

Coherent risk measures and infinite expectation risks

Alejandro Balbás * Iván Blanco[†] José Garrido[‡]

April 14, 2015

EXTENDED ABSTRACT

Though there is a growing interest on risk measurement in Economics, Finance and Insurance, it is still under debate how to properly measure risks when expected losses are unbounded. In this paper, a general methodology for introducing coherent risk measures for risks with infinite expectation is proposed. It is broadly recognized that the inclusion of risks with unbounded expectation (for instance, risks with a Cauchy or a Pareto distribution) presents many mathematical problems when extending the notion of coherent or expectation bounded risk measure. Unfortunately, previous literature has addressed this caveat by losing some desirable mathematical properties. For instance, if we use the value at risk (VaR) as a risk measure, we lose continuity and subadditivity. Though there are risk measures for heavy tailed risks recovering sub-additivity along the literature, we still lose continuity in all of them. Admittedly, using continuous sub-additive risk measures has many important analytical advantages, since the optimization of such functions is much simpler and many classical financial and actuarial problems (pricing and hedging, portfolio choice, equilibrium, optimal reinsurance, etc.) become much easier to tackle.

For these reasons, it may be worthwhile to looking for general solutions which allow us to extend the concept of coherent risk measures and deal with infinite expectation risks, while still preserving continuity and sub-additivity. To sum up, the main contribution of this paper is to extend the Artzner et al (1999) approach by allowing risks generating unbounded expected losses preserving desirable mathematical properties as continuity and sub-additivity. Furthermore, based on data of yearly claims in the French business insurance branch (Zajdenweber, 1996), we provide numerical examples and applications to classical insurance and operational risk problems. We develop coherent extensions of the Conditional Value at Risk (CVaR) to deal simultaneously with both, bounded (exposure to equity market risks) and unbounded (generalized Pareto distributions associated to operational risk) losses, and analyze actuarial insurance applications such as extensions of the Expected Value Premium Principle.

KEYWORDS: coherent risk measures, infinite expectation risk, heavy tails, unbounded losses, operational risk, expected value premium principle

Speaker IME session: Iván Blanco

*University Carlos III of Madrid. C/ Madrid, 126. 28903 Getafe (Madrid, Spain). alejandro.balbas@uc3m.es

[†]University Carlos III of Madrid. C/ Madrid, 126. 28903 Getafe (Madrid, Spain). ivan.blanco@uc3m.es.
Telephone: +34 616 13 97 91

[‡]Concordia University. Department of Mathematics and Statistics. 1455 de Maisonneuve Blvd. West, LB-901, Montreal, Quebec, Canada H3G 1M8. jose.garrido@concordia.ca