Almost Every Mating is a Carpet

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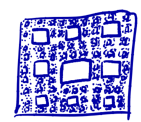
Liverpool 6/22/22

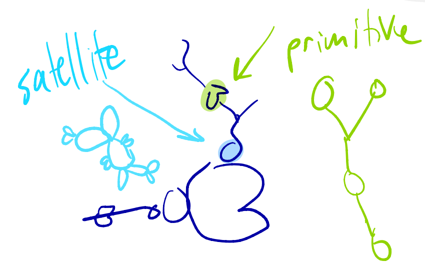
I] Background

a) hyperbolic components of the Mandelbrot set

b) matings

c) carpets





II] Statements of Results

(joint w/ Insong Park)
95% in prep.

III] Outline of Proof

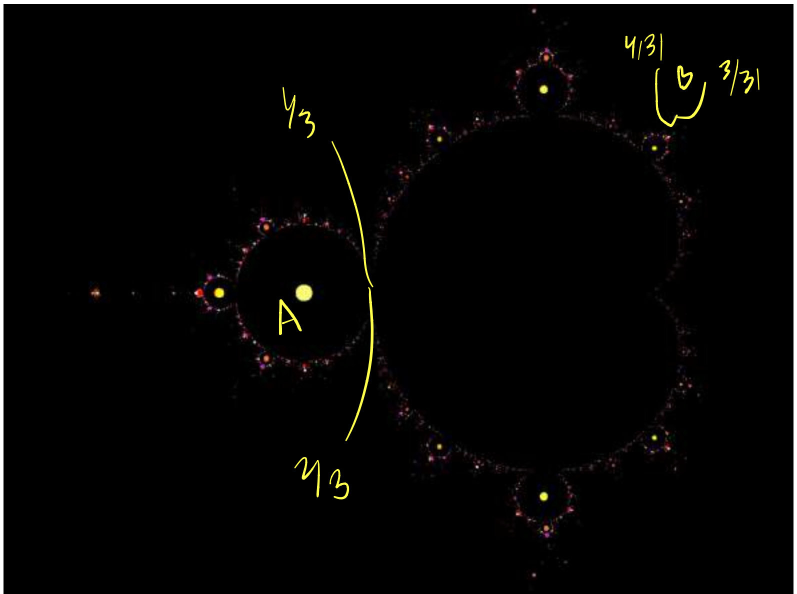
a) carpet criterion
b) stable carpet obstructions
c) unstable carpet obstructions

IV] Interpretation to $Per_n(0)$

(if time permits)

Ia] Background: Hyperbolic components of the Mandelbrot set

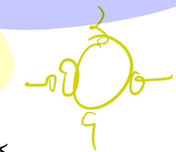
☆ everything in our talk today will be quadratic, PCF, hyperbolic



PCF hyperbolic guys really "see" all of M!

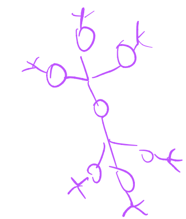
- Two kinds of hyperbolic PCF:

(1) satellite $\rightarrow \exists$ touching Fatou components



(A)

(2) primitive $\rightarrow \forall$ disjoint Fatou components



(B)

- By the numbers:

approx. 2^{n-1} n-cycles in total

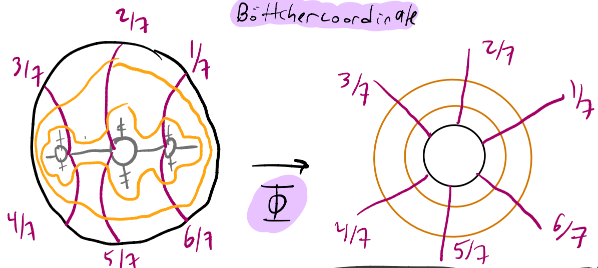
$\sim 2^{n-1}$ primitive vs $< 2^{\frac{n}{2}-1}$ satellite

Ib) Background: Matings of Polynomials

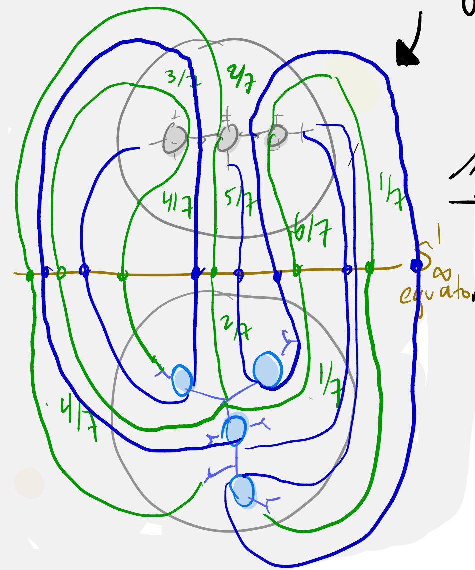
Key: If f is locally connected:

$$f \circlearrowright J_f \xrightarrow{\Phi} \overline{\mathbb{D}} \circlearrowright z^2$$

Böttchercoordinate



formal mating



$\simeq$

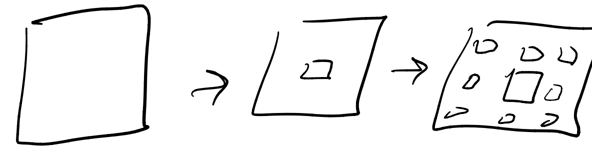
geometric mating

To **mate** f & g:

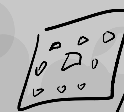
Glue J_f and J_g along (conjugate) external rays

Chéritat's slow mating videos!!!

Ic) Background: Carpets



Most basic example of Sierpinski carpet: square carpet

Topologically (Whyburn's criterion for $\Sigma \neq \emptyset$ to be homeo to )

1. Σ is connected → follows for matings
2. Σ is compact → free in rat'l dynamics
3. Σ is nowhere dense → true if $J_f \neq \hat{\mathbb{C}}$
4. Σ is locally connected → MSS: yes if $\overline{P_f} \cap J_f = \emptyset$
5. Σ complement regions $\{U_i\}$ satisfy
   - a. $\overline{U_i} \cap \overline{U_j} = \emptyset$!
   - b. ∂U_i is a simple closed curve → true for bdd Fatou comp's

Lemma (D-P):

Suppose $\Sigma = J_{f \sqcup g}$ then

$\boxed{5a} \iff \neg\exists$ chain of periodic rays connecting Fatou components of f or g

II] Statements of Results

$P(\le k) := \{\text{Polynomials w/ period} \le k\}$

Let f, g be PCF hyperbolic quadratic polynomials with $f \in P(\le n)$ & $g \in P(\le m)$.

Suppose $f \sqcup g$ is rational.

Let $P_{n,m} = \text{Prob}(f \sqcup g \text{ is a carpet})$

$$:= \frac{|\{f \sqcup g \text{ is a carpet}\}|}{|P(\le n)||P(\le m)|}$$

Thm ① As $n, m \to \infty$, $P_{n,m} \to 1$

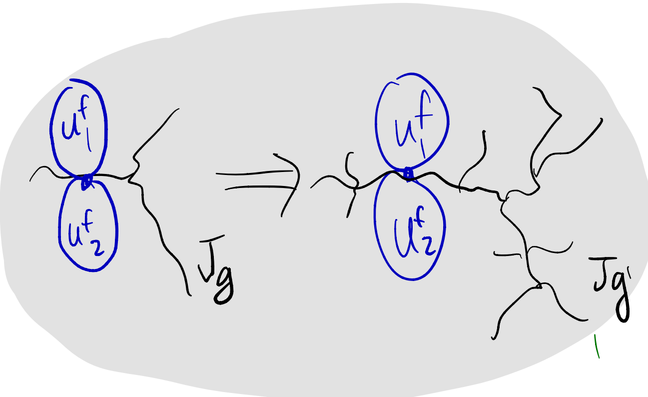
Let $P_m(f) = \text{Prob}\left(\begin{array}{c} g \in P(\le m): \\ f \sqcup g \text{ is a carpet} \end{array}\right)$

Thm ② A.E. f, as $m \to \infty$,

$$P_m(f) \to 1$$

ex] $P(\text{Airplane}) = 1 - 2/7$

III a Outline of Proof: Core Cases



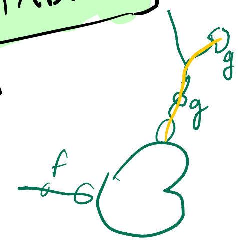
Milnor: wake containment

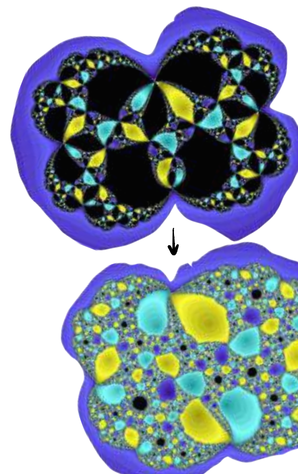
$\Rightarrow$

If 2 Fatou components of f touch on J_g, Then they also touch on $J_{g'}$, $\forall g' > g$

$\rightsquigarrow$ $\boxed{f \sqcup g \text{ " } \Downarrow f \sqcup g' \text{ " }}$

(f)-STABLE





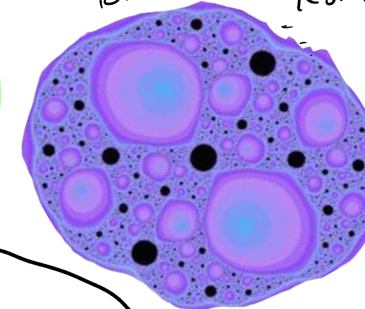
$f \sqcup g$

$\downarrow$

$f \sqcup g'$

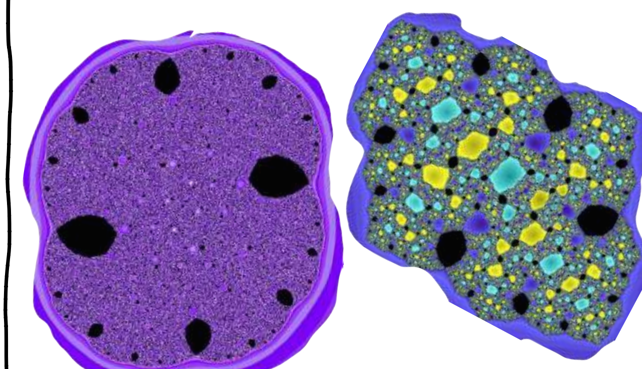
f or g satellite

Basilica ⊔ Koko



UNSTABLE

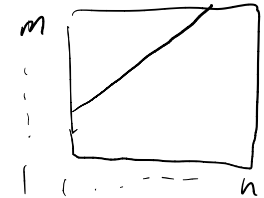
A component of f touches a component of g



III b — Outline of Proof: Doing the Count

① **Lemma**: All f-stable obstructions have "originating" f-stable obstruction: $f \sqcup g_0$

Moreover, g_0 is satellite



② **Lemma**: unstable $\Rightarrow$ $n = k\,m$

③ $$\boxed{P_{n,m} = 1 - \frac{\{f \text{ or } g \text{ satellite}\}}{P(\le n)\,P(\le m)} - \frac{\{\text{stable}\}}{P(\le n)\,P(\le m)} - \frac{\{\text{unstable}\}}{P(\le n)\,P(\le m)}}$$

grows slower than $2^{\frac{n-1}{2}}\, 2^{\frac{m-1}{2}}$

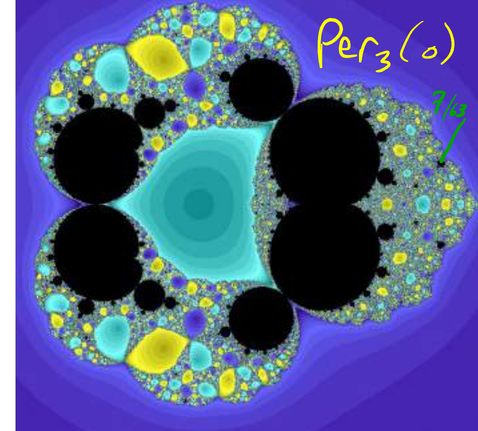
similar: Lemma $\Rightarrow$ "factors through" satellite

grows slower than $\mathrm{poly}(n,m) \cdot (2-\varepsilon)^n$

④ Denominators grow like $2^{n-1}\, 2^{m-1}$, so each term $\searrow 0$

IV

Interpretation in $\mathrm{Per}_n(0)$



Recall: $\mathrm{Per}_n(0)$ = all Quadratic rational maps w/ a superattracting n-cycle

↳ includes mating locus, in particular $ML(f)$ f w/ $\mathrm{per}(f)=n$

1. Interp of $P(f) = 1$ ⇝ no satellite shared matings

2. Buried hyp. comp ⟺ carpet (in progress)

3. Matings w/ angle landing on periodic Fatou component ⇒ a) unstable carpet obstruction

   b) hyp component off a bitransitive