Exploring the role of water stress and soil on banana-plantain production: a mathematical modelling approach

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Outline

1. Context
2. Biological background
3. Mathematical modeling and numerical simulation
4. Conclusion and Perspective

Context

Introduction

Agriculture and climate change are internally correlated with each other in various aspects, as climate change is the main cause of biotic and abiotic stresses, which have adverse effects on the agriculture of a region.

Motivation

- Bananas constitute the fourth most important crop of the world after rice, wheat, and maize and they form the world's second most important traded fruit after citrus.
- Water stress influence Production rate of banana-plantain.

Objectives

- Improve the understanding of the impact of water stress on banana plantain
- Propose solutions in terms of modelling, monitoring, prediction and prevention of water stress in the culture.

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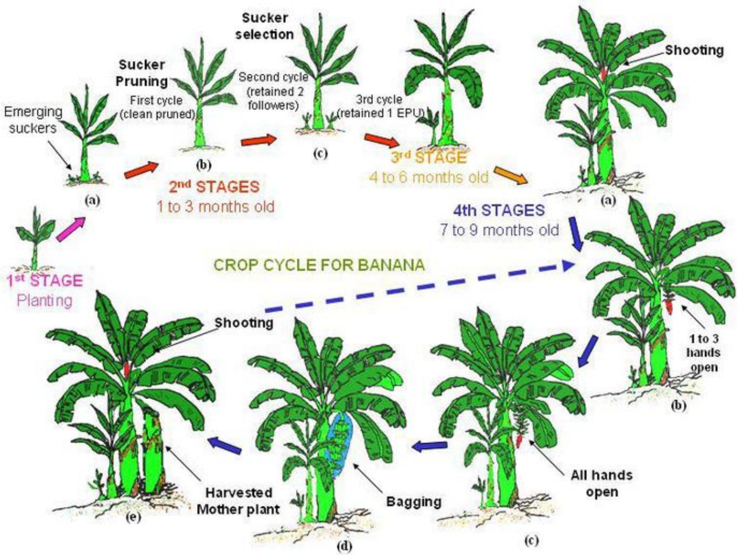
Specific objective

The goal is to come up with innovative solutions to increase the production of plants within a plantation.

Biological background

Figure: Banana plantain plant: Technical Manual

Biological background: Cycle of Banana-Plantain [1]



[1] "Crop cycle for banana plant" Ms. Gretel Guerrero done 2005, Lapanday.

Biological background

Fruits

Figure: The fruits of the banana plantain

Biological background

Water stress of banana plantain

Water stress of banana plantain is a term used to describe when demand for water is greater than the amount of water available at a certain period in time, and also when water is of poor quality and this restricts its usage.

Figure: Plant affect by water stress: Technical Manual

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Mathematical model

Hypotesis

- The recruitment of water comes from rain and irrigation.
- water has the same effect on the plants.

Variable

- E_s: Concentration of water in the soil
- P_v: Vegetative banana-plantain plants
- P_M: Unstressed mature banana-plantain plants
- P_s: Stressed banana-plantain plants
- V: Plant compensation
- P_{r1}: Production of unstressed plants
- P_{r2}: Production of stressed plants

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Flowchart

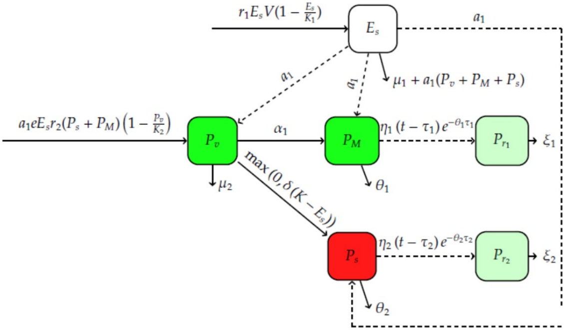

Figure: Flowchart of the model where $\theta_1 = \dfrac{\mu_3}{a_1 e E_s(t) P_M(t) + 1}$ and

$\theta_2 = \mu_4 + \dfrac{\theta}{a_1 e E_s(t) P_s(t) + 1}$. of the model.

Equations and basic properties

Equations

$$
\begin{cases}
\dot{E}_s(t) = r_1 E_s(t)\left(1 - \dfrac{E_s(t)}{K_1}\right) V(t) - a_1\left(P_v(t) + P_M(t) + P_s(t)\right) E_s(t) \\
-\mu_1 E_s(t), \\
\dot{P}_v(t) = r_2 e a_1 E_s(t)\left(P_M(t) + P_s(t)\right)\left(1 - \dfrac{P_v(t)}{K_2}\right) - (\alpha_1 + \mu_2) P_v(t) \\
-\max\left(0, \delta(K - E_s(t))\right) P_v(t), \\
\dot{P}_M(t) = \alpha_1 P_v(t) - \dfrac{\mu_3}{a_1 e E_s(t) P_M(t) + 1} P_M(t), \\
\dot{P}_s(t) = \max\left(0, \delta(K - E_s(t))\right) P_v(t) - \left(\mu_4 + \dfrac{\theta}{a_1 e E_s(t) P_s(t) + 1}\right) P_s(t), \\
\dot{V}(t) = \alpha(1 - V(t)) + \beta P_s(t), \\
\dot{P}_{r_1}(t) = \eta_1 e^{-\theta_1 \tau_1} P_M(t - \tau_1) - \varepsilon_1 P_{r_1}(t), \\
\dot{P}_{r_2}(t) = \eta_2 e^{-\theta_2 \tau_2} P_s(t - \tau_2) - \varepsilon_2 P_{r_2}(t).
\end{cases} \tag{1}
$$

Basic Properties

Positivity of solution and boundedness of trajectories

Model system (1) is a dynamical system on the biologically feasible compact domain:

$$\Omega = \Omega_E \times \Omega_P \times \Omega_V \times \Omega_{P_r}, \tag{2}$$

$$\Omega_E = \left\{ E_s \in \mathbb{R}^+, E_s \leq K_1 \right\}, \tag{3}$$

$$\Omega_P = \left\{ (P_v, P_M, P_s) \in \mathbb{N}^3, P_v \leq K_2,\ P_M = \frac{K_2 \alpha_1}{\mu_3} \text{ and } P_s \leq \frac{K_2 K \delta}{\mu_4} \right\}, \tag{4}$$

$$\Omega_V = \left\{ V \in \mathbb{R}_+ V \leq \frac{\alpha \mu_4 + \beta \omega K K_2}{\alpha \mu_4} \right\}, \tag{5}$$

$$\textit{and } \Omega_{P_r} = \left\{ (P_{r_1}, P_{r_2}) \in \mathbb{R}_+^+, P_{r_1} \leq \frac{\eta_1 K_2 \alpha_1}{\mu_3 \varepsilon_1} \text{ and } P_{r_2} \leq \frac{\eta_2 K_2 K \delta}{\mu_4 \varepsilon_2} \right\}, \tag{6}$$

Mathematical analysis

Case when there is water stress within the plantation i.e. $\max(0, \delta(K - E_s(t))) = \delta(K - E_s(t)$

$$
\begin{cases}
\dot{E}_s = r_1 E_s \left(1 - \dfrac{E_s}{K_1}\right) V - a_1(P_v + P_M + P_s)E_s - \mu_1 E_s, \\
\dot{P}_v = r_2 a_1 e E_s(P_M + P_s)(1 - \dfrac{P_v}{K_2}) - (\alpha_1 + \mu_2)P_v - \delta(K - E_s)P_v, \\
\dot{P}_M = \alpha_1 P_v - \dfrac{\mu_3}{a_1 e E_s(t) P_M(t) + 1} P_M, \\
\dot{P}_s = \delta(K - E_s)P_v - \left(\dfrac{\theta}{a_1 e E_s P_s + 1} + \mu_4\right) P_s, \\
\dot{V} = \alpha(1 - V) + \beta P_s.
\end{cases}
\tag{7}
$$

Equilibria and stabilities

Let:

$$E_s^0 = k_1 \left(1 - \frac{r_1}{\mu_1}\right) \quad (8)$$

Equilibria

- With only water: $Q_0^{(1)} = (E_s^0, 0, 0, 0, 1)^T$.
- with water and plants: $\overline{Q} = (\overline{E}_s, \overline{P}_v, \overline{P}_M, \overline{P}_s, \overline{V})$.

Threshold

$$\mathcal{N}_0^{(1)} = r_2 e a_1 E_s^0 \frac{\alpha_1(\mu_4 + \theta) + \delta \mu_3 (K - E_s^0)}{(\theta + \mu_4)\mu_3(\mu_2 + \alpha_1 + \delta(K - E_s^0))}. \quad (9)$$

Where $\mathcal{N}_0^{(1)}$ present the average quantity of water that plant must absorb for its growth.

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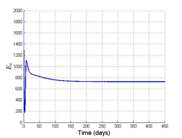
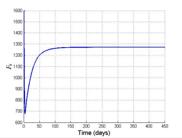
Numerical Analysis

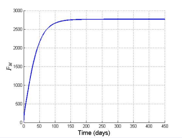
Numerical simulation



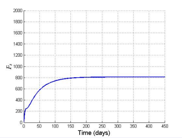
(E_s)



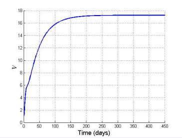
(P_v)



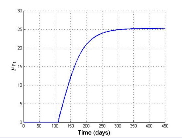
(P_M)



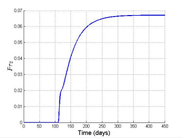
(P_s)



(V)



(P_{r1})



(P_{r2})

Figure: Simulation of model system (7) (so that $\mathcal{N}_0^{(1)} = 13.8739$).

Production

For 1600 regets of bananas plantains planted on 1*hectare* the field can produce about 25.16*tons* of stressed banana plantains and about 0.06*tons* of unstressed banana plantains.

Mathematical analysis

Case when there is no water stress within the plantation

In this case, $\max(0, \delta(K - E_s)) = 0$ which implies that $P_s = 0$, $V = 1$ and system (1) reduces to

$$\begin{cases} \dot{E}_s(t) = r_1 E_s(t)\left(1 - \dfrac{E_s(t)}{K_1}\right) - a_1\left(P_v(t) + P_M(t)\right)E_s(t) - \mu_1 E_s(t), \\ \dot{P}_v(t) = r_2 e a_1 E_s(t) P_M(t)\left(1 - \dfrac{P_v(t)}{K_2}\right) - (\alpha_1 + \mu_2) P_v(t), \\ \dot{P}_M(t) = \alpha_1 P_v(t) - \dfrac{\mu_3}{a_1 e E_s(t) P_M(t) + 1} P_M(t). \end{cases} \tag{10}$$

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Equilibria

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- With water and plants: $Q^* = (E_s^*, P_v^*, P_M^*)$

Threshold

$$\mathcal{N}_0^{(2)} = \frac{\alpha_1 e a_1 r_2 E_s^0}{\mu_2 \mu_3 + \alpha_1 \mu_3} \tag{11}$$

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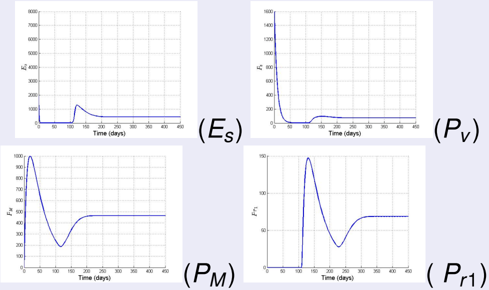


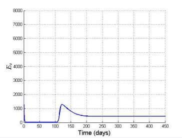
Figure: Simulation of model system (10) (so that $\mathcal{N}_0^{(1)} = 3.7787$).

Production

This means that we are in ideal situation for a good production of banana-plantains. For 1600 regets of banana-plantains planted on 1 *hectare*, the banana-plantains production is approximately 68.65 *tons*.

Numerical analysis

Numerical simulation



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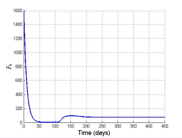

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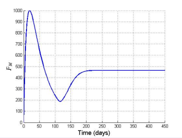
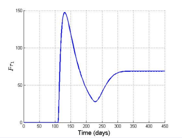


(P_M)



$(\ P_{r1})$

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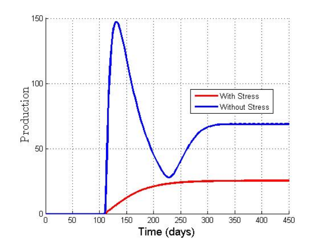
Impact of water stress on banana-plantains production



(a)

Figure: Impact of water stress on banana-plantains production

Table: Impact of water stress on banana-plantain production.

1600 **regets**	**Without water stress**	**With water stress**
Production (tons)	68.65	25.22
Percent (%)	100	36,73

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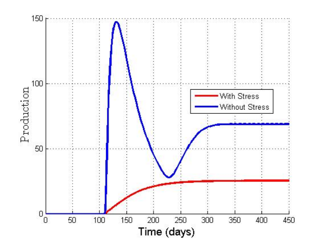

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Irrigation

Equation

$$\left\{\begin{array}{rcl}
\dot{E}_s(t) & = & g(t) + r_1 E_s(t)\left(1 - \frac{E_s(t)}{K_1}\right) V(t) - \mu_1 E_s(t) \\
& & -a_1 \left(P_v(t) + P_M(t) + P_s(t)\right) E_s(t), \\
\dot{P}_v(t) & = & r_2 e a_1 E_s(t)\left(P_M(t) + P_s(t)\right)\left(1 - \frac{P_v(t)}{K_2}\right) - (\alpha_1 + \mu_2) P_v(t) \\
& & -\max\left(0, \delta(K - E_s(t))\right) P_v(t), \\
\dot{P}_M(t) & = & \alpha_1 P_v(t) - \frac{\mu_3}{e a_1 E_s(t) P_M(t) + 1} P_M(t), \\
\dot{P}_s(t) & = & \max\left(0, \delta(K - E_s(t))\right) P_v(t) - \left(\mu_4 + \frac{\theta}{a_1 e E_s(t) P_s(t) + 1}\right) P_s(t), \\
\dot{V}(t) & = & \alpha(1 - V(t)) + \beta P_s(t), \\
\dot{P}_{r_1}(t) & = & \eta_1 e^{-\theta_1 \tau_1} P_M(t - \tau_1) - \varepsilon_1 P_{r_1}(t), \\
\dot{P}_{r_2}(t) & = & \eta_2 e^{-\theta_2 \tau_2} P_s(t - \tau_2) - \varepsilon_2 P_{r_2}(t).
\end{array}\right. \tag{12}$$

Irrigation function

1. If $t \leq T_1$
   - On $[0, T[$

$$g(t) = \begin{cases} f & \text{if} \quad t < \tau, \\ 0 & \text{otherwise.} \end{cases} \tag{13}$$

   - For all $n \in \mathbb{N}^*$ such that $t \in [nT, (n+1)T[$,

$$g(t) = \begin{cases} f\sqrt{n} & \text{if} \quad t - nT < \tau, \\ 0 & \text{else if.} \end{cases} \tag{14}$$

2. If $t > T_1$ and $\forall n \in \mathbb{N}^*$, $t \in [nT, (n+1)T]$,

$$g(t) = \begin{cases} f\sqrt{\dfrac{T_1}{N}} & \text{if} \quad t < \tau, \\ 0 & \text{else if.} \end{cases} \tag{15}$$

Irrigation

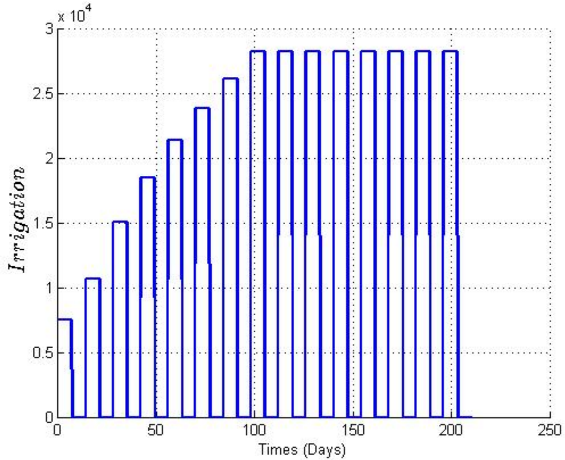

Figure: Time evolution of the concentration of water coming from the a period $T = 14$

Numerical analysis

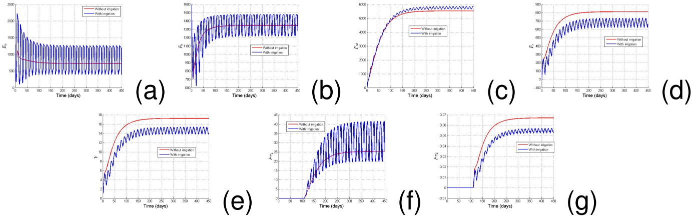

Figure: Simulation of model system (12).

Table: The impact of irrigation on banana plantain production.

	Without water stress	With water stress	With irrigation
Harvest (Kg)	68 650	25 220	41 550
PU (Fcfa)	150	150	150
PT (Fcfa)	10 297 500	3 783 000	6 232 500
Loss (Fcfa)	-	6 514 500	4 065 000
Gaim (Fcfa)	-	-	2 449 500

Impact of soil on banana-plantains crops affected by water stress

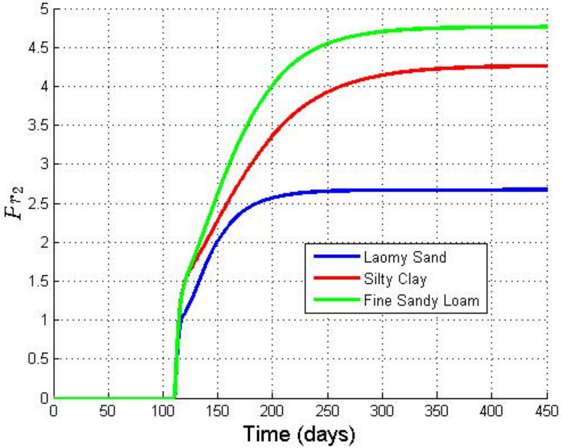

Figure: Banana-plantains production for three different type of soil in the presence of water stress

Impact of soil on banana-plantains crops affected by water stress

Table: The impact of soil

Soil	Loamy sand	Silty clay	Fine Sandy Loam
Harvest (Kg)	25 900	33 500	36 890
PU (Fcfa)	150	150	150
PT (Fcfa)	3 885 000	3 850 500	5 533 500
Loss (Fcfa)	6 390 000	6 424 500	4 741 500

Conclusion and Perspective

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Perspective

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- Use the tools of artificial intelligence.

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Acknowledgements

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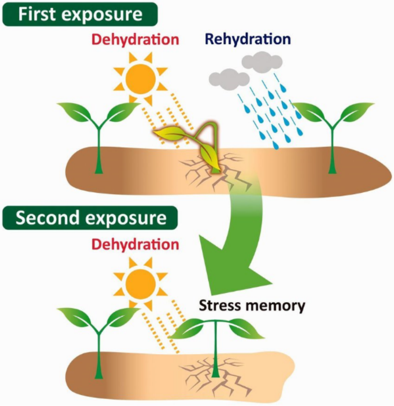

Figure: Stress plants (Source:Technical Manual).

Thank you for your kind attention!