Existence and uniqueness of chain ladder solutions

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The cross-classified chain ladder has a number of versions, depending on the distribution to which observations are subject. The simplest case is that of Poisson distributed observations, and then maximum likelihood estimates of parameters are explicit.

Most other cases, however, lead to implicit solutions of these parameter estimates, raising questions as to their existence and uniqueness. The present paper investigates these questions in the case where observations are distributed according to some member of the exponential dispersion family.

It is shown that a solution always exists provided only that the data array meets a regularity condition concerned with which (if any) cells are missing. A condition for uniqueness of that solution is also found. It is shown that this condition reduces to a simple form in the case that the EDF distribution falls within the Tweedie sub-family.

It is further found that uniqueness always occur when the Tweedie dispersion index lies in the interval [1,2]. Uniqueness is not established for larger values of the dispersion index, but a condition for uniqueness is found. It depends on the closeness of the array to "proportionality", i.e. all rows (or columns) proportional.

The investigation is then widened to Bayesian models, in which the parameters defining the data likelihoods described above are randomised with conjugate priors. Again, it is shown that a solution always exists in the general case (subject to some technical conditions) and conditions for uniqueness are found, which largely parallel those for the non-Bayesian model.

A numerical example is analysed, in which the solutions of the non-Bayesian maximum likelihood equations are found analytically. The behaviour of the log-likelihood is found to be complex even for a very simple data structure, and it is found that multiple solutions can indeed occur.