Scenario Weights for Importance Measurement

An R Package for Sensitivity Analysis

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joint work with Alberto Bettini, Pietro Millossovich and **Silvana Pesenti**

https://github.com/spesenti/SWIM

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Motivation

Complex quantitative models

- Capital modelling and beyond
- Granularity v opaqueness

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Sensitivity analysis

- Repeated model runs
- What to do with the results?

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- Where do model-risk vulnerabilities lie?

Sensitivity analysis

- ~~Repeated model runs~~ **Single model run**
- ~~What to do with the results?~~ **Consistent sensitivity measurement**

Overview

Example

Scenario Weights

The SWIM package in R

Example

A non-linear insurance portfolio

Portfolio consisting of

- Two lines of business
- Same multiplicative factor, e.g. inflation
- Reinsurance layer on the portfolio
- Reinsurance company can default

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	Input risk factors		Output
X_1	Claims from 1st LoB	Y	Portfolio loss
X_2	Claims from 2nd LoB		
X_3	Multiplicative factor		
X_4	% of RI recovery lost		

What if the portfolio VaR was 5% higher than in the current model?

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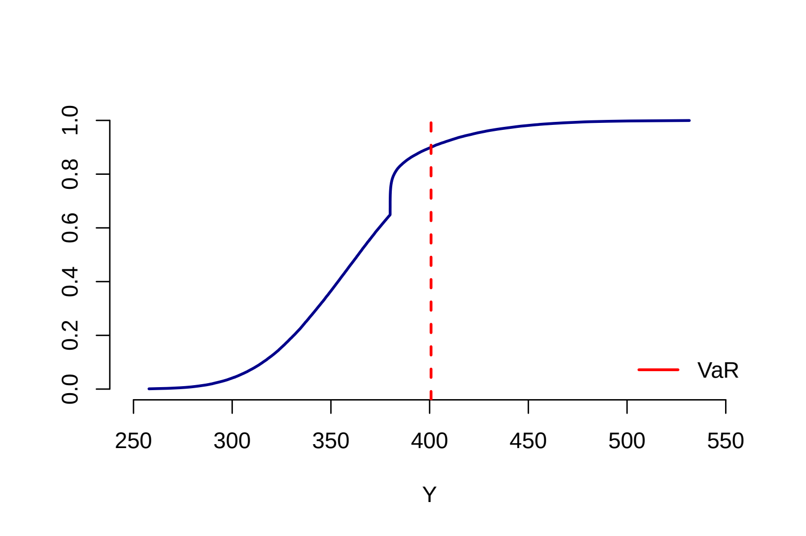
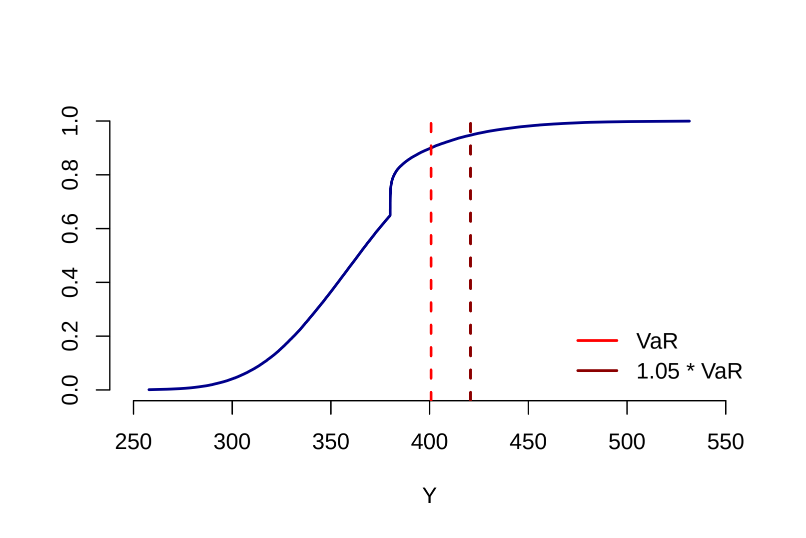
How would input factors reflect that change?

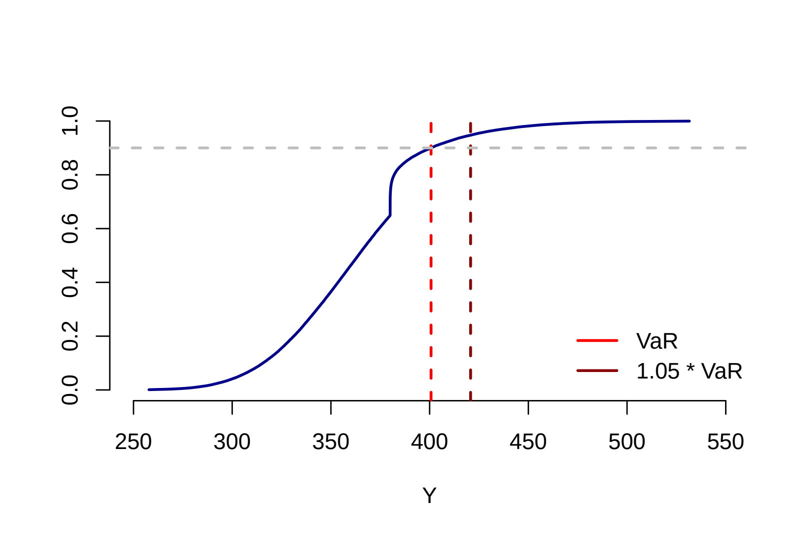
Distribution of portfolio loss



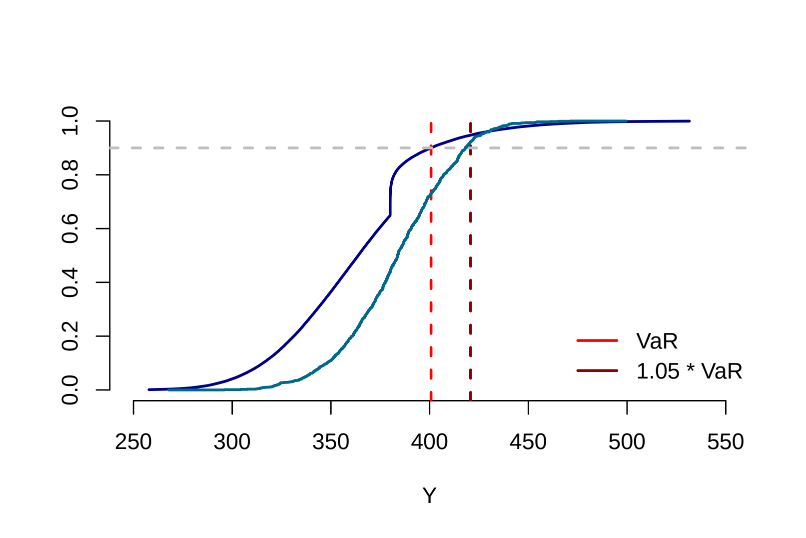
Distribution of portfolio loss



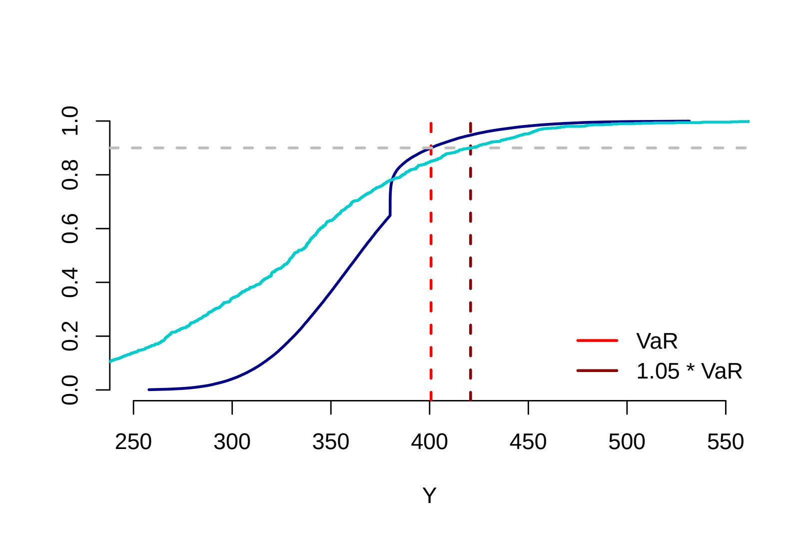
Distribution of portfolio loss



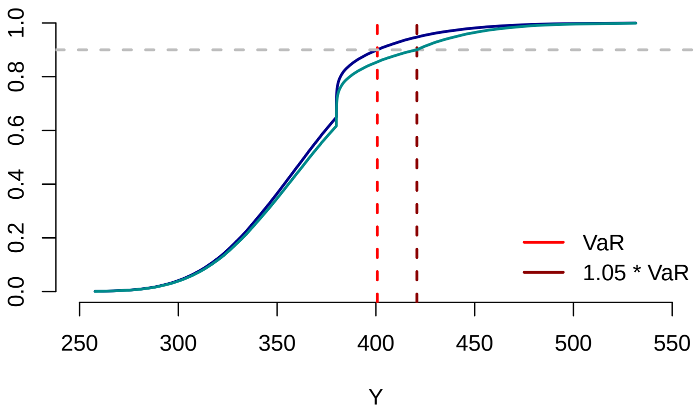
Distribution of portfolio loss



Distribution of portfolio loss



Distribution of portfolio loss



Scenario Weights

Constructing scenario weights

1. Define a **stress** on a random variable (risk factor or output) as a change in the value of a risk measure
   - Value-at-Risk, Expected Shortfall (TVAR)
   - Moments, probabilities, covariances

2. Derive **scenario weights** (change of measure) such that
   - re-weighted output fulfills the required stress
   - most plausible / least distorting, by minimising relative entropy

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▷ Typically we have a Monte Carlo sample and work with the empirical distribution.

Literature

Stressing moments:

- [Csiszár, 1975]

Stressing VaR and ES:

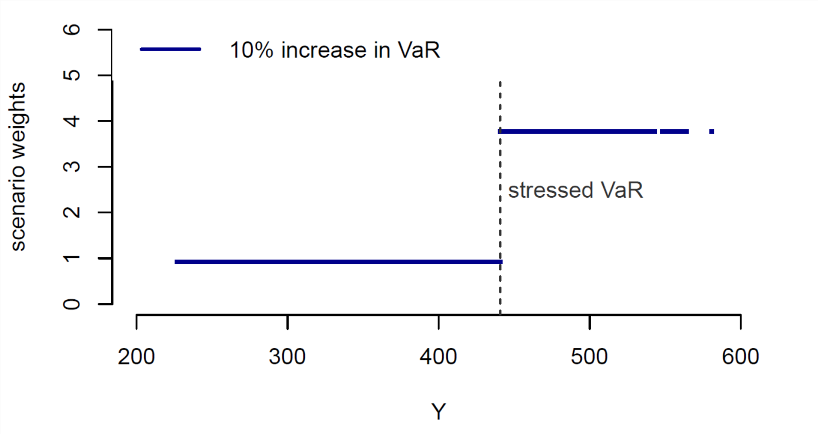
- [Pesenti et al., 2019]

See also:

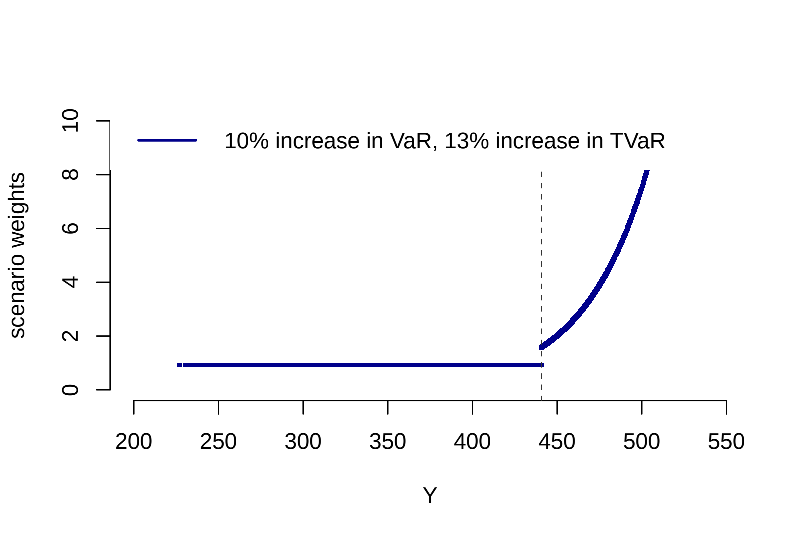
- [Weber, 2007], [Glasserman & Liu, 2010], [Breuer et al., 2012] [McNeil & Smith, 2012], [Cambou & Filipović, 2017]

Scenario weights for a stress on VaR

$$\frac{Prob(\text{Scenario } i \mid \text{high } Y)}{Prob(\text{Scenario } i \mid \text{low } Y)} = 4.10$$

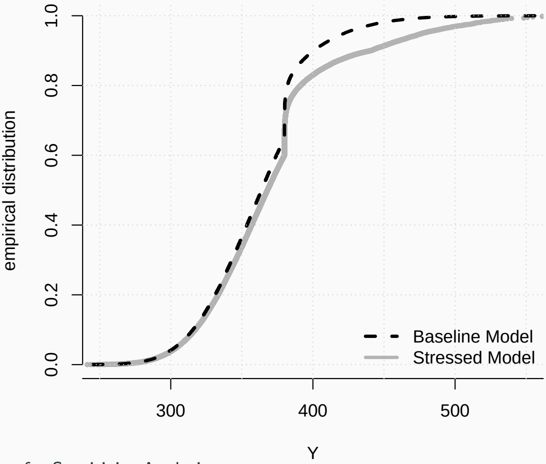


Scenario weights for a stress on VaR and TVaR

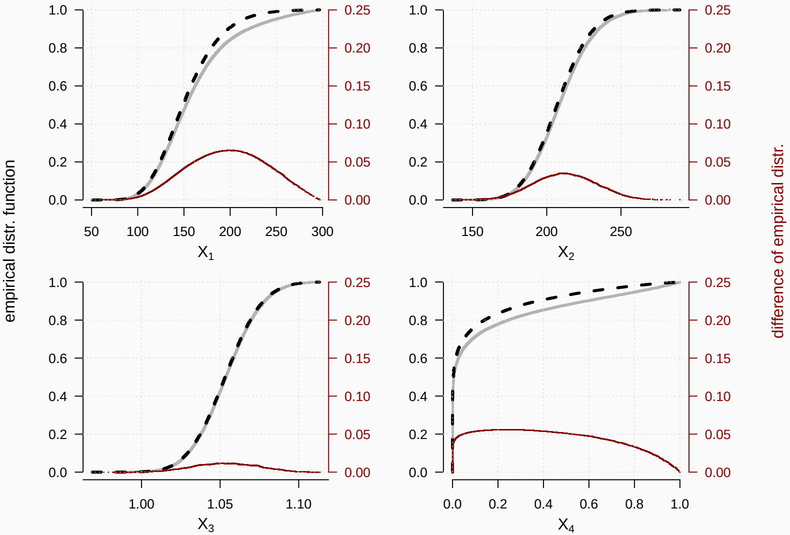


Insurance portfolio - Output

Stress VaR by 10% and TVaR by 13%, at level 0.95



Insurance portfolio - Inputs



Insurance portfolio - statistics

	X_1	X_2	X_3	X_4	Y
Mean	150	200	1.05	0.10	362
Mean, stressed	157	202	1.05	0.14	371
Relative increase	5%	1%	0%	44%	3%
Standard deviation	35	20	0.02	0.20	36
Standard deviation, stressed	43	21	0.02	0.26	50
Relative increase	25%	5%	1%	30%	38%

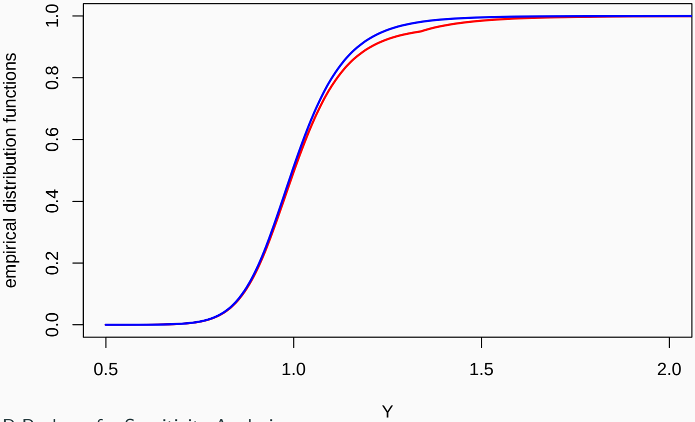
A sensitivity measure

Sensitivity measure for input risk factor X_i

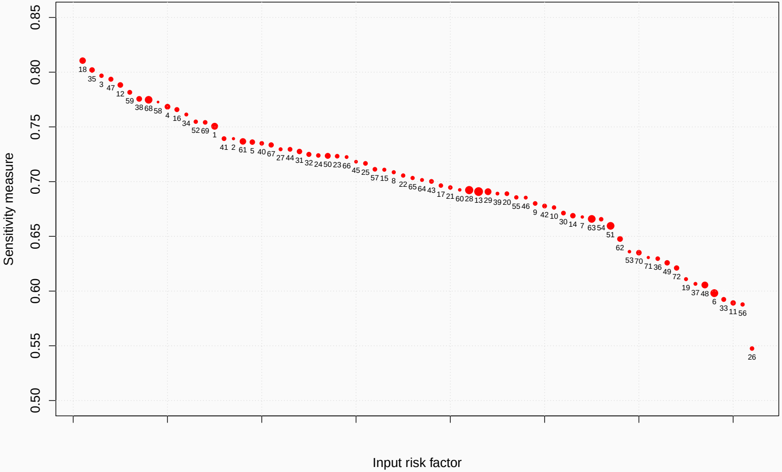
$$\Gamma_i = \frac{E^{\text{stressed}}(X_i) - E(X_i)}{\text{normalised}}$$

Real-data example

Distribution of the portfolio loss (blue) and after re-weighting (red).



Real-data example



The SWIM package in R

State of play

Current location of the package:

- https://github.com/spesenti/SWIM

  `install_github("spesenti/SWIM")`

Coming soon

- CRAN & vignette

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From the editors of the *Annals of Actuarial Science*

- Special issue on Insurance Data Science

  https://www.cambridge.org/core/journals/annals-of-actuarial-science/information/call-for-papers

Stress

```
stress(type = c("VaR", "VaR ES", "mean", "mean
        sd", "moment", "prob", "user"), ...)
```

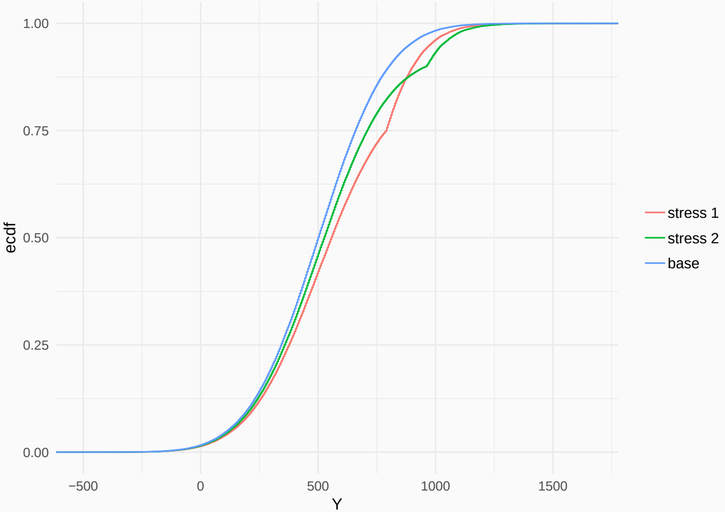
Stress

```
stress(type = c("VaR", "VaR ES", "mean", "mean
        sd", "moment", "prob", "user"), ...)
```

```
stress(type = "VaR", x, alpha = c(0.75, 0.9),
        q_ratio = c(1.2, 1.2), k = 1)
```

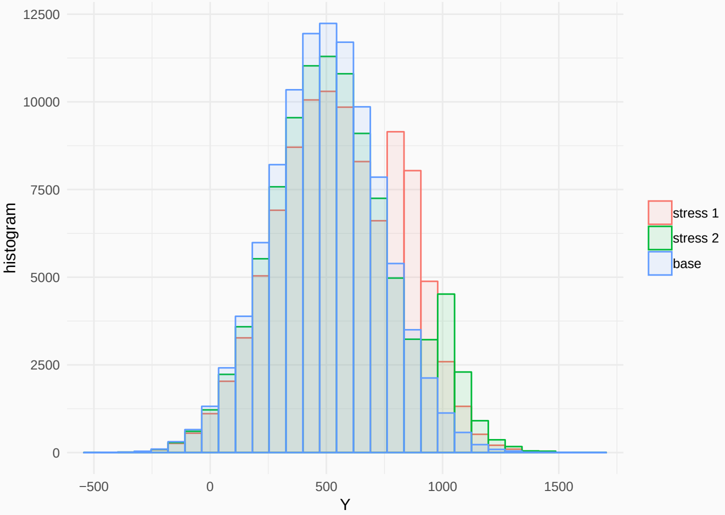
Plotting

`plot_cdf()`



Plotting

`plot_hist()`



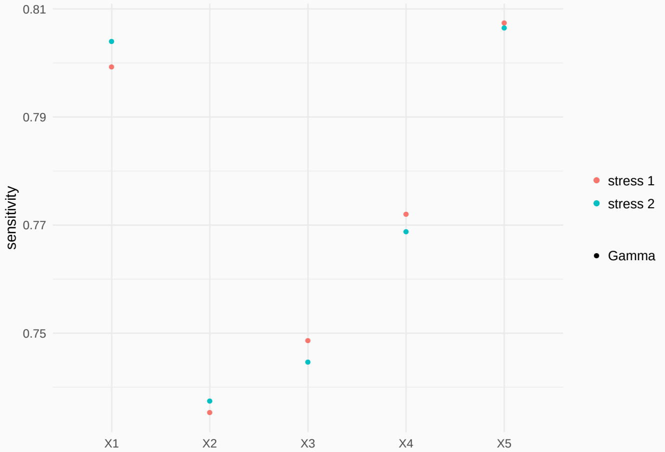
Statistics

```
summary()
```

```
$`stress 1`
                   Y     X1     X2     X3     X4     X5
mean          563.41 116.05 108.75 109.98 112.15 116.48
sd            263.89  81.20  46.24  52.49  63.53  81.39
skewness       -0.05  -0.03  -0.01  -0.02  -0.03  -0.03
ex kurtosis    -0.43  -0.22  -0.16  -0.16  -0.17  -0.23
1st Qu.       374.89  59.70  77.01  73.80  68.30  59.76
Median        555.48 116.12 108.81 109.99 112.20 116.35
3rd Qu.       788.79 173.01 140.67 146.25 156.63 174.25
```

Sensitivity Measures

`sensitivity()` `importance_rank()` `plot_sensitivity()`



THANK YOU FOR YOUR ATTENTION!

`https://github.com/spesenti/SWIM`

```
install_github("spesenti/SWIM")
```

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