Ambiguity on the Insurer's Side: The Demand for Insurance

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In Arrow's classical problem of demand for insurance indemnity schedules, it is well-known that the optimal insurance indemnification for an insurance buyer is a *deductible* contract, when the insurer is a risk-neutral Expected-Utility (EU) maximizer, when the DM is a risk-averse EU-maximizer, and with the two parties share the same probabilistic beliefs about the realizations of the underlying insurable loss. However, empirical evidence suggests that ambiguity (rather than risk) is prevalent in insurance pricing and underwriting, and that often insurers tend to exhibit more ambiguity than the insured individuals (Hogarth and Kunreuther (1989)). Motivated by these findings, we re-examine Arrow's problem in a setting where the insurer has ambiguous beliefs about the realizations of the insurable loss, whereas the insured is an EU-maximizer. Formally, we examine a problem similar to that of Arrow, with the sole difference that the beliefs of the insurer are represented by a capacity (non-additive probability measure) rather than a probability measure, as in Schmeidler (1989).

We first show that if the parties' beliefs satisfy a certain compatibility condition, then optimal indemnity schedules exist and are monotonic, in the sense that they are nondecreasing functions of the realizations of the loss random variable. This property rules out *ex post* moral hazard issues that could arise from the possibility that the insurer could misreport the actual amount of loss suffered (Huberman, Mayers and Smith (1983)).

We then consider the case where the insurer is either ambiguity-seeking or ambiguity-averse in the sense of Schmeidler (1989). We show that in both cases, an optimal indemnity schedule can be replicated by an optimal indemnity obtained from an insurance problem in which both the insured and the insurer are EU-maximizers, but have different beliefs about the realizations of the insurable random loss. Such problems have been recently studied by Ghossoub (2013).

Finally, under additional assumptions, we obtain an explicit characterization of the optimal indemnity schedule as a function of the underlying data. In the case of an ambiguity-seeking insurer whose capacity is a distortion of the probability measure of the insured, we show that the optimal indemnity schedule takes the form

$$Y^* = \min\left[X, \max\left(0, X - d\left(T\right)\right)\right],$$

where T is the concave probability distortion function of the insurer, and $d\left(T\right)$ is a state-contingent deductible that depends on the state of the world only through the function T. In the case of an ambiguity-averse insurer whose capacity has a core consisting of probability measures with the monotone likelihood ratio (MLR) property, we show that the optimal indemnity schedule is a state-contingent deductible of the form

$$Y^* = \min\left[X, \max\left(0, X - d\left(LR\right)\right)\right],$$

where LR denotes a function of the likelihood ratios of the probabilities in the core of the supermodular capacity over the probability of the insured. In both cases, we determine the state-contingent deductible d explicitly. Arrow's solution obtains as a limit case from both settings: when the distortion function T becomes the identity function in the ambiguity-seeking case and when the core collapses to the probability measure of the insured in the ambiguity-averse case.