

Pricing and Hedging Inflation-linked Annuities Considering Inflation, Interest rate Risk and Longevity Risk

Sharon S. Yang¹, Fen-Ying Chen², Hong-Chih Huang³

¹ Corresponding author, Department of Finance, National Central University,
Taoyuan, Taiwan, R.O.C., Email: syang@ncu.edu.tw. TEL: 886-2-29396207.

² Department of Finance, Shih Hsin University; Address: #111, Sec. 1, Mu-Cha Rd.,
Taipei 116, Taiwan, R.O.C., Email: fyichen@cc.shu.edu.tw

³ Department of Risk Management and Insurance, National Chengchi University,
Taipei, Taiwan, R.O.C., Email: jerry2@nccu.edu.tw

Abstract

After financial crisis in 2008, it has highlighted the sensitivity and vulnerability of financial markets to inflation risk, which could reduce the value of money and affect the net returns of financial instruments. In response to this, investors who are concerned with maintaining their investment's purchasing power rather than its market value are resorting to inflation-linked products to hedge their inflation risk. Consequently, the financial products associating with inflation structure have been rapidly growing for the last decade and has further increased their potential growth worldwide. As for in the annuity market, the increase of life expectancy has addressed the inflation risk as an important issue in order to ensure that future consumption will not fall below a minimum acceptance standard of living because of longer horizons. According to a retirement survey by the Society of Actuaries (2011), 71% of preretirees and 58% of retirees in the US are concerned with the ability to keep the value of their savings and investments up with rising inflation. Therefore, to protect the retirees from inflation risk, the annuity products associated with the inflation protection is common. In UK, annuity may be subject to LPI indexation, i.e. increased in line with inflation, subject to an annual cap. Thus, interest rate risk, longevity risk and inflation risk are three important risk factors regarding the inflation-linked annuity products. Longevity risk may affect the inflation risk because the increase of

life expectancy may cause greater inflation risk. Thus, we can't ignore the longevity risk when pricing inflation-linked annuity products. However, there is no research dealing with these three risk factors simultaneously for pricing such annuity products. Tiong (2013) presents various inflation-linked variable annuities which are designed to help investors protect their portfolios from inflation risk. This paper focuses on the inflation rate dynamics and obtains closed-form pricing formulas for these inflation-linked annuity products. To fill this gap, this research attempts to consider the effects of longevity risk, inflation risk and interest rate risk on pricing and hedging inflation-linked annuities. We assume the dynamics of interest rate term structure and inflation rate follow the Heath-Jarrow-Morton framework (HJM, 1992) and the mortality rate dynamic is captured by a stochastic mortality model using logarithm normal distribution. As a result, we obtain the closed form solution for pricing the exotic structures of inflation-linked annuities and the inflation-linked longevity bonds. We first propose an inflation-linked longevity bond for insurers to hedge inflation risk, longevity risk and interest rate risk and the pricing formula is a general form which can be reduced to price traditional longevity bonds or inflation-linked longevity bonds with a cap or a floor separately. It is helpful for insurers to hedge inflation risk and longevity risk efficiently. The effects of these three risk factors are examined numerically. We also conduct the sensitivity analysis. The fair prices of

inflation-linked annuities or bonds are much higher than the traditional ones. Thus, the insurers need to increase the longevity hedging cost for such inflation-linked annuities.

Keywords: Inflation Risk, Longevity Risk, HJM Model, Interest Rate Risk, Annuity.

