Insurance Risk Models with Premiums Dependent on Surplus

Jing Wang

Prof. Corina Constantinescu

University of Liverpool

Insitute of Financial and Actuarial Mathematics

27/06/2019
2019 Actuarial Teachers' and Researchers' Conference, Liverpool

Risk Models with Surplus-dependent Premiums

The classic risk model describes surplus process $U(t)$ of an insurance portfolio at time t by

$$U(t) = u + ct - \sum_{k=1}^{N(t)} X_k.$$

Let premium amounts depend on the surplus, the risk process is replaced by

$$U(t) = u + \int_0^t p(U(s))ds - \sum_{k=1}^{N(t)} X_k.$$

Risk Models with Surplus-dependent Premiums

The classic risk model describes surplus process $U(t)$ of an insurance portfolio at time t by

$$U(t) = u + ct - \sum_{k=1}^{N(t)} X_k.$$

Let premium amounts depend on the surplus, the risk process is replaced by

$$U(t) = u + \int_0^t p(U(s))ds - \sum_{k=1}^{N(t)} X_k.$$

Explicit Results for Constant Premium Cases

(i) Exp(λ) distributed interarrival times with Exp(μ) distributed claims sizes,

$$\psi(u) = \frac{\lambda}{c\mu} e^{-\frac{c\mu-\lambda}{c}u}.$$

(ii) Erlang($2, \lambda$) distributed interarrival times with Exp(μ) distributed claims sizes,

$$\psi(u) = \frac{\lambda^2}{c^2\sigma^2 - 2\lambda c\sigma + \lambda^2} e^{\sigma u},$$

where $\sigma = -\frac{c\mu - 2\lambda + \sqrt{c^2\mu^2 + 4c\lambda\mu}}{2c}$.

Explicit Results for Constant Premium Cases

(iii) Exp(λ) distributed interarrival times with Erlang($2, \mu$) distributed claims sizes,

$$\psi(u) = \gamma_1 e^{\sigma_1 u} + \gamma_2 e^{\sigma_2 u},$$

where

$$\sigma_1 = -\frac{2c\mu - \lambda + \sqrt{(2c\mu - \lambda)^2 + c\mu(8\lambda - 4c\mu)}}{2c},$$
$$\sigma_2 = -\frac{2c\mu - \lambda - \sqrt{(2c\mu - \lambda)^2 + c\mu(8\lambda - 4c\mu)}}{2c}$$

and

$$\gamma_1 = \frac{\frac{2\lambda^2}{c^2\mu} - \frac{\lambda}{c} - \frac{2\lambda}{c\mu}\sigma_2}{\sigma_1 - \sigma_2},$$
$$\gamma_2 = \frac{2\lambda}{c\mu} - \frac{\frac{2\lambda^2}{c^2\mu} - \frac{\lambda}{c} - \frac{2\lambda}{c\mu}\sigma_2}{\sigma_1 - \sigma_2}.$$

.

Asymptotic Results for Surplus-dependent Premium Cases (linear premium $p(u) = c + \varepsilon u$)

(i) Erlang$(2, \lambda)$ distributed interarrival times with Exp(μ) distributed claims sizes,

$$\psi(u) \sim \gamma\Big(\frac{\mu}{\varepsilon}\Big)^{-1-\frac{\varepsilon+2\lambda+\varepsilon\sqrt{1+\frac{4\lambda}{\varepsilon}}}{2\varepsilon}} \int_u^{\infty} e^{-\mu v} \cdot (c+\varepsilon v)^{-2} dv.$$

(ii) Exp(λ) distributed interarrival times with Erlang$(2, \mu)$ distributed claims sizes,

$$\psi(u) \sim \gamma \cdot 2^{\frac{\lambda}{2\varepsilon}-1} \pi^{-\frac{1}{2}} (\lambda\mu)^{\frac{1}{4}-\frac{\lambda}{\varepsilon}} \varepsilon^{-\frac{3}{2}+\frac{3\lambda}{2\varepsilon}} \int_u^{\infty} e^{-\frac{\mu}{\varepsilon}z+\frac{2}{\varepsilon}(\lambda\mu z)^{\frac{1}{2}}-1} \cdot z^{\frac{1}{4}} \Big\{1 - \frac{4\varepsilon(-1+\frac{\lambda}{\varepsilon})^2 - \varepsilon}{16(\lambda\mu z)^{\frac{1}{2}}}\Big\} dz.$$

References

[1] M. Abramowitz and I. A. Stegun, *Handbook of mathematical functions with formulas, graphs, and mathematical tables*, National Bureau of Standards Applied Mathematics Series, 55, U.S. Government Printing Office, Washington, D.C., 1964.

[2] H. Albrecher, C. Constantinescu, G. Pirsic, G. Regensburger and M. Rosenkranz, *An algebraic approach to the analysis of Gerber-Shiu functions*, Insurance: Mathematics and Economics, 46(1) (2010), pp. 42-51.

[3] H. Albrecher, C. Constantinescu, Z. Palmowski, G. Regensburger and M. Rosenkranz, *Exact and asymptotic results for insurance risk models with surplus-dependent premiums*, SIAM Journal on Applied Mathematics, 73(1) (2013), pp. 47-66.

Thank You !