Swimming Phytoplankton in Turbulence

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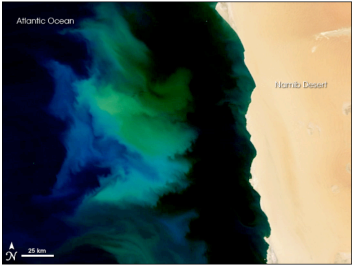
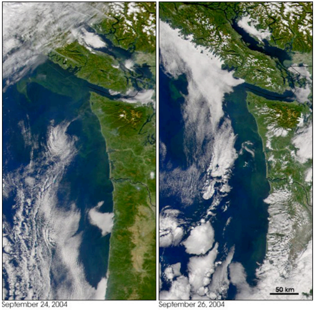
Introduction

Motivation: modelling the evolution of the spatial distribution of phytoplankton in the ocean.



Phytoplankton bloom off coast of Namibia.



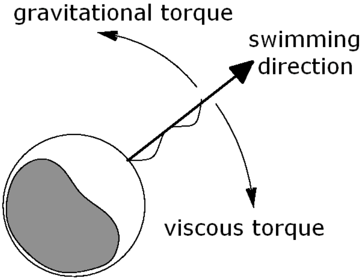
Harmful algal bloom (HAB) off NW coast of United States.

From *http://visibleearth.nasa.gov/*

Aim of present work: Develop a population-level model of motile phytoplankton in turbulent flow fields.

Gyrotaxis

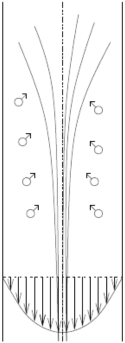
Many motile algae are bottom heavy, and so undergo gyrotaxis:

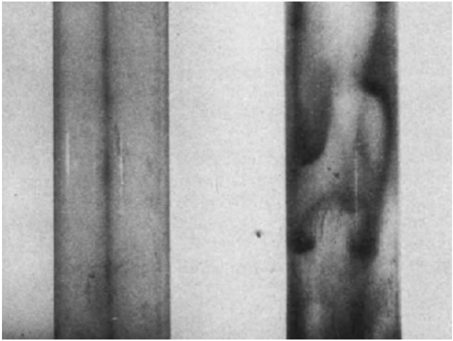


What is the average swimming direction?

Gyrotactic focussing

Example interaction of this mechanism:





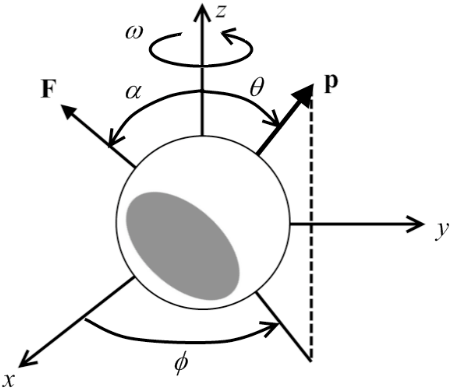
Kessler, J.O., *Nature*, 313: 218-220 (1985)

Gyrotaxis in simple flow fields

Write the orientation **p** as

$$\mathbf{p} = (\cos\phi \sin\theta, \sin\phi \sin\theta, \cos\theta)$$

using spherical polar angles θ and ϕ:



Gyrotaxis in simple flow fields

Gyrotaxis force-balance:

$$\dot{\mathbf{p}} = \overbrace{\frac{1}{2B}\left(\mathbf{F} - (\mathbf{F}.\mathbf{p})\mathbf{p}\right)}^{\text{gravitational torque}} + \underbrace{\frac{1}{2}\boldsymbol{\omega} \wedge \mathbf{p}}_{\text{viscous torque}}$$

where B is reorientation time (bottom heaviness) and $\boldsymbol{\omega}$ is the fluid vorticity. See, for instance, Pedley, T. J. & Kessler, J. O., *Ann. Rev. Fluid Mech.*, 24: 313-358 (1992)

In terms of θ and ϕ:

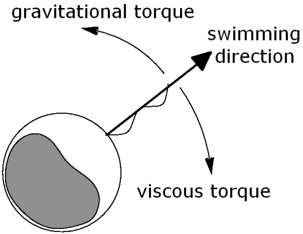
$$\dot{\theta} = -\frac{1}{2B}\left(\sin\theta\cos\alpha - \cos\theta\sin\alpha\cos\phi\right)$$

$$\dot{\phi} = \Omega - \frac{1}{2B}\frac{\sin\phi\sin\alpha}{\sin\theta}$$

Equilibria for $\alpha = \pi/2$

If the restoring force and the vorticity are perpendicular, a stable equilibrium only exists if $\lambda = 2\Omega B < 1$:

$$\cos\theta_{eq} = 0$$
$$\sin\phi_{eq} = \lambda$$



For $\lambda \geq 1$, the vorticity is too strong to allow organisms to reach a stable orientation and so tumbling occurs.

Equilibria for $\alpha \neq \pi/2$

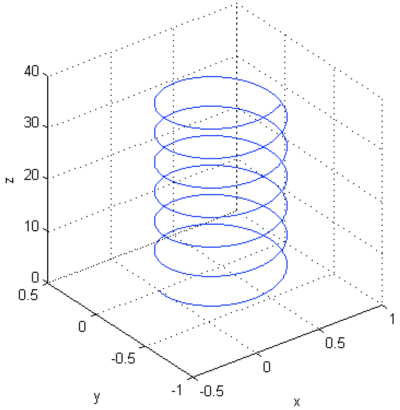
Key NEW result: for $\alpha \neq \pi/2$, a stable equilibrium direction always exists:

$$\cos\theta_{eq} = \sqrt{\frac{\lambda^2 - 1 + \sqrt{(\lambda^2-1)^2 + 4\lambda^2\cos^2\alpha}}{2\lambda^2}}$$

$$\tan\phi_{eq} = \frac{\lambda\cos\theta_{eq}}{\cos\alpha}.$$

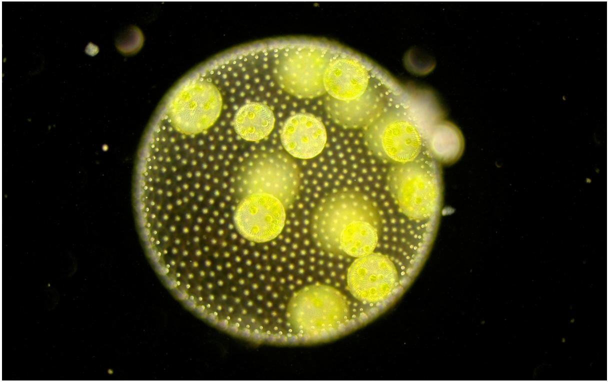
Long-term transport

For a solid body rotation flow, i.e. $\mathbf{U}(\mathbf{x}) = (\Omega y, -\Omega x, 0)$, and $\alpha \neq \pi/2$, the cell trajectory is helical with transport along vorticity vector:



Experimental work

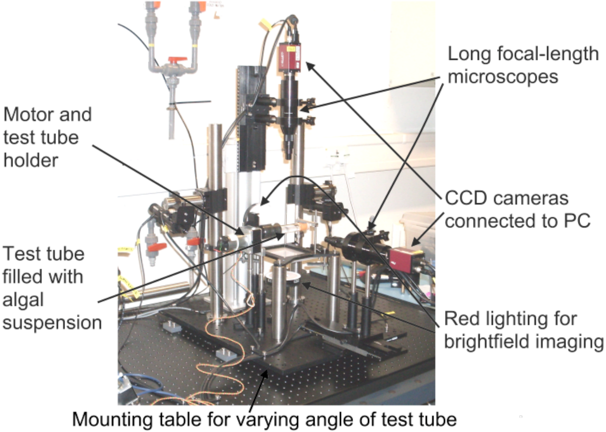
3D tracking of the spherical alga *Volvox carteri* in solid-body rotation:



Diameter of alga between 150-700 μm: individuals are bottom-heavy due to daughter colonies (*gonidia*) in posterior.

Experimental setup

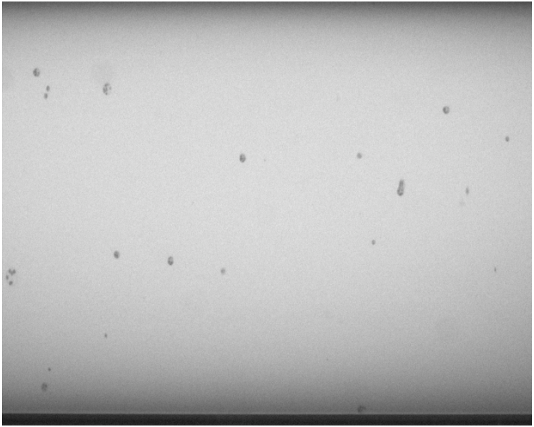
Experimental setup:



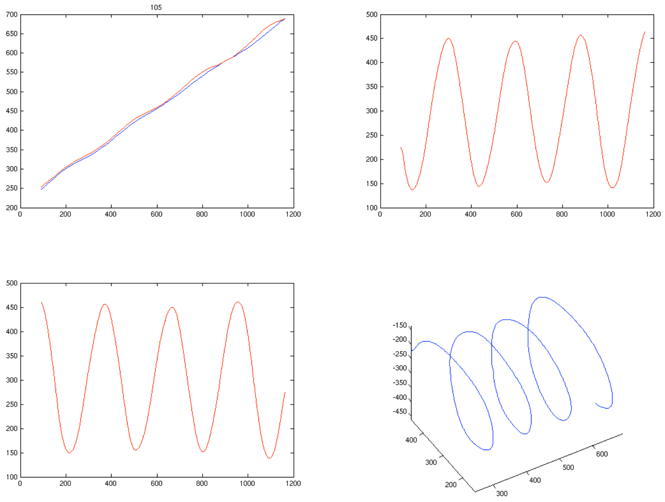
Angle of test tube and rotational speed can be changed independently.

Experimental video

For $\alpha = 2\pi/3$, $\Omega = 0.75$:

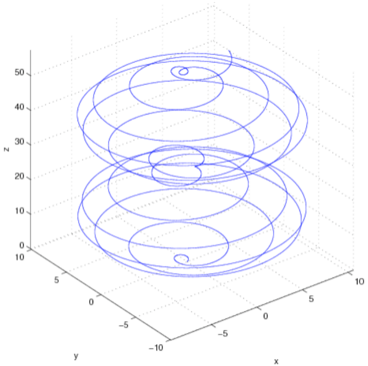


Reconstructed track



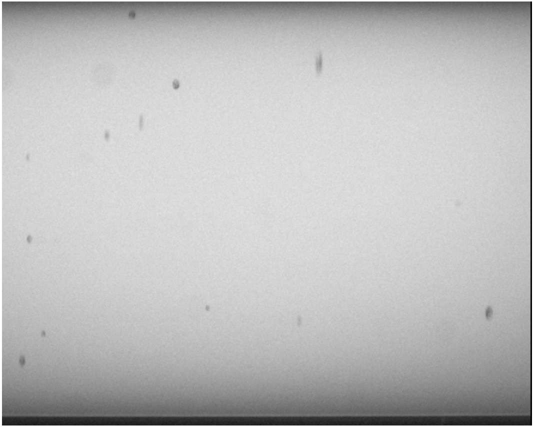
What happens if there is no stable equilibrium?

In this case ($\alpha = \pi/2$, $2\Omega B > 1$), there is no stable equilibrium so cells tumble and trajectories are more complicated, there is no closed form:



Experimental video showing this trajectory

For $\alpha = \pi/2$, $\Omega = 1$:

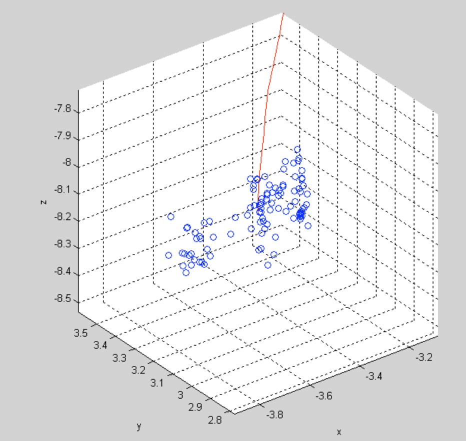


Simulations of gyrotactic algae in turbulent flow

- Compute velocity field of turbulent flow for particular turbulent strength.
- Compute trajectories of individuals undergoing stochastic gyrotaxis in this flow field.
- Compute population-level statistics such as mean position, and mean dispersion.

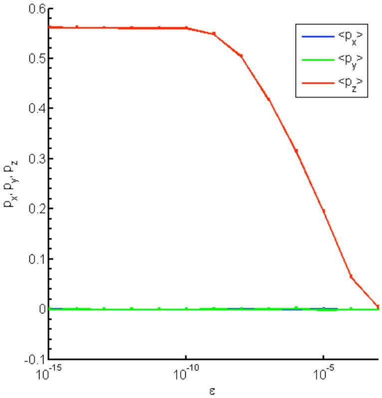
Simulation results

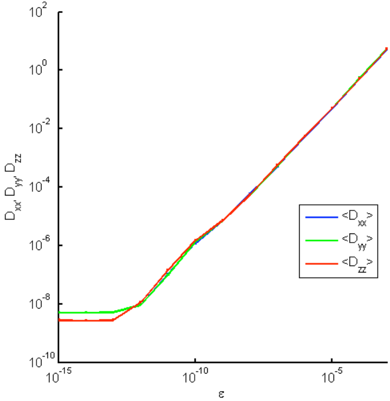
For 100 algae initially around the origin in a flow with $\epsilon = 10^{-3}$ m s^{-2}:



Numerical results

Plot of mean swimming and mean diffusion against strength of turbulence ϵ:





Summary

- Obtained new expression for mean swimming direction of gyrotactic cells in 3D flows
- Calculated individual cell trajectories in simple flow fields
- Good agreement between experiments and theoretical predictions
- Computed population-level transport in numerical simulation of individual cells in turbulence