

# Large Duration Asymptotics in Bivariate Survival Models with Unobserved Heterogeneity

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In some survival analysis, characterizing tail properties is of vital importance because of the huge economic cost associated with large duration samples. This paper derives tractable asymptotic distributional properties, in a bivariate survival model with bivariate unobserved heterogeneity (frailty). Then I discuss the possibility of identifying and consistently estimating the parameters of the asymptotic distribution, from the observation of large duration survivors only, for different observability cases. Proof of asymptotic consistency is also proved.

I also show that, under these same conditions, the heterogeneity distribution among survivors converges to a limit semi-parametric distribution. This distribution is more parsimonious and flexible, thus it is a serious competitor to the current non-parametric and parametric heterogeneity specifications in proportional hazard survival models with heterogeneity.

This framework has two important applications in Insurance. First, it can be used to characterize the joint long-term care and mortality risks at advanced ages. Current long-term care databases are usually of reduced size, making the estimation of joint tail risk difficult. Our paper provides an asymptotically consistent non-parametric estimator of the joint dependence structure. Compared to the traditional extreme value theory, our model is both more flexible and easier to estimate. In particular, it is valid in the presence of random censoring.

Second, the new heterogeneity distribution can be used to capture the dependence structure of mortality rates due to cancer and cardiovascular diseases. In particular, we will address the puzzle of "war on cancer" initiated by (Honore and Lleras-Muney, 2006, *Econometrica*). We show that our model allows for disentangling the dependence due to the common longevity improvements, that is the macro-level dependence, and that due to correlated unobserved heterogeneity of the two diseases, that is the micro-level dependence.