Infinite-time Absolute Ruin in Dependent Renewal Risk Model with Constant Force of Interest

Jiajun Liu\textsuperscript{a}(Speaker) Yang Yang\textsuperscript{b,c} *

Abstract

Consider a renewal risk model with a constant premium and a constant force of interest rate, where the claim sizes and inter-arrival times follow certain dependence structures via some restriction on their copula function. Under the assumption that the distribution of the claim-size belongs to the intersection of the class $S(\gamma)$, $\gamma \geq 0$ and the class $\mathcal{R}_{-\infty}$, or a larger intersection class of O-subexponential distribution, class $\mathcal{L}(\gamma)$ and $\mathcal{R}_{-\infty}$, the infinite-time absolute ruin probabilities are derived.

Keywords: Asymptotics; Class $S(\gamma)$; Class $\mathcal{L}(\gamma)$; Dependence; Constant force of interest; Farlie-Gumbel-Morgenstern distribution; Heavy tails; O-subexponential distribution class; Renewal risk model; Ruin probability.

1 Introduction

Consider the following renewal risk model with constant premium rate and constant force of interest. Denote by $W_r(t)$, the total reserve up to time $t$, satisfies

$$W_r(t) = xe^{rt} + c \int_0^t e^{r(t-y)}dy - \sum_{i=1}^{N(t)} X_i e^{r(t-\tau_i)}, \quad t \geq 0.$$ 

Inspired by the literature, we define the probability of infinite-time absolute ruin as

$$\psi(x, \infty) = \Pr \left( \inf_{t \geq 0} W_r(t) < -\frac{c}{r} \bigg| W_r(0) = x \right), \quad x \geq 0, \tag{1.1}$$

which can also be rewritten as

$$\psi(x, \infty) = \Pr \left( \sum_{i=1}^{\infty} X_i \prod_{j=1}^{i} Y_j > x + \frac{c}{r} \right), \quad x \geq 0, \tag{1.2}$$

where $Y_i = e^{-\theta_i}, i \geq 1$, can be considered as discount factor according to the constant force of interest $r$.

Definition 1.1 The corresponding survival copula is defined as

$$\overline{C}(u, v) = u + v - 1 + C(1 - u, 1 - v), \quad (u, v) \in [0, 1]^2.$$ 

Assume that the copula function $C(u, v)$ is absolutely continuous, denote by $C_1(u, v) := \frac{\partial}{\partial u} C(u, v)$, $C_2(u, v) := \frac{\partial^2}{\partial^2 u} C(u, v)$, and $C_{12}(u, v) := \frac{\partial^2}{\partial u \partial v} C(u, v)$, then

$$\overline{C}_2(u, v) := \frac{\partial}{\partial v} \overline{C}(u, v) = 1 - C_2(1 - u, 1 - v), \overline{C}_{12}(u, v) := \frac{\partial^2}{\partial u \partial v} \overline{C}(u, v) = C_{12}(1 - u, 1 - v).$$

Assumption 1.1 The relation

$$\overline{C}_2(u, v) \sim u \overline{C}_{12}(0+, v), \quad u \downarrow 0,$$

holds uniformly on $(0, 1]$. *\textsuperscript{a}Department of Mathematical Sciences, University of Liverpool, Peach Street, Liverpool, L69 7ZL, United Kingdom. E-mail: J.Liu9@liverpool.ac.uk
2 Main results

**Theorem 2.1** In the renewal risk model with constant force of interest rate \( r > 0 \), assume that \((X_i, \theta_i)\), \(i \in \mathbb{N}\), are a sequence of i.i.d. random pairs with generic random pair \((X, \theta)\) satisfying Assumption ?? with \( F \in S(\gamma) \cap R_{-\infty} \) for some \( \gamma \geq 0 \), then \( \mathbb{E} e^{\gamma S_\infty} < \infty \), where \( S_\infty = \sum_{i=1}^{\infty} X_i e^{-r \tau_i} \), and
\[
\psi(x, \infty) \sim e^{-\gamma c/r} \mathbb{E} e^{\gamma S_\infty} \int_0^\infty F(x e^{rt}) G_{\theta_c}(dt), \tag{2.1}
\]
where \( G_{\theta_c}(dt) = C_{12} [1 - G_{\theta(t)}] G_\theta(dt) \).

**Theorem 2.2** In the renewal risk model with constant force of interest rate \( r > 0 \), assume that \((X_i, Y_i)\), \(i \in \mathbb{N}\), are a sequence of i.i.d. random pairs with generic random pair \((X, Y)\) under Assumption ?? with \( F \in S(\gamma) \cap R_{-\infty} \) for some \( \gamma \geq 0 \), then \( \mathbb{E} e^{\gamma S_\infty} < \infty \), where \( S_\infty = \sum_{i=1}^{\infty} X_i \prod_{j=1}^{i-1} Y_j, \ Y_j = e^{-r \theta_i} \), and
\[
\psi(x, \infty) \sim \mathbb{E} e^{\gamma S_\infty} \Pr(\{XY_c > x + c/r\}), \tag{2.2}
\]
where \( Y_c \) is distributed by \( G_c(dy) = C_1[1-G(dy)] = C_{12}[1-G(y)] G(dy) \).

**Theorem 2.3** In the renewal risk model with constant force of interest rate \( r > 0 \), assume that \((X, \theta)\) or \((X, Y)\) is dependent according to Assumption ?? with \( F \in L(\gamma) \cap OS \cap R_{-\infty} \) for some \( \gamma \geq 0 \), then relation (?) or (?) holds with \( \mathbb{E} e^{\gamma S_\infty} < \infty \).

**Theorem 2.4** In the renewal risk model with constant force of interest rate \( r > 0 \), assume that \((X_i, Y_i)\), \(i \in \mathbb{N}\), are a sequence of i.i.d. random pairs with generic random pair \((X, Y)\) following a common bivariate FGM distribution function with \( \rho \in [-1, 1] \). If \( F \in S(\gamma) \cap R_{-\infty} \) for some \( \gamma \geq 0 \), then \( \mathbb{E} e^{\gamma S_\infty} < \infty \), where \( S_\infty = \sum_{i=1}^{\infty} X_i \prod_{j=1}^{i-1} Y_j \), and
\[
\psi(x, \infty) \sim \mathbb{E} e^{\gamma S_\infty} \Pr(\{XY_{\rho} > x + c/r\}),
\]
where \( Y_{\rho} \) is distributed by \( G_{\rho} \) with \( G_{\rho}(y) = (1-\rho) G(y) + \rho G^2(y) \).

**References**


