Monte Carlo valuation of the initiation option in a GLWB variable annuity

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Agenda

- intro - GLWB VAs
- framework
- fair valuation of the optimal initiation and surrender option
- solution with LSMC
- some examples

Variable annuities (VAs)

- unit linked-type vehicles
  - popular in North-America, & Asia since new century
  - complicated structure due to the presence of several riders ($\equiv$ options)
  - VAs jargon: GMxB $=$ Guaranteed Minimum Benefit of type x; $x =$ A(ccumulation), $=$ D(eath), $=$ I(ncome); $=$ W(ithdrawal)
  - **GLWB** $=$ Guaranteed Lifelong Withdrawal Benefit
- need realistic models for pricing & risk management purposes $\rightsquigarrow$ policyholder behaviour is crucial
  - practitioners frequently relies on intuitive but simple rules $\rightsquigarrow$ risk of underestimation / mismanagement

Guaranteed Lifelong Withdrawal Benefits

- GLWBs features
  - ▷ single initial investment; accumulation phase followed by (**guaranteed**) decumulation phase
  - ▷ **benefits**: annuity payments after contract is initiated, personal account (if any) at death
  - ▷ **personal account**: same as in a unit-linked
    $\uparrow\downarrow$ reference fund returns,
    $\downarrow$ (insurance and management) fees,
    $\downarrow$ annuity payments,
    ruin?
  - ▷ **base amount**: used to calculate annuity instalments
    $\uparrow$ bonus rate during accumulation phase,
    $\uparrow$ reset by personal account (**ratchet**),
    $\downarrow$ excessive withdrawals
  - ▷ **withdrawals initiation**: complex decision driven by moneyness and / or other factors

Guaranteed Lifelong Withdrawal Benefits

- Wide literature on pricing and risk management of VAs using
  - ▷ closed form (few cases, GMDB)
  - ▷ MC (all GMxB but GMWBs)
  - ▷ trees, dynamic programming
- some references on GLWBs:
  - ▷ [Shah and Bertsimas (2008)]
  - ▷ [Piscopo (2009)]
  - ▷ [Kling et al. (2011)]
  - ▷ [Piscopo and Haberman (2011)]
  - ▷ [Steinorth and Mitchell (2012)]

  few focuses on the optimal initiation decision...

Optimal Initiation

- [Huang et al. (2014)] using PDEs, solve for (portfolio) value and analyse initiation decision based on moneyness
- [Huang et al. (2017)] solve using dynamic programming combined with Fourier analysis to approximate the value function, assuming full dynamic withdrawals and initiation
- both can hardly be generalized to high dimensional models

"*. . . Monte Carlo (MC) methods could therefore in principle be used to calculate* [the fair value] v^0, *except that the optimization over* [the initiation time] τ *is hard to implement using simulation.*"

[Huang, Milevsky and Salisbury (2014)]

Is it really hard?

Our contribution

- we show how (LS)MC can be used to calculate the **initiation option** $\rightsquigarrow$ non standard as the initiation decision will generate a **stream of random cash flows**
  - ratchet
  - remaining fund value in case of death
- extend to allow for **surrender** (usually admitted as total withdrawal); distinguish between
  - early surrender
  - full surrender

  $\rightsquigarrow$ (double) optimal stopping problem $\rightsquigarrow$ transform into a two-stage problem
- advantage: can easily accommodate complicate models and contract features
- preliminary results: early surrender valuable; full surrender less so if contract can be optimally initiated

Optimal annuitization

- Wide literature on optimal (timing of) annuitization in pension / insurance products
  - [Yaari (1965)]
  - [Milevsky (1998)]
  - [Milevsky et al. (2006)]
  - [Milevsky and Young (2007)]
  - Di Giacinto and Vigna (2012)]
  - [Gerrard et al. (2012)]
  - [Hainaut and Deelstra (2014)]
- GLWB differ because
  - maintain access to the fund
  - surrender
  - random cash-flows after annuitization (initiation)
  - fair valuation vs max utility

Framework

- time grid: $\mathbb{T} = \{0, 1, \ldots, N\}$
  (eg $N =$ extremal age $-$ ph's initial age)
- ph's stopping times in $\mathbb{T}$:
  - ▷ ph's time of **death**: τ
  - ▷ **initiation** time: λ, $0 \leq \lambda \leq \tau$
  - ▷ **surrender** time: π, $1 \leq \pi \leq \tau$
- constraints on (λ, π):
  - ▷ $\lambda < \pi \leq \tau$ (initiation and eventual surrender)
  - ▷ $\pi < \lambda = \tau$ (early surrender, no initiation)
  - ▷ $\lambda = \pi = \tau$ (no initiation or surrender)
  - ▷ convention: $\lambda = \tau$ or $\pi = \tau \rightsquigarrow$ no action is taken

Framework

- all processes defined on (or restricted to) $\mathbb{T} = \{0, 1, \ldots, N\}$
  - ▷ **reference fund** value: S_t
  - ▷ **personal** account: X_t
  - ▷ **base** amount (determines annuity payments): M_t
  - ▷ **state variables** (other than X_t and M_t),

$$Z_t = (X_t, M_t, \widehat{Z}_t), \qquad \widehat{Z}_t = (\text{other state variables})$$

    eg $\widehat{Z}_t = (r_t,\ \mu_t,\ V_t, \ldots)$, with r_t short rate, μ_t stochastic mortality, V_t stochastic volatility, ...
  - ▷ discount factor $B_{t,u} = \exp\left(-\int_t^u r_v \mathrm{d}v\right)$

Framework

- Two distinctive features: **roll-up** (until initiation) and **ratchet** of base amount
- personal account and base amount **dynamics** given (λ, π):

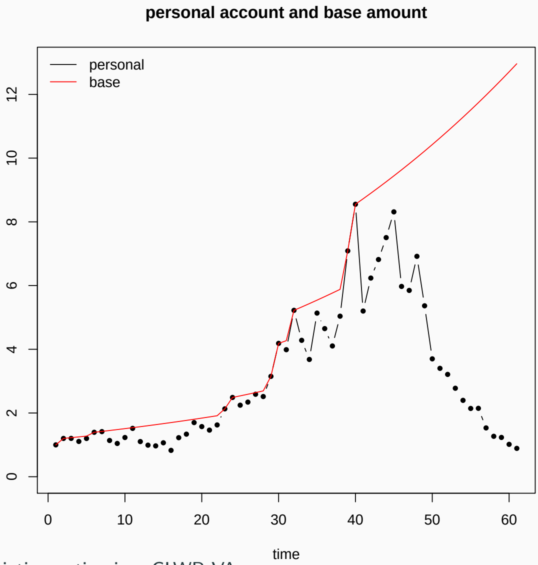
$$X_{t+1} = \max\left\{X_t\left(\frac{S_{t+1}}{S_t} - \psi\right) - (\phi + g_\lambda 1_{\{\lambda < t\}})M_t, 0\right\} 1_{\{\pi > t\}}$$

$$M_{t+1} = \max\{M_t(1 + \beta 1_{\{\lambda \geq t\}}), X_{t+1}\} 1_{\{\pi > t\}}$$

with $X_0 = M_0 = 100$

  - ▷ **management** fee: ψ
  - ▷ **insurance** fee: ϕ
  - ▷ **roll-up** rate: β
  - ▷ **annuity** rate if rider is initiated at t: g_t

Personal and base account



Valuation problem

- valuation (risk neutral) measure: Q
- fair value of GLWB:

$$V_0 = \sup_{\lambda,\pi} E^Q \left[\text{pv of cashflows generated by GLWB contract}\right]$$

  over all possible initiation-surrender strategies (λ, π) available to ph
  $\rightsquigarrow$ **(double) optimal stopping problem**:
  - $\lambda < \pi \leq \tau$ (initiation and eventual surrender), or
  - $\pi < \lambda = \tau$ (early surrender, no initiation) , or
  - $\lambda = \pi = \tau$ (no initiation or surrender)

  convention: $\lambda = \tau$ or $\pi = \tau$ $\rightsquigarrow$ no action is taken
- here focus on initiation and surrender (no dynamic withdrawals!)
- two special cases: **initiation or early surrender** (V_0'), **initiation, no surrender** (V_0''). Clearly

$$V_0'' \leq V_0' \leq V_0$$

Least Square Monte Carlo

- combine MC with regression to calculate conditional expectations $\rightsquigarrow$ compute continuation values $\rightsquigarrow$ solve dynamic programming problems
- if
  - ▷ $e = (e_1, \ldots, e_k)'$ is a (truncated) L^2 basis function,
  - ▷ $Y_t =$ **pv at t of future cash flows if a given action is taken**,
  - ▷ Z_t state variables at t,

  then

$$E_t^Q[Y_t] = E^Q[Y_t|Z_t] \approx \delta \cdot e(Z_t)$$

- estimate δ using simulated values $Y_t^{(h)}$, $Z_t^{(h)}$: $\widetilde{\delta}$ solves

$$\arg\min_{\delta \in \mathbb{R}^k} \sum_h \left(Y_t^{(h)} - \delta \cdot e(Z_t^{(h)}) \right)^2$$

Least Square Monte Carlo . . .

- early exercise
  - ▷ American options: [Carriére (1996)], [Tsitsiklis and Van Roy (1999)], [Longstaff and Schwarz (2001)], . . .
  - ▷ surrender option in life insurance contracts: [Andreatta and Corradin (2003)], [Bacinello et al. (2010)], in VAs [Bacinello et al. (2011)], . . .
- Solvency II, calculation of NAV
  - ▷ [Bauer et al. (2012)], [Floryszczak et al. (2016)], . . .
- theoretical results, convergence, number of simulations vs number of basis functions
  - ▷ [Clément et al. (2002)], [Moreno and Navas (2003)], [Stentoft (2004, 2012)], . . .

LSMC algorithm

- Calculate V_0' and V_0'' by LSMC $\rightsquigarrow$ recalculate at each date the pv of future cash flows implied by
  - (early) surrender
  - initiation
  - continuation

  compare and decide, then continue backward
- recalculation of future cash flows can be efficiently coded
- the case of initiation and late surrender is more complicate

Initiation with surrender

- recall the fair value of GLWB:

$$V_0 = \sup_{\lambda,\pi} E^Q \left[\text{pv of cashflows generated by GLWB contract}\right]$$

Proposition

Calculate V_0 through a **two-stage problem**:

[1st stage problem] For each initiation date t, find the optimal (late) surrender

$$F_t^{I*} = \sup_{\eta} E_\lambda^Q \left[\text{pv of cashflows of contract initiated at } t\text{, surrendered at } \eta\right]$$

[2nd stage problem] Calculate V_0 by

$$V_0 = \sup_{\lambda,\pi} E^Q[\text{pv of cashflows of contract optimally (late) surrendered}]$$

(π: early surrender)

Initiation with surrender ...

- 2nd stage is the same as the "initiation or early surrender" $\rightsquigarrow$ solve it with same algorithm
- 1st stage problem can be solved through **repeated applications of LSMC** $\rightsquigarrow$ calculate F_t^{I*} at each time t $\rightsquigarrow$ time consuming
- however, the same set of simulated state variables is used for all LSMC calculations in 1st and 2nd stages $\rightsquigarrow$ only recalculation of personal account and base amount is needed

Numerical example

- parameters and model close to [Huang et al. (2014)]
- S_t: GBM, $r = 3\%$, $\sigma = 20\%$
- τ: Gompertz force of mortality, ph's age = 60

$$\mu_t = \frac{1}{b} \mathrm{e}^{(x_0 + t - m)/b}$$

  $m = 87.25$, $b = 9.5$

- $\beta = 6\%$, $g(t) = g = 4\%$, $\psi = 0$, $\phi = 150$ bp
- surrender rate $p(t) = p = 2\%$
- LSMC: 10 batches of 10 000 simulations, basis functions: 3rd degree polynomial in 2 variables

Contract values

- $F^I_{0,\tau}$ - contract initiated at 0
- F^{I*}_0 - contract initiated at 0 optimally surrendered
- V''_0 - contract optimally initiated (no surrender)
- V'_0 - contract optimally initiated or early surrendered
- V_0 - contract optimally initiated and / or surrendered

Contract values: roll-up rate

β	$F^I_{0,\tau}$	F^{I*}_0	V''_0	V'_0	V_0
2.0%	97.11	99.75	97.11	98.17	99.75
4.0%	97.11	99.77	97.11	98.78	99.78
6.0%	97.11	99.83	97.72	100.85	100.90
8.0%	97.11	99.82	104.19	106.63	106.61
10.0%	97.11	99.89	119.66	120.79	120.87

Contract values: insurance fee

ϕ (bp)	$F^I_{0,\tau}$	F^{I*}_0	V''_0	V'_0	V_0
50	106.44	106.57	109.68	110.39	110.16
100	101.36	102.70	102.88	104.42	104.63
150	97.11	99.83	97.72	100.85	100.90
200	93.54	98.12	93.68	98.60	98.66
250	90.52	97.24	90.52	97.33	97.41

Contract values: annuity rate

g	$F^I_{0,\tau}$	F^{I*}_0	V''_0	V'_0	V_0
3.0%	84.98	96.46	84.98	96.39	96.49
3.5%	90.72	97.15	90.78	97.37	97.38
4.0%	96.86	99.50	97.43	100.43	100.55
5.5%	103.11	104.20	104.62	106.06	105.90
5.0%	109.96	110.42	112.21	112.76	112.77

Conclusions

- fair valuation of optimal initiation in GLWB using Monte Carlo methods is feasible
  - ▷ slow but flexible $\rightsquigarrow$ parallel computing, memory issues?
  - ▷ good with many state variables
  - ▷ include surrender before (and after!) initiation
- extensions:
  - ▷ effects of stochastic mortality, stochastic volatility and interest rates
  - ▷ reversionary annuities
  - ▷ age-increasing annuity rates and other realistic contract features