SMITH–PURCELL RADIATION FROM A CHAIN OF SPHERES

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Introduction:

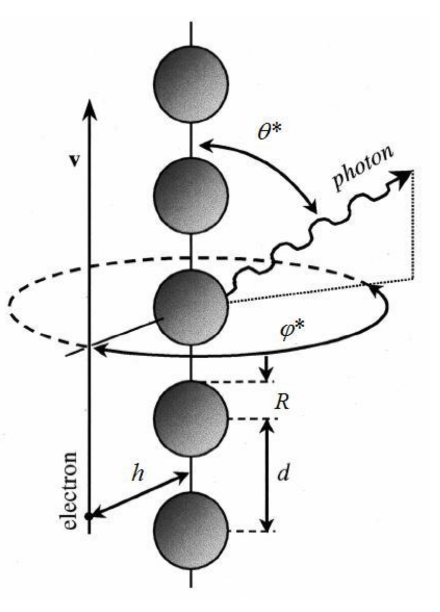
Smith-Purcell and diffraction radiation were investigated [1]. These types of radiation, appearing when a charged particle moves close to a conducting target, are the most promising techniques for the applications, as a non-invasive beam diagnostics.

Of late years, close attention has been paid to investigation of Smith-Purcell and diffraction radiation from two and one-dimensional systems, that consist of small spherical particles [2-4]. In [2] radiation emission probability and electron energy loss spectra for finite and infinite strings of Al and silica spheres were obtained by solving Maxwell's equations in the frames of a multipole expansion approach. The results obtained in Ref. [2] are rather general, but they are expressed in terms of spherical harmonic expansion, which makes the formulas rather complicated. Besides, nonrelativistic case only is analysed in this work.

Another way to describe electromagnetic processes in matter consists in calculation of the local field that acts on a single particle in a physical system. This work is devoted to development of theoretical basis of interaction between a moving charge and a chain of spherical particles characterized by arbitrary dielectric function. The local field theory for the chain of spheres was developed. Local field effects relate to physical interaction between scattering particles and also proved to lead to a sharp increase of the radiation intensity at some frequencies [3, 5].

Spectral and angular distribution of diffraction radiation from the non-periodic chain of spheres is obtained analytically; local field effects are discussed. Analytical expression for distribution of Smith-Purcell radiation from the periodic chain of spheres is obtained as well. For the first time it has been shown, that Smith-Purcell radiation for such a system distributed irregularly along the cone. The results are investigated for particles of different sizes, dielectric and metal, and for both ultrarelativistic and nonrelativistic cases.

Problem's statement:



- Spherical particles are assumed to be of different sizes, dielectric or metal.
- In dipole approximation a wavelength of emitted radiation must be much greater than a size of particles: $\lambda_{rad} >> R$
- Electron's energy losses are assumed to be negligible to it's kinetic energy: $E_{los} \ll E_{kin}$

Using microscopic Maxwell's equations exact microscopic field can be obtained as:

$$\mathbf{E}_{mic}(\mathbf{r},\omega)=\mathbf{E}_0(\mathbf{r},\omega)-\frac{1}{2\pi^2}\int\frac{d^3l}{l^2-\frac{\omega^2}{c^2}}\sum_b\left[\mathbf{l}\left[\mathbf{l},\mathbf{d}(\mathbf{R}_b,\omega)\right]\right]\exp\left(i\mathbf{l}(\mathbf{r}-\mathbf{R}_b)\right) \qquad (1)$$

where $\mathbf{d}(\mathbf{R_b},\omega)$ is spherical particle's dipole moment.
The main idea of local field theory consists in replacement of exact microscopic field by some effective field, called local field:

$$\mathbf{E}_{loc}(\mathbf{r},\omega)=\mathbf{E}_0(\mathbf{r},\omega)-\frac{1}{2\pi^2}\int\frac{d^3l}{l^2-\frac{\omega^2}{c^2}}\sum_b\left[\mathbf{l}\left[\mathbf{l},\mathbf{d}(\mathbf{R}_b,\omega)\right]\right]\exp\left(i\mathbf{l}(\mathbf{r}-\mathbf{R}_b)\right) \qquad (2)$$

The value of the local field can be obtained as a result of averaging over distribution of all the rest particles relative to the certain a-th particle.

Local field effects in diffraction radiation from the non-periodic chain of spherical particles:

Distribution of emitted diffraction radiation with respect to an angle and a frequency can be written as:

$$d^2E(\mathbf{k},\omega)=(2\pi)^6\frac{\omega^2 d\omega d\Omega}{c^3}\left|\left[\mathbf{n},\mathbf{j}(\mathbf{k},\omega)\right]\right|^2, \qquad (3)$$

where the expression for the current density, generated in the chain was obtained as:

$$j_k(\mathbf{k},\omega)=-\frac{i\omega}{(2\pi)^3}\alpha(\omega)\left\{\sum_{b=1}^{N-1}\frac{1}{1+\frac{\Theta(\omega)}{2}}\delta_{ki}E_i^0(\mathbf{R}_b,\omega)+\right.$$

$$\left.+\frac{3}{2}\sum_{b=1}^{N-1}\frac{\Theta(\omega)}{\left(1+\frac{\Theta(\omega)}{2}\right)\left(1-\Theta(\omega)\right)}e_k e_i E_i^0(\mathbf{R}_b,\omega)\right\}\exp(-i\mathbf{k}\mathbf{R}_b); \qquad (4)$$

where N is number of spherical particles in the chain and:

$$\Theta(\omega)=\frac{\alpha(\omega)}{L}\iint dl_x dl_z\, f(l_x)\left[1-\Phi\left(\sqrt{\frac{l_x^2+l_z^2}{2}}\Delta\right)\right]\frac{l_x^2}{\sqrt{l_x^2+l_z^2}}\exp\left(\frac{l_x^2\Delta^2}{2}\right); \qquad (5)$$

$\alpha(\omega)=R^3\frac{\varepsilon(\omega)-1}{\varepsilon(\omega)+2}$ is polarizability of every single sphere in the chain [6,7];
R is spherical particles' diameter; Φ is errors' function;
$\varepsilon(\omega)$ is dielectric conductivity function of particles' material;
Δ is particles' offset along y, z axes and L is chain's length.

The value of (5) can have an imaginary part, so the denominators in (3) and (4) are not zero. However, they can be very small, that implies the peaks of radiation intensity.

Local field effects in Smith-Purcell radiation from the periodic chain of spherical particles:

In this section Smith-Purcell effect will be considered. It is suggested that:

- The chain of spheres is periodic.
- In the first approximation on each particle in the chain acts electron's self-field only.

Expression for distribution of emitted Smith-Purcell radiation with respect to an angle and a frequency was obtained as:

$$\frac{d^2E(\mathbf{n},\omega)}{d\omega d\Omega}=\frac{e^2}{c}\frac{k^6R^6}{\pi^2\beta^4}\left(\frac{\varepsilon(\omega)-1}{\varepsilon(\omega)+2}\right)^2\frac{\sin^2\left((\beta^{-1}-n_x)\frac{kdN}{2}\right)}{\sin^2\left((\beta^{-1}-n_x)\frac{kd}{2}\right)}\times$$

$$\left\{\left(\frac{n_y}{\gamma}K_1\left(\frac{kh}{\beta\gamma}\right)\right)^2+\left(\frac{n_x}{\gamma}K_1\left(\frac{hk}{\beta\gamma}\right)-\frac{n_z}{\gamma^2}K_0\left(\frac{kh}{\beta\gamma}\right)\right)^2+\left(\frac{n_y}{\gamma^2}K_0\left(\frac{kh}{\beta\gamma}\right)\right)^2\right\}, \qquad (6)$$

where R is spherical particles' diameter, $\varepsilon(\omega)$ is dielectric conductivity function of spheres' material, h is impact parameter or distance between the chain and the charged particle, d is spacing between spherical particles, K_0,K_1 are Bessel's functions of zero and first orders respectively.

Expressions for ultrarelativistic and nonrelativistic cases can be performed as well. In nonrelativistic case, when $kh<<\beta<<1$, (6) can be transformed as:

$$\frac{d^2E(\mathbf{n},\omega)}{d\omega d\Omega}=\frac{e^2}{c}\frac{k^4R^6}{\pi^2h^2\beta^2}\left(\frac{\varepsilon(\omega)-1}{\varepsilon(\omega)+2}\right)^2\frac{\sin^2\left((\beta^{-1}-n_x)\frac{kdN}{2}\right)}{\sin^2\left((\beta^{-1}-n_x)\frac{kd}{2}\right)}(1-n_z^2), \qquad (7)$$

where n_x,n_z are x and z components of the unit vector between $\mathbf{v}$ and $\mathbf{k}$.

In ultrarelativistic case, when $\gamma \gg 1$, (6) can be written as:

$$\frac{d^2E(\mathbf{n},\omega)}{d\omega d\Omega}=\frac{e^2}{c}\frac{k^4R^6}{\pi^2h^2}\left(\frac{\varepsilon(\omega)-1}{\varepsilon(\omega)+2}\right)^2\frac{\sin^2\left((1-n_x)\frac{kdN}{2}\right)}{\sin^2\left((1-n_x)\frac{kd}{2}\right)}(1-n_z^2)\left(\frac{kh}{\gamma}K_1\left(\frac{kh}{\gamma}\right)\right)^2 \qquad (8)$$

Taking into account the condition for the local field effects:

$$\frac{\sin^2\left(\frac{pfN}{2}\right)}{\sin^2\left(\frac{pf}{2}\right)}\xrightarrow{N\gg 1}2pN\sum_m d(pf-2pm) \qquad (9)$$

when calculating (6), leads us to the definition of the Smith-Purcell relation:

$$\frac{d}{\lambda}(\beta^{-1}-n_x)=m,\quad m=1,2... \qquad (10)$$

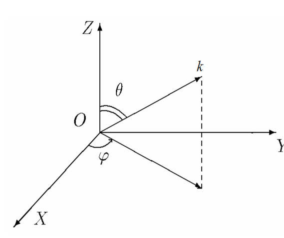
where m is order of emitted radiation.

Graphical analysis:

Distribution of Smith-Purcell radiation from the chain of polyethylene spheres is considered in two geometries.

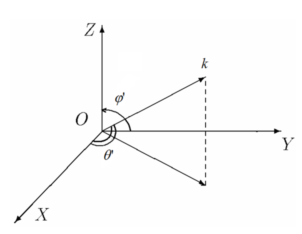
The first one is:

$$n_x=\sin\theta\cos\phi,\quad n_y=\sin\theta\sin\phi,\quad n_z=\cos\theta.$$



The second one is:

$$n_x=\cos\theta',\quad n_y=\sin\theta'\cos\phi',\quad n_z=\sin\theta'\sin\phi'.$$



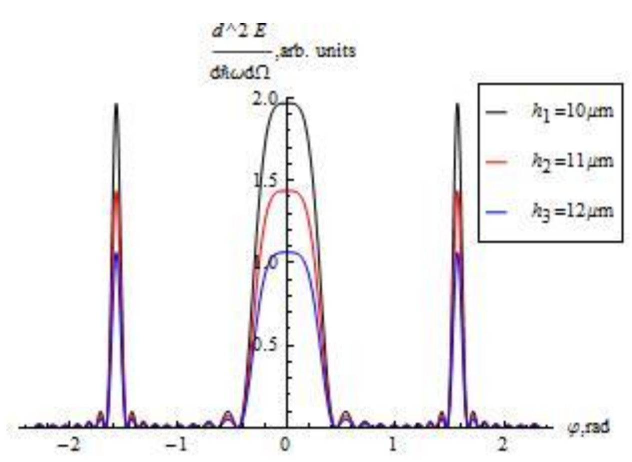


Fig1. Angular distribution of Smith-Purcell radiation for $\lambda=700nm$ in ultrarelativistic case. Distribution of radiation is presented in first geometry for three different values of the impact parameter and the rest parameters chosen as: $\gamma=100, d=700nm, R=100nm, N=10, \varepsilon_{HDPE}=2{,}37, m=1, \theta=\frac{\pi}{2}, \phi\in[-\pi,\pi]$.

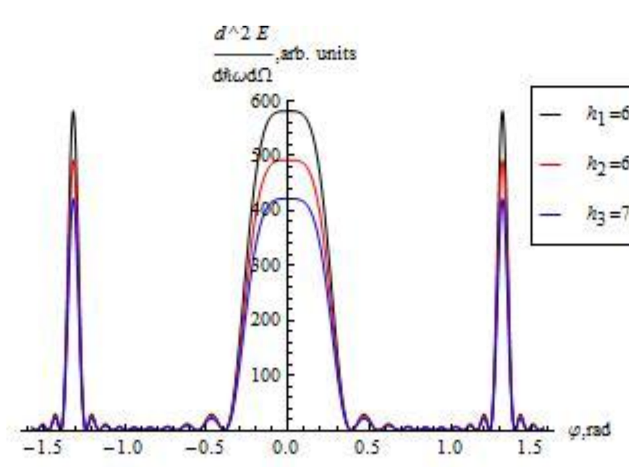


Fig.2 Angular distribution of Smith-Purcell radiation for $\lambda=3mm$ in ultrarelativistic case. Distribution of Smith-Purcell radiation is presented in first geometry for three different values of the impact parameter and the rest parameters chosen as: $\gamma=100, d=4mm, R=0{,}5mm, N=10, \varepsilon_{HDPE}=2{,}37, m=1, \theta=\frac{\pi}{2}, \phi\in[-\pi,\pi]$.

Analysis of the peaks positions, that correspond to the maximums of Smith-Purcell radiation in the different planes of the first geometry, lets us assume that Smith-Purcell radiation is distributed over the cone. It means that switching to the second geometry helps us to find out how exactly the Smith-Purcell radiation is distributed over the cone.

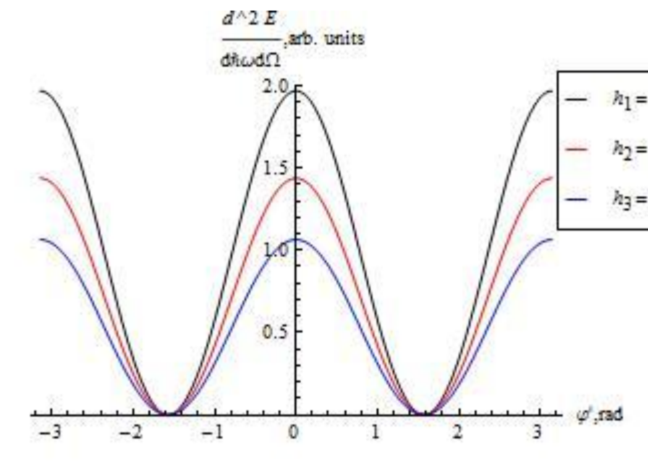


Fig.3 Angular distribution of Smith-Purcell radiation over the cone in the second geometry for $\lambda=700nm$. This distribution corresponds to the peaks of Smith-Purcell radiation in the first geometry, presented on Fig.1.

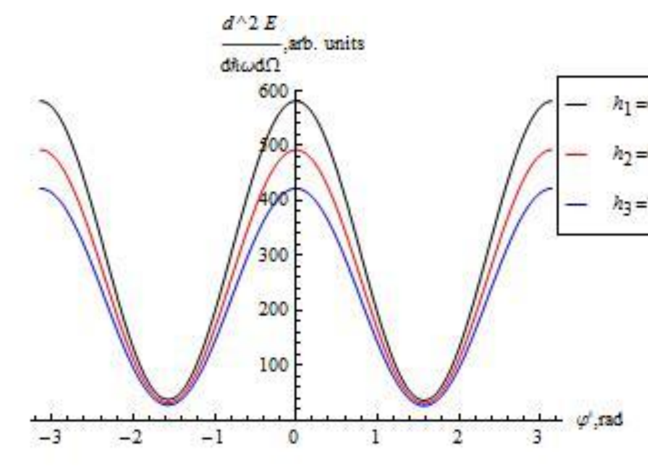


Fig.4 Angular distribution of Smith-Purcell radiation over the cone in the second geometry for $\lambda=3mm$. This distribution corresponds to the peaks of Smith-Purcell radiation in the first geometry, presented on Fig.2.

Results' region of validity and local field criterion:

From (10) it is possible to derive region of validity for (6). Smith-Purcell radiation for nonrelativistic charged particle ($\beta \ll 1$) is possible when:

$$\lambda \approx \frac{d}{\beta m} \gg d, \qquad (11)$$

and for ultrarelativistic charge ($\gamma \gg 1$) when:

$$\frac{\gamma^{-2}}{2}\frac{d}{m} \leq \lambda \leq 2\frac{d}{m} \qquad (12)$$

As was pointed out earlier, local field effects relate to physical interaction between scattering particles and also proved to lead to a sharp increase of the radiation intensity at some frequencies. These frequencies can be found from the system:

$$\begin{cases}1+\frac{\theta(\omega)}{2}=0;\\ 1-\theta(\omega)=0.\end{cases} \qquad (12)$$

where

$$\Theta(\omega)=\frac{\alpha(\omega)}{L}\iint dl_x dl_z\, f(l_x)\left[1-\Phi\left(\sqrt{\frac{l_x^2+l_z^2}{2}}\Delta\right)\right]\frac{l_x^2}{\sqrt{l_x^2+l_z^2}}\exp\left(\frac{l_x^2\Delta^2}{2}\right).$$

Conclusion:

- The local field theory for the chain of spheres was developed.
- Spectral and angular distribution of diffraction radiation from the non-periodic chain of spheres was obtained analytically.
- Smith-Purcell effect for the case of periodic chain of spherical particles was investigated and analytical expression for the distribution of Smith-Purcell radiation was obtained.
- For the first time it has been shown, that Smith-Purcell radiation, for such a system, is distributed irregularly along the cone.

References:

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