Optimal mix between pay-as-you-go and funding for DC pension schemes in an overlapping generations model[1]

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Topic Coverage

Topic coverage

Aim

The aim of this paper is twofold:

- Show whether diversification between pay-as-you-go and funding is optimal in a multi-generational setting.
- Study whether we can add sensitive constraints which ensure individual's wealth-maximisation as well as system's equilibrium.

Overview of pension systems

- Basic financing techniques
  - Pay as you go (PAYG): current contributors pay current pensioners (Unfunded schemes)
  - Funding: contributions are accumulated in a fund which earns a market interest rate (Funded schemes)
- Benefit formulae
  - Defined Benefit: Pension is calculated according to a fixed formula which usually depends on the members salary and the number of contribution years.
  - Defined Contribution: Pension is dependent on the amount of money contributed and their return.

The financing choice is present for both DB and DC pension schemes.

	Pay-as-you-go	Funding
DB	Classical social security	Classical Employee DB Plan
DC	Notional Accounts (NDCs)	Pension savings accounts

⇒ What if we want to mix PAYG **and** Funding?

Why mixing?

Why mixing?

- Funding may have a higher expected return than payg return (especially in countries with negative population growth!)
- Mixing $\Rightarrow$ diversification ? (and under which conditions?)

In practice:

- Sweden, Latvia and Poland (Chłón-Domińczak et al. (2012) and Könberg et al. (2006)) already split individual contributions between funding and PAYG.
- Sweden allocates 86,5% of the contributions to PAYG, whereas Latvia and Poland allocate 70% and 62.6% respectively.

Literature review

What has been done:

- Dutta et al. (2000): Mean-variance framework with two generations.
- Merton (1983) and Storesletten et al. (1999): General equilibrium models but explicit allocations can't be derived.
- Devolder and Melis (2015): They obtain the optimal constant allocation in a stochastic framework for three generations.

Contributions to the literature:

- Allocation are both age and time dependent.
- The problem is studied in a multi-generational framework.
- We maximize individual's capital while ensuring liquidity.

Remark

How this problem differs between a two risky-asset allocation problem?

- Self-financing strategy hypothesis doesn't hold!
- Pay-as-you-go is a non-traded (but measurable) asset.
- Once allocated it has to be held until maturity (buy-and-hold strategy)

Topic coverage

Multi-generations setting

- Each individual aged x at time t retires at age x_r at time r_x and starts its career when aged x_0 at time s_x.
- Population aged x at time t is denoted by $L(x, t)$.
- The probability of attaining age x at time t, while being $x_0 < x$ now is $p_{(x_0,x)}(t)$.
- There are $M = x_r - x_0$ contributing generations and N retired generations coexisting at every moment and they all have finite lives.

Allocation problem

- A π percentage of their salary $W(t)$ is payed to the Public pension system.
- A $a(x,t)$ part of this contribution goes to funding, while the remaining part $1-a(x,t)$ goes to the PAYG system.
- PAYG contributions are denoted by:

$$C^P(t) = \pi W(t) \sum_{x=x_0}^{x_r-1} (1-a(x,t))L(x,t) \tag{1}$$

- and the PAYG pension expenditures by $O^P(t)$.
- The PAYG system is liquid if $C^P(t)+F^P(t) \geq O^P(t)$ where $F^P(t)$ denotes the PAYG buffer fund.

Pension portfolio (1/2)

The pension portfolio of the individual will be denoted by $Z(x, t, s_x)$ and is based on the relative wealth:

$$RW(x_0, s_{x_0}, r_{x_0}) = \frac{X(x_0, s_{x_0}, r_{x_0})}{L(x_r, r_{x_0})W(r_{x_0})} = \frac{\pi}{p_{(x_0,x_r)}(r_{x_0})} Z(x_0, s_{x_0}, r_{x_0})$$

The wealth is given by:

$$X(x_0, s_{x_0}, r_{x_0}) = \pi \underbrace{L(x_0, s_{x_0})}_{\text{initial cohort pop.}} \underbrace{W(s_{x_0})}_{\text{initial salary}} \sum_{i=s_{x_0}}^{r_{x_0}-1} \underbrace{p_{(x_0, x_0+i-s_{x_0})}(i)}_{\text{survival prob.}} \prod_{k=s_{x_0}+1}^{i} \underbrace{(1+g_k)}_{\text{salary increase}}$$

$$\times \left[a(x_0 + i - s_{x_0}, i) \prod_{j=i+1}^{r_{x_0}} \underbrace{(1+i_j)}_{\text{fund return}} + (1 - a(x_0 + i - s_{x_0}, i)) \prod_{j=i+1}^{r_{x_0}} \underbrace{(1+nr_j)}_{\text{PAYG return}} \right]$$

Pension portfolio (2/2)

The pension portfolio can be rewritten according to a moment of calculation different from the one associated to the start of the career.

$$
\begin{aligned}
Z(x,t,r_x(t)) = & \underbrace{\Big(Z^F(x,s_x(t),t-1) + p_{(x_0,x)}(t)a(x,t)\Big)}_{\text{current fund value}} \underbrace{F_{t+1,r_x(t)}}_{\text{deflated market rate}} \\
& + \underbrace{\Big(Z^P(x,s_x(t),t-1) + p_{(x_0,x)}(t)(1-a(x,t))\Big)}_{\text{current payg value}} \underbrace{D_{t+1,r_x(t)}}_{\text{deflated payg rate}} \\
& + \sum_{i=t+1}^{r_x-1} \underbrace{p_{(x_0,x+i-t)}(i)}_{\text{survival prob.}} \underbrace{\left(D_{i+1,r_x}(t) + a(x+i-t,i)(F_{i+1,r_x(t)} - D_{i+1,r_x(t)})\right)}_{\text{future uncertainty}}
\end{aligned}
$$

$\Rightarrow$ Expectation and variance can easily be calculated according to the convenient filtration.

Optimisations performed

1. Life-cycle optimisation (named ex-ante)
   - At the beginning of the career we calculate the life-cycle *deterministic* strategy with the information available.
   - We let this life-cycle depend on age and time.
2. Cross-sectional optimisation (named ex-post)
   - We maximize the contribution received by each cohort up until retirement.
   - We use the information gathered at time of calculation (filtration t).
   - We add liquidity constraints (contributions=outcome).
   - *Shortcoming*: Proportions are calculated as if there wouldn't be future contributions (whereas we have a high certainty about it).

We maximise in both cases the mean-variance function as follows:

$$U_t(Z) = \mathrm{E}_t[Z] - \frac{\gamma}{2}\mathrm{Var}_t[Z]$$

where γ is the risk aversion parameter.

Ex-ante optimal allocation

Proposition

Our maximisation problem can be stated as follows:

$$\max_{\{a_1\}} U_{s_{x_0}}[Z(x_0, s_{x_0}, r_{x_0})] \tag{2}$$

And the optimal solution is:

$$\begin{aligned} a_1(x_0 + i - s_{x_0}, i) = & \frac{\mathrm{Var}_{s_{x_0}}[D_{i+1,r_{x_0}}] - \mathrm{Cov}_{s_{x_0}}[F_{i+1,r_{x_0}}, D_{i+1,r_{x_0}}]}{\mathrm{Var}_{s_{x_0}}[F_{i+1,r_{x_0}} - D_{i+1,r_{x_0}}]} \\ & + \frac{1}{\gamma} \frac{\mathrm{E}_{s_{x_0}}[F_{i+1,r_{x_0}} - D_{i+1,r_{x_0}}]}{p_{(x_0, x_0+i-s_{x_0})}(i)\mathrm{Var}_{s_{x_0}}[F_{i+1,r_{x_0}} - D_{i+1,r_{x_0}}]} \end{aligned} \tag{3}$$

- These proportions are deterministic.
- They only account for the information gathered at start of the career.

Ex-post optimal allocation (multiple cohorts) (1/3)

- The previous allocations don't take into account the path of returns.
- They can't take into account both liquidity and cohort's preferences.

In order to take these aspects into account, we can change our problem to:

$$\max_{\{a_2\}} \sum_{x=x_0}^{x_r-1} U_t[Z^*(x,t,s_x)] \tag{4}$$
$$\text{s.t. } C^P(t) = O^P(t)$$

where:

- $U_t[Z^*(x,t,s_x)]$ is the mean-variance value of the pension portfolio of an individual aged x at time t
- Z^* is the censored pension portfolio which ignores future contributions.

$\Rightarrow$ We maximise value at retirement simultaneously for all contributors in the economy.

Ex-post optimal allocation (multiple cohorts) (2/3)

Proposition

The solution of the maximisation problem (4) is:

$$a_2(x,t)^* = \underbrace{a_2(x,t)}_{\textit{unconstrained}} - \frac{\pi W(t) L(x,t)}{\gamma p_x(t)^2 \mathrm{Var}_t[F_{t+1;r_x} - D_{t+1;r_x}]} \cdot \underbrace{\frac{\overbrace{O^P(t) - \pi W(t) \sum_{j=x_0}^{x_r-1} (1 - a_2(j,t)) L(j,t)}^{C^P(t)}}{\sum_{j=x_0}^{x_r-1} \frac{\pi W(t) L(j,t)^2}{\gamma p(x_0,j,s_j)^2 \mathrm{Var}_t[F_{t+1;r_j} - D_{t+1;r_j}]}}}_{\textit{Lagrange multiplier}}$$

where,

$$a_2(x,t) = \frac{\left(Z^P(x,s_x,t-1) + p_{(x_0,x)}(t)\right) \mathrm{Var}_t[D_{t+1,r_x}] - Z^F(x,s_x,t-1) \mathrm{Var}_t[F_{t+1,r_x}]}{p_{(x_0,x)}(t) \mathrm{Var}_t[F_{t+1,r_x} - D_{t+1,r_x}]}$$
$$- \frac{\mathrm{Cov}_t[F_{t+1,r_x}, D_{t+1,r_x}] \left(Z^P(x,s_x,t-1) - Z^F(x,s_x,t-1) + p_{(x_0,x)}(t)\right)}{p_{(x_0,x)}(t) \mathrm{Var}_t[F_{t+1,r_x} - D_{t+1,r_x}]}$$
$$+ \frac{1}{\gamma} \frac{\mathrm{E}_t[F_{t+1,r_x} - D_{t+1,r_x}]}{p_{(x_0,x)}(t) \mathrm{Var}_t[F_{t+1,r_x} - D_{t+1,r_x}]}$$

Ex-post optimal allocation (multiple cohorts) (3/3)

The proportion invested in the **constrained** PAYG problem depends on the expenditure on pensions $O^P(t)$, the income from PAYG contributions $C^P(t)$ and the *unconstrained* allocations:

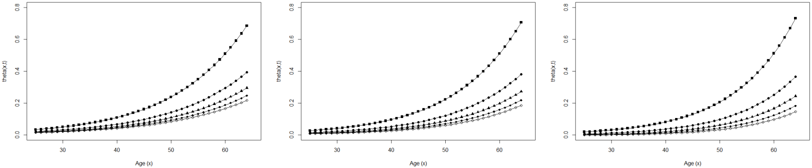
- If $C^P(t) = O^P(t) \rightarrow 1 - a_2(x,t)^* = 1 - a_2(x,t)$ and the liquidity constraint isn't necessary.
- If $C^P(t) > O^P(t) \rightarrow 1 - a_2(x,t)^* < 1 - a_2(x,t)$. The excess on contributions leads to a lower proportion invested in PAYG.
- If $C^P(t) < O^P(t) \rightarrow 1 - a_2(x,t)^* > 1 - a_2(x,t)$. The excess on expenditure leads to a higher proportion invested in PAYG.

Topic coverage

Numerical illustration

Figure: Ex-ante optimal proportions in funding for an entering cohort in presence of correlation. The proportions decrease as risk aversion increases from 1 to 5.

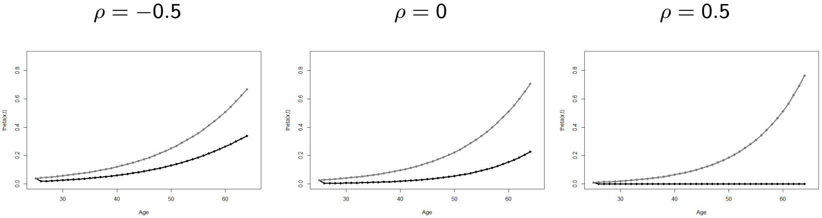
$\rho = -0,25$ $\rho = 0$ $\rho = +0,25$



- Not significantly affected by the correlation between the demographic and the financial processes.
- Funding allocation $\nearrow$ with age ('horizon effect' - uncertainty decreases the closer the individual is to retirement)
- Risk aversion does not affect the shape, but the level.

Numerical illustration

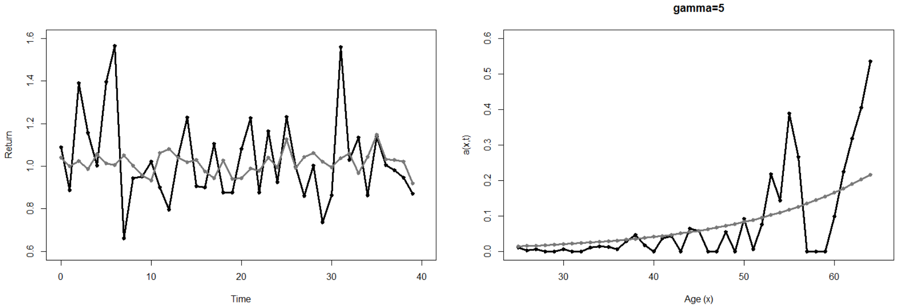
Figure: Ex-ante (grey line) versus Ex-post (black line) optimal allocations in funding: comparison for different levels of correlation (unconstrained case) and $\gamma = 1$



- Ex-post allocations vary more sharply.
- The proportion in funding goes from very high for negative correlations, to close to zero for positive correlations (reduced diversification if the demographic and financial asset are positively correlated).
- Ex-post sizably lower than ex-ante. The values narrow for $\gamma = 3$ and 5.

Numerical illustration

Figure: Ex-ante (circle) versus Ex-post (rhombus) optimal allocations: comparison for one simulated path and risk aversion coefficient set to $\gamma = 3$ (unconstrained case)



How do our proportions compare to reality?

Table: Weighted arithmetic mean of the allocation to funding ($\rho = -0.25$)[2]

	Base[3]		Low[4]		High[5]	
	Ex-ante	Ex-post	Ex-ante	Ex-post	Ex-ante	Ex-post
$\varphi = 1$	26.77%	11.01%	35.00%	5.40%	20.14%	20.25%
$\varphi = 3$	11.99%	10.52%	12.99%	4.51%	12.83%	20.06%
$\varphi = 5$	9.03%	10.42%	8.58%	4.33%	11.36%	20.03%

- Sweden, Latvia and Poland have age and time *independent* proportions to funding; namely, 13,5%, 30% and 34,4% respectively.
- Our values fall within the boundaries of what is done in practice.

[2] The empirical research shows that population and financial markets are negatively correlated (Poterba 2001).

[3] 'Base' case population growth: $\mu_D = 2\%$ and $\sigma_D = 5\%$.

[4] Low population growth: $\mu_D = 0.5\%$ and $\sigma_D = 3\%$.

[5] High population growth: $\mu_D = 3.5\%$ and $\sigma_D = 7\%$.

Topic coverage

Conclusion

- The ex-post proportion is much less affected by the risk aversion coefficient than its ex-ante counterpart.
- The correlation doesn't significantly affect the ex-ante proportions while it affects substantially the ex-post proportions.
- The analysis suggests also that there are no diversification benefits in the ex-post case when the correlation between the financial and demographic asset is positive.
- Liquidity constraints can be successfully added in the optimisation problem and imply sensible consequences (if the system is liquid it implies lower PAYG allocations and viceversa).
- Transition should be sensitively made in order to avoid the mixed system to become only PAYG through the liquidity constraint.

Thank you for your attention!

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