Unbounded Backward Stochastic Differential Equations with Quadratic Growth and Applications

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What is the BSDE?

- For a positive real number T, $0 \leqslant T < \infty$, those BSDE's are:

$$\begin{cases} Y(t) = \xi + \int_t^T f(s, Y_s, Z_s)ds - \int_t^T Z_s dW_s, \\ Y(T) = \xi \end{cases}$$

- $(\Omega, \mathcal{F}, \mathcal{F}_t, \mathbb{P})$ be a given filtered probability space on which a Brownian motion $(W(t), t \geq 0)$ is defined.
- ξ is a given $\mathcal{F}_T$ measurable random variable which is the terminal condition at the terminal time T.
- The generator f is a measurable function.
- The BSDE solution is the pair of processes (Y_t, Z_t) which are required to be adapted with respect to the filtration of the Brownian motion.

The problem of existence of a solution to BSDEs was solved :

⇒ Under the global Lipschitz condition on the generator f with lipschitz constants. (Pardaoux and Peng, 1990)

⇒ Under the global Lipschitz condition with stochastic lipschitz processes. (El Karoui and Huang, 1997)

⇒ Under the quadratic growth on the control process Z with bounded coefficients.(Kobylanski, 2000)

⇒ Under only local Lipschitz condition. (K. Bahlali, 2001)

⇒ Under the linear growth in Y and Z with unbounded coefficients.(Gashi and Li, 2018)

⇒ We have obtained the solvability for quadratic BSDEs with unbounded coefficients under certain assumptions.

The interest in BSDEs with possibly unbounded coefficients is not only theoretical, but is also motivated by applications in mathematical finance.

- An important interest rate models are given by BSDEs with possibly unbounded coefficients.
- The problem of pricing and hedging contingent claims are given in by BSDEs with possibly unbounded coefficients.
- They appear in optimal investment problems as exponential and power utility optimization problems.

THANK YOU FOR LISTENING!

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