Optimal Bonus-Malus Systems Using Generalized Additive Models for Location, Scale and Shape

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March 28, 2015

Abstract

A basic interest of actuarial literature is the construction of optimal Bonus-Malus Systems (BMS), defined as systems obtained through Bayesian analysis, see Lemaire (1995). The inclusion of important a priori rating variables for the number and/or costs of claims plays a major role, see for example Dionne and Vanasse (1989, 1992), Denuit et al. (2007), Mahmoudvand and Hassani (2009), Tzougas and Frangos (2014) and Tzougas, Frangos and Vrontos (2014). However, the commonly used specification is that only the mean frequency and/or severity is modelled as a function of risk factors. In this respect the expected relative risk level revised on the basis of past experience does not account for the effect coming from the other parameters of the distribution of the response due to the unobserved heterogeneity changes with explanatory variables. This paper extends this setup presenting the construction of optimal BMS with frequency and severity components based on the generalized additive models for location, scale and shape (GAMLSS) approach of Rigby and Stasinopoulos (2005). Specifically, for the frequency component we employ the Negative Binomial Type I, Sichel, Poisson Inverse Gaussian (a special case of the Sichel) and finite Poisson mixture GAMLSS models, while for the severity component we employ the Pareto and finite Exponential mixture GAMLSS models. Within the adopted framework all the parameters and mixing probabilities of these models are modelled in terms of covariates. Applying Bayes theorem, we update the posterior mean and posterior probability of the policyholders' classes of risk. The aforementioned models are compared on the basis of a sample of the automobile portfolio of a major insurance company employing the Generalized Akaike Information Criterion (GAIC) which is valid for both nested or non-nested model comparisons (as suggested by Rigby and Stasinopoulos, 2005 and 2009). Our analysis shows that the employment of more advanced models can provide a measure of uncertainty regarding the credibility updates of claim frequency/severity of each specific risk class and the difference in the premium that they imply can act as a cushion against adverse experience. Finally, these "tailor-made" premiums are compared to those which correspond to the 'univariate',without regression components, models.

Keywords: Optimal BMS; Claim frequency; Claim severity; Generalized Additive Models for Location, Scale and Shape; Mixtures of Distributions.

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