# Utility Risk Portfolio Selection

Kwok Chuen Wong [1] [2]

## Abstract

In this talk, we provide a comprehensive study on utility-risk portfolio selection problem. By considering the first order condition for the objective function, we derive a primitive static problem, called Non-linear Moment Problem, subject to a set of constraints involving non-linear functions of corresponding "mean-field terms", to completely characterize the optimal terminal wealth. Under a mild assumption on utility, we establish the existence of the optimal solutions for both utility-downside-risk and utility-strictly-convex-risk problems, their positive answers have long been missing in the literature. In particular, the existence result in utility-downside-risk problem is in contrast with that of mean-downside-risk problem considered in Jin et al. [1], in which they actually demonstrated the non-existence of optimal solution instead. Besides, in the case of mean-risk problem, in which the first order derivative of utility function is a constant, the corresponding Non-linear Moment Problem possesses a unique explicit solution, which coincides with those first obtained in Jin et al. [1]. Finally, we also revisit the non-existence of optimal solution for mean-semivariance problem via our proposed approach by considering the corresponding Nonlinear Moment Problem. This paper is a joint work with Phillip Yam (The Chinese University of Hong Kong) and Harry Zheng (Imperial College London).

Keywords: Nonlinear Moment Problem; Mean-field type perspective; Utility; Risk function; Downside risk; Portfolio selection.

# References

[1] H. Jin, J. Yan, and X. Y. Zhou (2005) Continuous-time mean-risk portfolio selection. *Ann. Inst. H. Poincaré* Probab. Stat., 41, pp. 559–580.

[1]Department of Statistics and Actuarial Science, The University of Hong Kong, Pokfulam, Hong Kong (`wongkc89@hku.hk`, +852 39178152)

[2]Department of Mathematics, Imperial College London, London SW7 2AZ, United Kingdom.