Optimal investment and consumption in a market with random coefficients and different rates for lending and borrowing

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June 27, 2019

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Problem formulation

- Market with two different asset types (risky and risk-less assets).
- Bank account and stock with one dimensional standard Brownian motion.
- Short selling is not allowed, stock can be acquired by borrowing at lending rate $R(t)$, which is higher than the interest rate $r(t)$ i.e. $R(t) > r(t)$, $\forall t \in [0, T]$.
- According to Fleming & Zariphopoulou (1991), the investor's wealth is given by

$$\begin{cases} dy(t) = \Big[r(t)y(t) + (\mu(t) - r(t))\pi(t) \\ \qquad\qquad - (R(t) - r(t))\phi(t) - c(t) \Big] dt + \sigma(t)\pi(t)dW, \quad t \in [0, T], \\ y(0) = y_0 > 0. \end{cases} \tag{1}$$

Problem formulation

- The objective is to maximise the expected utility from consumption and terminal wealth under the following control variables:

$$J\Big(\pi(\cdot), c(\cdot), \phi(\cdot)\Big) := -\,\mathbb{E}\Big[\int_0^T c^{\gamma}(t)dt + y^{\gamma}(T)\Big], \qquad (2)$$

where $\gamma \in (0,1)$. Hence, we can write our optimization problem in the following form

$$\begin{cases} \min\limits_{(\pi,c,\phi)\in\mathcal{A}} J\Big(\pi(\cdot), c(\cdot), \phi(\cdot)\Big), \\ s.t.\ (1), \end{cases} \qquad (3)$$

- Two types of utility functions are considered, the power utility and the logarithmic utility.

References

FLEMING, W.H. & ZARIPHOPOULOU, T. (1991) An optimal investment/ consumption model with borrowing. *Mathematics of Operations Research*, **16**, 802-822.

Thank You!