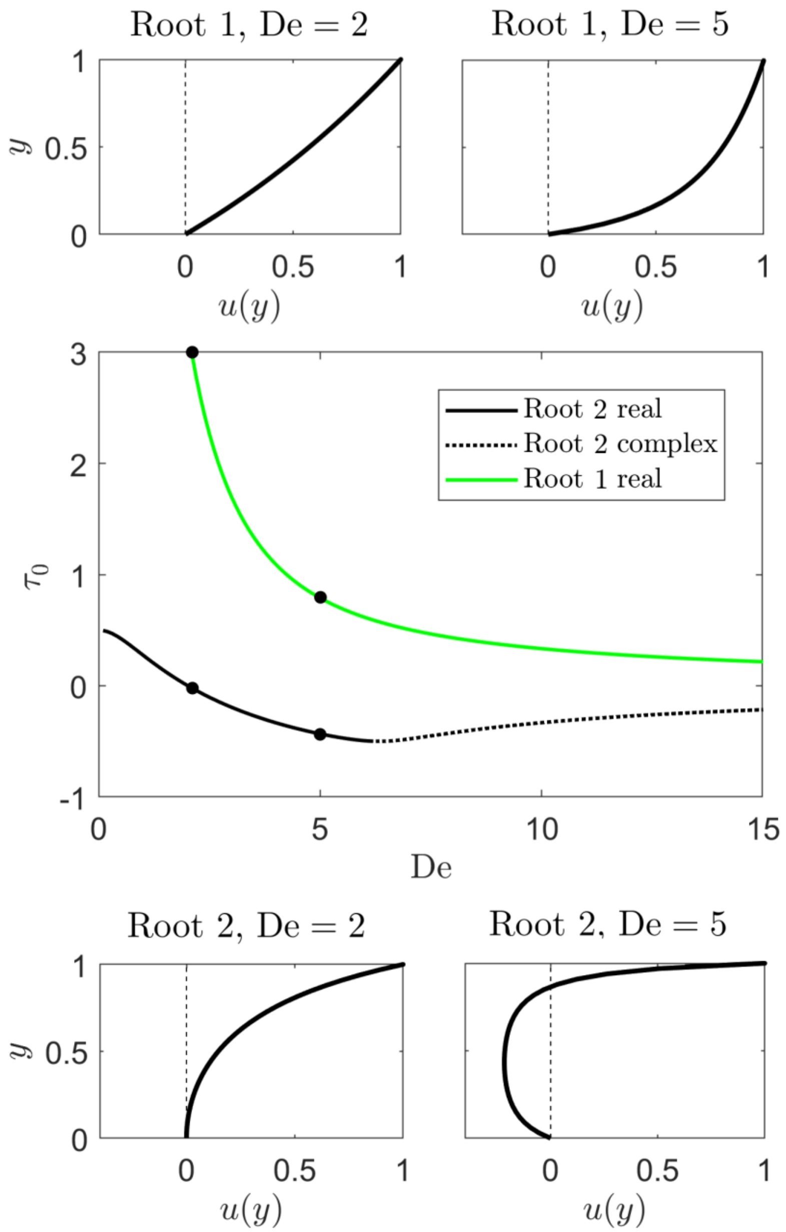


$$\frac{2\alpha-1}{2\alpha\mathrm{De}^2 G}\ln\left(\frac{1-\alpha\mathrm{De}^2\tau_0^2}{1-\alpha\mathrm{De}^2(\tau_0+G)^2}\right) - \frac{(\alpha-1)(G+2\tau_0)}{(1-\alpha\mathrm{De}^2\tau_0^2)(1-\alpha\mathrm{De}^2(\tau_0+G)^2)} - 1 = 0$$

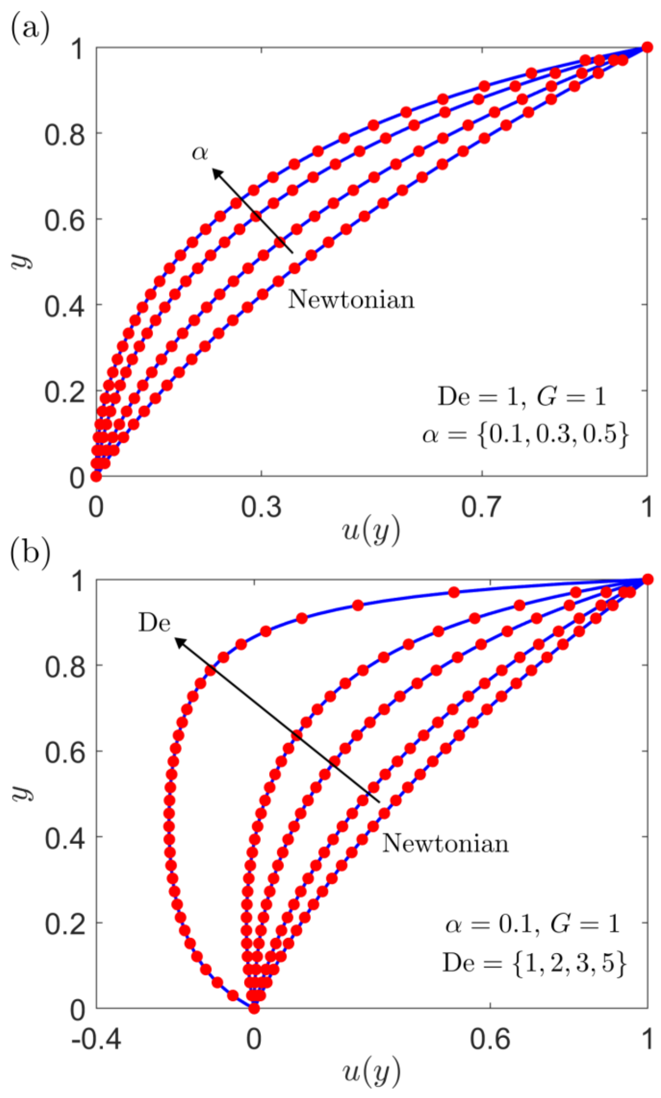
A. Raisi. *Rheol. Acta.* 2008.

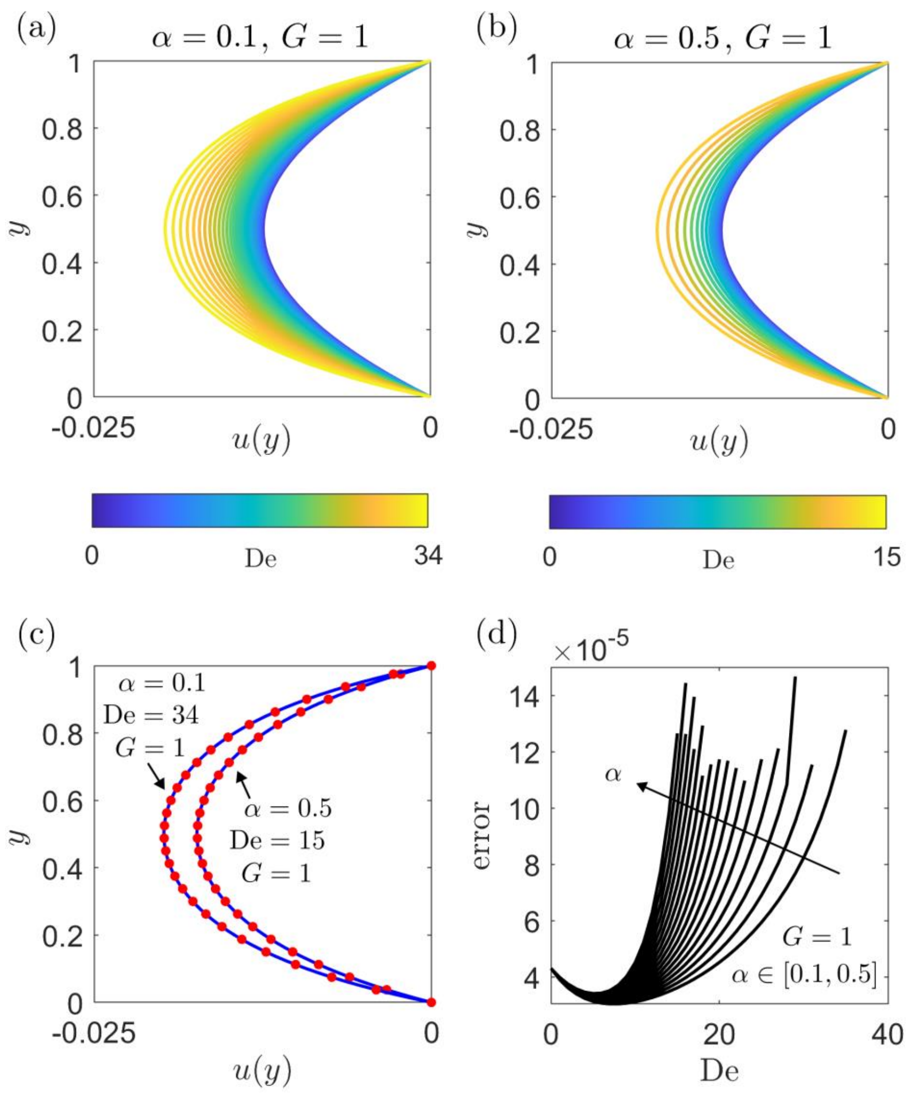
Extracting the 'physical' flow solution





Comparing the analytical and numerical solutions





Conclusions & future work

Conclusions

- Using a hybrid computational approach, we can efficiently simulate sperm in shear-thinning fluids. A similar technique will allow for the study of bacteria in fluids that exhibit both shear-thinning and viscoelastic properties.

- For sperm cells, shear-thinning rheology tends to hinder propulsion compared to swimming in a Newtonian fluid. This is likely due to flagellar shape changes emerging from fluid-flagellum interactions.

- Even for simple flow problems, obtaining the flow profile of viscoelastic fluids can be difficult. For the Couette-Poiseuille flow of a Giesekus fluid, multiple analytic solutions exist, although our analysis determines that only one solution is physical.

Future work

- To fully implement the hybrid method for the case of modelling bacteria in shear-thinning viscoelastic fluids (and near solid boundaries).

- To explore the effect of non-Newtonian fluids on bacterial locomotion and biofilm development.

Thanks for listening!

Questions?