Mathematics  MMath

COURSE DETAILS

- A level requirements: AAB
- UCAS code: G101
- Study mode: Full-time
- Length: 4 years

KEY DATES

- Apply by: 25 January 2023
- Starts: 25 September 2023

Course overview

Studying Mathematics at Liverpool is an excellent foundation for a wide range of careers. This four year programme means you will graduate with a Master’s qualification and is the ideal path to a PhD or industry research post. You will also have the option to spend an academic year abroad on this course.

INTRODUCTION

Mathematics is a fascinating, beautiful and diverse subject to study. It underpins a wide range of disciplines; from physical sciences to social science, from biology to business and finance. At Liverpool, our programmes are designed with the needs of employers in mind, to give you a solid foundation from which you may take your career in any number of directions.

A Mathematics degree at the University of Liverpool is an excellent investment in your future. We have a large department with highly qualified staff, a first-class reputation in teaching and research, and a great city in which to live and work. You will see a broad range of degree programmes at Liverpool – Mathematics can be combined with many other subjects to widen your options even further.

In the first two years of this programme, you will study a range of topics covering important areas of both pure and applied mathematics, no assumptions are made about whether or not you have previously studied mechanics or statistics, or have previous experience of the use of computers. The modules studied in year one help to get all students at the same level, studying fundamental ideas and reinforcing A level work.

This programme also has a year abroad option, an incredible opportunity to spend an academic year at one of our partner universities.
Students graduating from this programme are well placed to go on to a PhD or take a research post in industry.

**WHAT YOU’LL LEARN**

- Pure and applied mathematics
- Mechanics
- Statistics
- Teamwork

- Problem solving
- How to communicate and present clearly
Course content
Discover what you’ll learn, what you’ll study, and how you’ll be taught and assessed.

YEAR ONE
In year one you will study a range of compulsory modules.

COMPULSORY MODULES

CALCULUS I (MATH101)
Credits: 15 / Semester: semester 1
At its heart, calculus is the study of limits. Many quantities can be expressed as the limiting value of a sequence of approximations, for example the slope of a tangent to a curve, the rate of change of a function, the area under a curve, and so on. Calculus provides us with tools for studying all of these, and more. Many of the ideas can be traced back to the ancient Greeks, but calculus as we now understand it was first developed in the 17th Century, independently by Newton and Leibniz. The modern form presented in this module was fully worked out in the late 19th Century. MATH101 lays the foundation for the use of calculus in more advanced modules on differential equations, differential geometry, theoretical physics, stochastic analysis, and many other topics. It begins from the very basics – the notions of real number, sequence, limit, real function, and continuity – and uses these to give a rigorous treatment of derivatives and integrals for real functions of one real variable.

CALCULUS II (MATH102)
Credits: 15 / Semester: semester 2
This module, the last one of the core modules in Year 1, is built upon the knowledge you gain from MATH101 (Calculus I) in the first semester. The syllabus is conceptually divided into three parts: Part I, relying on your knowledge of infinite series, presents a thorough study of power series (Taylor expansions, binomial theorem); part II begins with a discussion of functions of several variables and then establishes the idea of partial differentiation together with its various applications, including chain rule, total differential, directional derivative, tangent planes, extrema of functions and Taylor expansions; finally, part III is on double integrals and their applications, such as finding centres of mass of thin bodies. Undoubtedly, this module, together with the other two core modules from Semester 1 (MATH101 Calculus I and MATH103 Introduction to linear algebra), forms an integral part of your ability to better understand modules you will be taking in further years of your studies.

INTRODUCTION TO LINEAR ALGEBRA (MATH103)
Credits: 15 / Semester: semester 1
Linear algebra is the branch of mathematics concerning vector spaces and linear mappings between such spaces. It is the study of lines, planes, and subspaces and their intersections using algebra.

Linear algebra first emerged from the study of determinants, which were used to solve systems of linear equations. Determinants were used by Leibniz in 1693, and subsequently, Cramer's Rule for solving linear systems was devised in 1750. Later, Gauss further developed the theory of solving linear systems by using Gaussian elimination. All these classical themes, in their modern interpretation, are included in the module, which culminates in a detailed study of eigenproblems. A part of the module is devoted to complex numbers which are basically just planar vectors. Linear algebra is central to both pure and applied mathematics. This module is an essential pre-requisite for nearly all modules taught in the Department of Mathematical Sciences.

**INTRODUCTION TO STATISTICS USING R (MATH163)**

**Credits: 15 / Semester: semester 2**

Students will learn fundamental concepts from statistics and probability using the R programming language and will learn how to use R to some degree of proficiency in certain contexts. Students will become aware of possible career paths using statistics.

**MATHEMATICAL IT SKILLS (MATH111)**

**Credits: 15 / Semester: semester 1**

This module introduces students to powerful mathematical software packages such as Maple and Matlab which can be used to carry out numerical computations or to produce a more complicated sequence of computations using their programming features. We can also do symbolic or algebraic computations in Maple. These software packages have built-in functions for solving many kinds of equations, for working with matrices and vectors, for differentiation and integration. They also contain functions which allow us to create visual representations of curves and surfaces from their mathematical descriptions, to work interactively, generate graphics and create mathematical documents. This module will teach students many of the above-mentioned features of mathematical software packages. This knowledge will be helpful in Years 2, 3 and 4 when working on different projects, for example in the modules MATH266 and MATH371.

**INTRODUCTION TO STUDY AND RESEARCH IN MATHEMATICS (MATH107)**

**Credits: 15 / Semester: semester 1**
This module looks at what it means to be a mathematician as an undergraduate and beyond. The module covers the discussion of mathematics at university, research mathematics and careers for mathematicians as well as core elements of mathematical language and writing such as logic, proofs, numbers, sets and functions. The activities include sessions delivered by staff on their research areas, sessions by alumni and other mathematicians working outside academia on careers for mathematicians and sessions by careers services. The module also provides key tools needed for studying mathematics at university level. You will explore the core mathematical concepts in more detail in groups and individually and practice communicating mathematics in speech and writing.

NEWTONIAN MECHANICS (MATH122)

Credits: 15 / Semester: semester 2

This module is an introduction to classical (Newtonian) mechanics. It introduces the basic principles like conservation of momentum and energy, and leads to the quantitative description of motions of bodies under simple force systems. It includes angular momentum, rigid body dynamics and moments of inertia. MATH122 provides the foundations for more advanced modules like MATH228, 322, 325, 326, 423, 425 and 431.

NUMBERS, GROUPS AND CODES (MATH142)

Credits: 15 / Semester: semester 2

A group is a formal mathematical structure that, on a conceptual level, encapsulates the symmetries present in many structures. Group homomorphisms allow us to recognise and manipulate complicated objects by identifying their core properties with a simpler object that is easier to work with. The abstract study of groups helps us to understand fundamental problems arising in many areas of mathematics. It is moreover an extremely elegant and interesting part of pure mathematics. Motivated by examples in number theory, combinatorics and geometry, as well as applications in data encryption and data retrieval, this module is an introduction to group theory. We also develop the idea of mathematical rigour, formulating our theorems and proofs precisely using formal logic.

Programme details and modules listed are illustrative only and subject to change.

YEAR TWO

In year two, you will choose some compulsory and some optional modules from the list below. Please note that we regularly review our teaching so the choice of modules may change.

COMPULSORY MODULES

DIFFERENTIAL EQUATIONS (MATH221)
Differential equations play a central role in mathematical sciences because they allow us to describe a wide variety of real-world systems and the mathematical techniques encountered in this module are useful to a number of later modules; this is why MATH201 is compulsory for a number of degree programmes. The module will aim to stress the importance of both theory and applications of ordinary differential equations (ODEs) and partial differential equations (PDEs), putting a strong emphasis on problem solving and examples. It has broadly 5 parts and each part contains two types of equations: those that can be solved by specific methods and others that cannot be solved but can only be studied to understand some properties of the underlying equations and their solutions. The main topics are first order ODEs, second order ODEs, systems of ODEs, first-order PDEs and some of the most well-known second-order PDEs, namely the wave, heat and Laplace equations.

VECTOR CALCULUS WITH APPLICATIONS IN FLUID MECHANICS (MATH225)

To provide an understanding of the various vector integrals, the operators div, grad and curl and the relations between them. To give an appreciation of the many applications of vector calculus to physical situations. To provide an introduction to the subjects of fluid mechanics and electromagnetism.

LINEAR ALGEBRA AND GEOMETRY (MATH244)

Linear algebra provides a toolbox for analysing phenomena ubiquitous in many areas of mathematics: linear maps, or linearity in general. In all of these situations it is essential to first identify the kind of objects which are mapped or behave in a linear way. To cover the many different possibilities the concept of an abstract vector space is introduced. It generalizes the real vector spaces introduced in MATH103 (Introduction to Linear Algebra) and the calculational techniques developed there can still be used. Applications of ideas from Linear Algebra appear in Geometry (MATH201, MATH349), in Algebra (MATH247, MATH343), in solving Differential Equations (MATH201, MATH221), which in turn model many physical systems (MATH323, MATH324), in Physics, especially Quantum Mechanics (MATH325, MATH421), in Biology (MATH335, MATH426) and in Statistics (MATH363).

STATISTICS AND PROBABILITY I (MATH253)

Analysis of data has become an essential part of current research in many fields including medicine, pharmacology, and biology. It is also an important part of many jobs in e.g. finance, consultancy and the public sector. This module provides an introduction to statistical methods with a strong emphasis on applying and interpreting standard statistical techniques. Since modern statistical analysis of real data sets is performed using computer power, a statistical software package is introduced and employed throughout.
COMPLEX FUNCTIONS (MATH243)
Credits: 15 / Semester: semester 1
This module introduces students to a surprising, very beautiful theory having intimate connections with other areas of mathematics and physical sciences, for instance ordinary and partial differential equations and potential theory.

OPTIONAL MODULES
CLASSICAL MECHANICS (MATH228)
Credits: 15 / Semester: semester 2
This module is concerned with the motion of physical bodies both in everyday situations and in the solar system. To describe motion, acceleration and forces you will need background knowledge of calculus, differentiation, integration and partial derivatives from MATH101 (Calculus I), MATH102 (Calculus II) and MATH103 (Introduction to Linear Algebra). Classical mechanics is important for learning about modern developments such as relativity (MATH326), quantum mechanics (MATH325) and chaos and dynamical systems (MATH322). This module will make you familiar with notions such as energy, force, momentum and angular momentum which lie at the foundations of applied mathematics problems.

METRIC SPACES AND CALCULUS (MATH242)
Credits: 15 / Semester: semester 2
This is a foundational module aimed at providing the students with the basic concepts and techniques of modern real Analysis. The guiding idea will be to start using the powerful tools of analysis, familiar to the students from the first year module MATH101 (Calculus I) in the context of the real numbers, to vectors (multivariable analysis) and to functions (functional analysis). The notions of convergence and continuity will be reinterpreted in the more general setting of metric spaces. This will provide the language to prove several fundamental results that are in the basic toolkit of a mathematician, like the Picard Theorem on the existence and uniqueness of solutions to first order differential equations with an initial datum, and the implicit function theorem. The module is central for a curriculum in pure and applied mathematics, as familiarity with these notions will help students who want to take several other subsequent modules as well as many projects. This module is also a useful preparation (although not a formal prerequisite) for MATH365 Measure theory and probability, a very useful module for a deep understanding of financial mathematics.

COMMUTATIVE ALGEBRA (MATH247)
Credits: 15 / Semester: semester 2
The module provides an introduction to the theory and methods of the modern commutative algebra (commutative groups, commutative rings, fields and modules) with some applications to number theory, algebraic geometry and linear algebra.

STATISTICS AND PROBABILITY II (MATH254)
This module provides an introduction to probabilistic methods that are used not only in actuarial science, financial mathematics and statistics but also in all physical sciences. It focuses on discrete and continuous random variables with values in one and several dimensions, properties of the most useful distributions (e.g. geometric, exponential, and normal), their transformations, moment and probability generating functions and limit theorems. This module will help students doing MATH260 and MATH262 (Financial mathematics). This module complements MATH365 (Measure theory and probability) in the sense that MATH365 provides the contradiction-free measure theoretic foundation on which this module rests.

FINANCIAL MATHEMATICS (MATH260)

Mathematical Finance uses mathematical methods to solve problems arising in finance. A common problem in Mathematical Finance is that of derivative pricing. In this module, after introducing the basic concepts in Financial Mathematics, we use some particular models for the dynamic of stock price to solve problems of pricing and hedging derivatives. This module is fundamental for students intending to work in financial institutions and/or doing an MSc in Financial Mathematics or related areas.

OPERATIONAL RESEARCH (MATH269)

The term "Operational Research" came in the 20th century from military operations. It describes mathematical methods to achieve the goal (or to find the best possible decision) having limited resources. This branch of applied mathematics makes use of and has stimulated the development of optimisation methods, typically for problems with constraints. This module can be interesting for any student doing mathematics because it concentrates on real-life problems.

MATHEMATICS EDUCATION AND COMMUNICATION (MATH291)

This module is designed to give students experience of communicating in a variety of media and in a variety of contexts. It will also introduce students to contemporary issues in education, and educational practice. This will be achieved by seminars, interactions with teachers and other educational professionals, and the design and delivery of enrichment materials, utilising the existing and highly successful outreach activity within the department.

NUMERICAL METHODS FOR APPLIED MATHEMATICS (MATH226)

This module provides an introduction to probabilistic methods that are used not only in actuarial science, financial mathematics and statistics but also in all physical sciences. It focuses on discrete and continuous random variables with values in one and several dimensions, properties of the most useful distributions (e.g. geometric, exponential, and normal), their transformations, moment and probability generating functions and limit theorems. This module will help students doing MATH260 and MATH262 (Financial mathematics). This module complements MATH365 (Measure theory and probability) in the sense that MATH365 provides the contradiction-free measure theoretic foundation on which this module rests.

FINANCIAL MATHEMATICS (MATH260)

Mathematical Finance uses mathematical methods to solve problems arising in finance. A common problem in Mathematical Finance is that of derivative pricing. In this module, after introducing the basic concepts in Financial Mathematics, we use some particular models for the dynamic of stock price to solve problems of pricing and hedging derivatives. This module is fundamental for students intending to work in financial institutions and/or doing an MSc in Financial Mathematics or related areas.

OPERATIONAL RESEARCH (MATH269)

The term "Operational Research" came in the 20th century from military operations. It describes mathematical methods to achieve the goal (or to find the best possible decision) having limited resources. This branch of applied mathematics makes use of and has stimulated the development of optimisation methods, typically for problems with constraints. This module can be interesting for any student doing mathematics because it concentrates on real-life problems.

MATHEMATICS EDUCATION AND COMMUNICATION (MATH291)

This module is designed to give students experience of communicating in a variety of media and in a variety of contexts. It will also introduce students to contemporary issues in education, and educational practice. This will be achieved by seminars, interactions with teachers and other educational professionals, and the design and delivery of enrichment materials, utilising the existing and highly successful outreach activity within the department.

NUMERICAL METHODS FOR APPLIED MATHEMATICS (MATH226)

This module provides an introduction to probabilistic methods that are used not only in actuarial science, financial mathematics and statistics but also in all physical sciences. It focuses on discrete and continuous random variables with values in one and several dimensions, properties of the most useful distributions (e.g. geometric, exponential, and normal), their transformations, moment and probability generating functions and limit theorems. This module will help students doing MATH260 and MATH262 (Financial mathematics). This module complements MATH365 (Measure theory and probability) in the sense that MATH365 provides the contradiction-free measure theoretic foundation on which this module rests.
Most problems in modern applied mathematics require the use of suitably designed numerical methods. Working exactly, we can often reduce a complicated problem to something more elementary, but this will often lead to integrals that cannot be evaluated using analytical methods or equations that are too complex to be solved by hand. Other problems involve the use of ‘real world’ data, which don’t fit neatly into simple mathematical models. In both cases, we can make further progress using approximate methods. These usually require lengthy iterative processes that are tedious and error prone for humans (even with a calculator), but ideally suited to computers. The first few lectures of this module demonstrate how computer programs can be written to handle calculations of this type automatically. These ideas will be used throughout the module. We then investigate how errors propagate through numerical computations. The focus then shifts to numerical methods for finding roots, approximating integrals and interpolating data. In each case, we will examine the advantages and disadvantages of different approaches, in terms of accuracy and efficiency.

Programme details and modules listed are illustrative only and subject to change.

**YEAR THREE**

In year three you will have the option to take a summer industrial research project.

**OPTIONAL MODULES**

**FURTHER METHODS OF APPLIED MATHEMATICS (MATH323)**

**Credits: 15 / Semester: semester 1**

Ordinary and partial differential equations (ODEs and PDEs) are crucial to many areas of science, engineering and finance. This module addresses methods for, or related to, their solution. It starts with a section on inhomogeneous linear second-order ODEs which are often required for the solution of higher-level problems. We then generalize basic calculus by considering the optimization of functionals, e.g., integrals involving an unknown function and its derivatives, which leads to a wide variety of ODEs and PDEs. After those systems of two linear first-order PDEs and second-order PDEs are classified and reduced to ODEs where possible. In certain cases, e.g., ‘elliptic’ PDEs like the Laplace equation, such a reduction is impossible. The last third of the module is devoted to two approaches, conformal mappings and Fourier transforms, which can be used to obtain solutions of the Laplace equation and other irreducible PDEs.

**CARTESIAN TENSORS AND MATHEMATICAL MODELS OF SOLIDS AND VISCOUS FLUIDS (MATH324)**

**Credits: 15 / Semester: semester 1**
This module provides an introduction to basic concepts and principles of continuum mechanics. Cartesian tensors are introduced at the beginning of the module, bringing simplicity and versatility to the analysis. The module places emphasis on the importance of conservation laws in integral form, and on the fundamental role integral conservation laws play in the derivation of partial differential equations used to model different physical phenomena in problems of solid and fluid mechanics. Some knowledge of Vector Calculus (e.g. MATH225 Vector calculus with applications in fluid mechanics) is useful.

QUANTUM MECHANICS (MATH325)

Credits: 15 / Semester: semester 1

The development of Quantum Mechanics, requiring as it did revolutionary changes in our understanding of the nature of reality, was arguably the greatest conceptual achievement of all time. The aim of the module is to lead the student to an understanding of the way that relatively simple mathematics (in modern terms) led Bohr, Einstein, Heisenberg and others to a radical change and improvement in our understanding of the microscopic world.

RELATIVITY (MATH326)

Credits: 15 / Semester: semester 1

Einstein's theories of special and general relativity have introduced a new concept of space and time, which underlies modern particle physics, astrophysics and cosmology. It makes use of, and has stimulated the development of modern differential geometry. This module develops the required mathematics (tensors, differential geometry) together with applications of the theory to particle physics, black holes and cosmology. It is an essential part of a programme in theoretical physics. Together with MATH325 (Quantum mechanics) it covers the basics of modern theoretical physics. Possible follow up modules in theoretical physics are MATH423 (Introduction to string theory), MATH425 (Quantum field theory) and MATH431 (Introduction to modern particle theory). MATH326 is essential for students who consider doing a project on black holes or cosmology. Students following a pure mathematics or applied mathematics pathway might be interested in MATH326 because of its applications of differential geometry, and take it together with MATH349 (Differential geometry).

NUMBER THEORY (MATH342)

Credits: 15 / Semester: semester 1

Number theory begins with, and is mainly concerned with, the study of the integers. Because of the fundamental role which integers play in mathematics, many of the greatest mathematicians, from antiquity to the modern day, have made contributions to number theory. In this module you will study results due to Euclid, Euler, Gauss, Riemann, and other greats: you will also see many results from the last 10 or 20 years. Several of the topics you will study will be familiar from MATH142 (Numbers, groups, and codes). We will go into them in greater depth, and the module will be self-contained from the point of view of number theory. However, some background in group theory (no more than is in MATH142) will be assumed.
GROUP THEORY (MATH343)

Credits: 15 / Semester: semester 1

The module provides an introduction to the modern theory of finite non-commutative groups. Group Theory is one of the central areas of Pure Mathematics. Being part of Algebra, it has innumerable applications in Geometry, Number Theory, Combinatorics and Analysis, but also plays a very important role in Theoretical Physics, Mechanics and Chemistry. The module starts with basic definitions and some well-known examples (the symmetric group of permutations and the groups of congruence classes modulo an integer) and builds up to some very interesting and non-trivial constructions, such as the semi-direct product, which makes it possible to construct more complicated groups from simpler ones. In the final part of the course, the Sylow theory and its applications to the classification of groups are considered.

DIFFERENTIAL GEOMETRY (MATH349)

Credits: 15 / Semester: semester 1

Differential geometry studies distances and curvatures on manifolds through differentiation and integration. This module introduces the methods of differential geometry on the concrete examples of curves and surfaces in 3-dimensional Euclidean space. The module MATH248 (Geometry of curves) develops methods of differential geometry on examples of plane curves. This material will be discussed in the first weeks of the course, but previous familiarity with these methods is helpful. Students following a pathway in theoretical physics might find this module interesting as it discusses a different aspect of differential geometry, and might take it together with MATH326 (Relativity), MATH410 (Manifolds, homology and Morse theory) and MATH446 (Lie groups and Lie algebras).

APPLIED PROBABILITY (MATH362)

Credits: 15 / Semester: semester 1

To give examples of empirical phenomena for which stochastic processes provide suitable mathematical models. To provide an introduction to the methods of probabilistic model building for dynamic events occurring over time. To familiarise students with the usual techniques in the area of probability modelling.

LINEAR STATISTICAL MODELS (MATH363)

Credits: 15 / Semester: semester 1

This module follows on directly from MATH263 (Statistical Theory and Methods I), extending the work there on linear regression and analysis of variance, and then going beyond these to generalised linear models. The module emphasises applications of statistical methods, while the companion module MATH361 (Theory of Statistical Inference) focuses on more theoretical aspects. Statistical software is used throughout as familiarity with its use is a valuable skill for those interested in a career in a statistical field. It is helpful, though not essential, to have taken MATH264 (Statistical Theory and Methods II).
GAME THEORY (MATH331)
Credits: 15 / Semester: semester 2
In this module you will explore, from a game-theoretic point of view, models which have been used to understand phenomena in which conflict and cooperation occur and see the relevance of the theory not only to parlour games but also to situations involving human relationships, economic bargaining (between trade union and employer, etc), threats, formation of coalitions, war, etc.

NUMERICAL METHODS FOR ORDINARY AND PARTIAL DIFFERENTIAL EQUATIONS (MATH336)
Credits: 15 / Semester: semester 2
Many real-world systems in mathematics, physics and engineering can be described by differential equations. In rare cases these can be solved exactly by purely analytical methods, but much more often we can only solve the equations numerically, by reducing the problem to an iterative scheme that requires hundreds of steps. We will learn efficient methods for solving ODEs and PDEs on a computer.

COMBINATORICS (MATH344)
Credits: 15 / Semester: semester 2
Combinatorics is a part of mathematics in which mathematicians deal with discrete and countable structures by means of various combinations, such as permutations, ordered and unordered selections, etc. The seemingly simple methods of combinatorics can raise highly non-trivial mathematical questions and lead to deep mathematical results, which are, in turn, closely related to some fundamental phenomena in number theory

THE MAGIC OF COMPLEX NUMBERS: COMPLEX DYNAMICS, CHAOS AND THE MANDELBROT SET (MATH345)
Credits: 15 / Semester: semester 2
A “dynamical system” is a system that changes over time according to a fixed rule. In complex dynamics, we consider the case where the state of the system is described by a single (complex) variable, and the rule of evolution is given by a holomorphic function. It turns out that this seemingly simple setting leads to very rich, subtle and intricate problems, some of which are still the subject of ongoing mathematical research, both at the University of Liverpool and internationally. This module will provide an introduction to this fascinating subject, and introduce students to some of these problems. In the course of this study, we will encounter many results about complex functions that may seem “magic” when compared with what might be expected from real analysis. A highlight of this kind is the theorem that every polynomial is “chaotic” on its Julia set. We will also see how this “magic” can help us understand phenomena that at first seem to have no connection with complex numbers at all.

TOPOLOGY (MATH346)
Credits: 15 / Semester: semester 2
Topology is the mathematical study of space. It is distinguished from geometry by the fact that there is no consideration of notions of distance, angle or other similar quantities. For this reason topology is sometimes popularly referred to as ‘rubber sheet’ geometry. It was introduced by Poincaré, under the name of analysis situs, in 1895 and became one of the most successful areas of 20th century mathematics. It continues to be an active research area to this day, and its insights and methods underlie many areas of modern mathematics. More recently, new applications of topological ideas outside mathematics have been developed, in particular to provide qualitative analysis of large data sets. This module introduces the basic notions of topological space and continuous map, illustrating them with many examples from different areas of mathematics. It also introduces homotopy theory, the study of paths in a space, which has become one of the most fundamental areas of modern mathematics.

**THEORY OF STATISTICAL INFEERENCE (MATH361)**

**Credits:** 15 / **Semester:** semester 2

This module introduces fundamental topics in mathematical statistics, including the theory of point estimation and hypothesis testing. Several key concepts of statistics are discussed, such as sufficiency, completeness, etc., introduced from the 1920s by major contributors to modern statistics such as Fisher, Neyman, Lehmann and so on. This module is absolutely necessary preparation for postgraduate studies in statistics and closely related subjects.

**MEDICAL STATISTICS (MATH364)**

**Credits:** 15 / **Semester:** semester 2

**MEASURE THEORY AND PROBABILITY (MATH365)**

**Credits:** 15 / **Semester:** semester 2

This module is important for students who are interested in the abstract theory of integrating and in the deep theoretical background of the probability theory. It will be extremely useful for those who plan to do MSc and perhaps PhD in Probability, including financial applications. If you plan to take level 4 module(s) on Financial Mathematics next year, MATH365 can be very helpful.

**MATHEMATICAL RISK THEORY (MATH366)**

**Credits:** 15 / **Semester:** semester 2

To provide an understanding of the mathematical risk theory used in practise in non-life actuarial depts of insurance firms, to provide an introduction to mathematical methods for managing the risk in insurance and finance (calculation of risk measures/quantities), to develop skills of calculating the ruin probability and the total claim amount distribution in some non-life actuarial risk models with applications to insurance industry, to prepare the students adequately and to develop their skills in order to be exempted for the exams of CT6 subject of the Institute of Actuaries (MATH366 covers 50% of CT6 in much more depth).
**NETWORKS IN THEORY AND PRACTICE (MATH367)**

**Credits: 15 / Semester: semester 2**

MATH367 aims to develop an appreciation of optimisation methods for real-world problems using fundamental tools from network theory; to study a range of ‘standard problems’ and techniques for solving them. Thus, network flow, shortest path problem, transport problem, assignment problem, and routing problem are some of the problems that are considered in the syllabus. MATH367 is a decision making module, which fits well to those who are interested in receiving knowledge in graph theory, in operational research, in economics, in logistics and in finance. It is a 100% written exam module, and full marks will be awarded for complete answers to all questions.

**STOCHASTIC THEORY AND METHODS IN DATA SCIENCE (MATH368)**

**Credits: 15 / Semester: semester 2**

This module raises the awareness of students on how mathematical methods from stochastics can help to deal with problems arising in a variety of areas, ranging from quantifying uncertainty, to problems in physics, to optimisation and decision making, among others. The module summarises probability theory, explain the basics of simulation and sampling and then focuses on learning theory and methods. Specific topics and examples will be presented along with the theory and computer experiments.

**STATISTICAL PHYSICS (MATH327)**

**Credits: 15 / Semester: semester 2**

Statistical Physics is a core subject in Physics and a cornerstone for modern technologies. To name just one example, quantum statistics is informing leading edge developments around ultra-cold gases and liquids giving rise to new materials. The module will introduce foundations of Statistical Physics and will develop an understanding of the stochastic roots of thermodynamics and the properties of matter. After successfully completing this module students will understand statistical ensembles and related concepts such as entropy and temperature, will understand the properties of classical and quantum gases, will be know the laws of thermodynamics and will be aware of advanced phenomena such as phase transition. The module will also develop numerical computer programming skills for the description of macroscopic effects such as diffusion by an underlying stochastic process.

**PROFESSIONAL PROJECTS AND EMPLOYABILITY IN MATHEMATICS (MATH390)**

**Credits: 15 / Semester: semester 1**

This module gives the opportunity to further develop skills of mathematical problem solving and the application of mathematical results to real-world scenarios through group activities. The module aims to develop skills that are needed when undertaking employment or research, such as working in-depth on a problem over an extended period, writing reports, communicating mathematical results to different audiences and working in collaboration with others. This module will provide employability skills experiences and develop students’ ability to articulate their skills, which will be useful to draw on when applying for jobs.
MATHS SUMMER INDUSTRIAL RESEARCH PROJECT (MATH391)
Credits: 15 / Semester: semester 1, summer
The research internship module is designed to give students the experience of working in a research environment or setting that is quite different from any project work that they undertake in the Department of Mathematics. It should provide an insight into how students may apply skills and experiences later in their career; whether working abroad, in industry or in a scientific setting.

MATHEMATICAL BIOLOGY (MATH335)
Credits: 15 / Semester: semester 1
In the current age of big data, mathematics is becoming indispensable in order for us to make sense of experimental results and in order to gain a deeper understanding into mechanisms of complex biological systems. Mathematical models can provide insights that cannot be gained through experimental work alone. This module will focus on teaching students how to construct and analyse models for a wide range of biological systems. Mathematical approaches covered will be widely applicable.

APPLIED STOCHASTIC MODELS (MATH360)
Credits: 15 / Semester: semester 1
Stochastic processes are ways of quantifying the dynamic relationships of sequences of random events. Stochastic models play an important role in elucidating many areas of the natural and engineering sciences. They can be used to analyse the variability inherent in biological and medical processes, to deal with uncertainties affecting managerial decisions and with the complexities of psychological and social interactions, and to provide new perspectives, methodology, models and intuition to aid in other mathematical and statistical studies. This module is intended as a beginning course in introducing continuous-time stochastic processes for students familiar with elementary probability. The objectives are: (1) to introduce students to the standard concepts and methods of stochastic modelling; (2) to illustrate the rich diversity of applications of stochastic processes in the science; and (3) to provide exercises in the applications of simple stochastic analysis to appropriate problems. The module is complementary to MATH362 (Applied probability), in which discrete-time processes are studied. Those who plan to go on to MSc study in financial mathematics will find this module a very useful preparation for modules such as MATH481 (Interest rate theory), MATH482 (Stochastic modelling in finance), MATH483 (Stochastic analysis and its applications) and MATH484 (Advanced numerical analysis for financial mathematics).

NETWORKS IN MATHEMATICAL BIOLOGY (MATH338)
Credits: 15 / Semester: semester 2
Networks are familiar to us from many real-world systems such as the internet, power grids, transportation and biological networks. The underpinning mathematical concept is called a graph and it is no surprise that the same issues arise in each area, whether this is to identify the most important or influential individuals in the network, or to prevent dynamics on the network (e.g. epidemics) or to make the network robust to the dynamics it supports (e.g. power grids and transportation). In this module, we learn about different classes of networks and how to quantify and describe them including their structures and their nodes. Much of our detailed understanding of networks and their features will come from analysis of idealised random networks which nevertheless are often good representations of those seen in the real world. We will consider real-world biological applications of network theory, in particular focusing on epidemics.

**LINEAR DIFFERENTIAL OPERATORS IN MATHEMATICAL PHYSICS (MATH421)**

**Credits: 15 / Semester: semester 1**

This module is concerned with linear partial differential equations (PDEs) that arise in mathematical physics, and advanced methods for solving them. There is a particular focus on methods that use singular solutions, which satisfy the PDE at all but a finite number of points. We will study three canonical PDEs: Laplace's equation, the heat equation and the wave equation. In each case we will see how the solution to complicated problems can be built up from solutions to simpler problems, typically in the form of an infinite series or an integral.

**QUANTUM FIELD THEORY (MATH425)**

**Credits: 15 / Semester: semester 1**

Quantum Field Theory provides the mathematical language of modern theoretical particle and condensed matter physics. Historically Quantum Field Theory was developed to be the consistent theory of quantum mechanics and special relativity. The mathematical techniques developed in this course form the theoretical basis for varied fields such as high energy particle physics or superconductivity.

**VARIATIONAL CALCULUS AND ITS APPLICATIONS (MATH430)**

**Credits: 15 / Semester: semester 1**

In the same way that calculus is concerned with the extremisation of functions, variational calculus deals with the extremisation of functionals, or "functions of functions". Variational calculus underpins much of modern mathematical physics and applied mathematics. This module provides the fundamental background theory on variational calculus, which is accompanied by a range of physical examples. The course is delivered via thirty-six lectures and twelve tutorials. There will be ten problem sheets, which will underpin your learning, enhance your understanding of the topics covered, and prepare you for the final examination. Each problem sheet will contribute 1% toward your final mark for this module. The final written examination contributes 90% toward your final mark for Math430.

**MANIFOLDS, HOMOLOGY AND MORSE THEORY (MATH410)**
CREDITS: 15 / SEMESTER: SEMESTER 1

HIGHER ARITHMETIC (MATH441)
CREDITS: 15 / SEMESTER: SEMESTER 2
This module provides an introduction to topics in Analytic Number Theory, including the worst and average case behaviour of arithmetic functions, properties of the Riemann zeta function, and the distribution of prime numbers.

REPRESENTATION THEORY OF FINITE GROUPS (MATH442)
CREDITS: 15 / SEMESTER: SEMESTER 2

PROBABILITY AND ANALYSIS (MATH465)
CREDITS: 15 / SEMESTER: SEMESTER 1
This module aims to teach students the modern theoretical and applicable methods and tools of the vast field of Probability Theory (Stochastics) that are at the intersection of many mathematical disciplines. It aims to help students obtain solid foundations in the field per se and its applications. There will be emphasis both on stochastic and analytical tools, as the interplay between them is the foundation of any solid knowledge of the subject.

SINGULARITY THEORY OF DIFFERENTIABLE MAPPINGS (MATH455)
CREDITS: 15 / SEMESTER: SEMESTER 1
This module is an introduction to the calculus side of Singularity Theory. Theory of singularities of differentiable maps is a far-reaching generalisation of the study of functions at maxima and minima. It has numerous applications in mathematics, the natural sciences and technology (as in the so-called theory of bifurcations and catastrophes). This module concentrates on the theory and stability of smooth maps, and classification techniques for critical points of smooth functions. Although not pre-requisites, any of MATH244 (Linear algebra and geometry), MATH248 (Geometry of curves), MATH343 (Group theory), MATH349 (Differential geometry) and MATH443 (Curves and singularities) would be helpful. MATH410 (Manifolds, homology and Morse theory) is a follow-up module but may be taken simultaneously.

STOCHASTIC ANALYSIS AND ITS APPLICATIONS (MATH483)
CREDITS: 15 / SEMESTER: SEMESTER 2
This module provides the foundations of stochastic analysis. Many of the basic results are considered in detail, in particular the ones that play a crucial role in applications such as mathematical finance. Students taking this module will study conditional expectations, martingales, Brownian motion, Brownian bridge, the reflection principle and scaling, stopping times, Ito's integral and stochastic calculus, stochastic differential equations (linear and nonlinear), martingale representation, Girsanov theorem, and Feynman-Kac formula. Applications include stochastic control, optimal investment, and mathematical finance. All the theoretical results are illustrated with numerical examples from various fields of applications.

**MATH499 – PROJECT FOR M.MATH. (MATH499)**

**Credits: 15 / Semester: whole session**

This is a one-semester module for Year 4 G101 Mathematics MMath students. Research is performed in an advanced topic in a particular area of Mathematics under the supervision of a member of staff, which is followed by preparation of a report and an oral presentation. It is hoped that this will provide further insights into advanced subjects and additional experience in handling specialist literature.

**ADVANCED TOPICS IN MATHEMATICAL BIOLOGY (MATH426)**

**Credits: 15 / Semester: semester 2**

Mathematics can be applied to a wide range of biological problems, many of which involve studying how systems change in space and time. In this module, an example selection of mathematical applications will be presented chosen from staff research interests involving developmental biology, epidemic dynamics & biological fluid dynamics.

**WAVES, MATHEMATICAL MODELLING (MATH427)**

**Credits: 15 / Semester: semester 2**

This module introduces some of the generic ideas that underpin the analysis of waves in physical systems. Both linear and nonlinear models are discussed. Quasi-linear hyperbolic first-order systems of equations are introduced leading to the study of Riemann invariants, simple waves and shock solutions. Some knowledge of Vector Calculus (e.g. MATH225 Vector calculus with applications in fluid mechanics) is useful.

**ASYMPTOTIC METHODS FOR DIFFERENTIAL EQUATIONS (MATH433)**

**Credits: 15 / Semester: semester 2**

This module provides an introduction into perturbation theory for partial differential equations. This theory has a wide, and growing, range of applications in the study of electromagnetism, elasticity, heat conduction, the propagation of waves, and the study of cracks in materials.

**ELLIPTIC CURVES (MATH444)**
This module provides an introduction to the problems and methods in the theory of elliptic curves.

RIEMANN SURFACES (MATH445)
Credits: 15 / Semester: semester 1
This module will introduce students to a beautiful theory at the core of modern mathematics. Students will learn how to handle some abstract geometric notions from an elementary point of view that relies on the theory of holomorphic functions. This will provide those who aim to continue their studies in mathematics with an invaluable source of examples, and those who plan to leave the subject with the example of a modern axiomatic mathematical theory.

GEOMETRY OF CONTINUED FRACTIONS (MATH447)
Credits: 15 / Semester: semester 2
This module covers a wide variety of methods of geometric continued fraction theory.

ALGEBRAIC GEOMETRY (MATH448)
Credits: 15 / Semester: semester 2
Algebraic geometry is a classical and nowadays vast area of mathematics. It deals with geometric figures given as roots of polynomial equations. Such figures live in projective spaces and are called algebraic varieties. Because of the algebraic nature of the defining equations, varieties can be considered over different ground fields.

Non-singular projective varieties over complex numbers are complex manifolds. As such, they are also smooth manifolds, so that the methods of complex-analytic and differential geometry are applicable to them. If varieties are given by polynomials with coefficients in the field of rational numbers, they encode deep arithmetical phenomena. This is why algebraic geometry marvelously merges different kinds of geometry and number theory into one big field. Algebraic geometry itself splits into several major subareas, such as birational geometry, intersection theory, deformation theory, moduli problems, arithmetic geometry, non-commutative geometry and much more. In the last decades the role of algebraic geometry in theoretical physics is steadily increasing. Within this advanced and demanding one-semester module the students will learn basics of algebraic geometry, being concentrated on the detailed elaboration of some instructive examples illustrating fundamental concepts and phenomena. Our purpose would be to train our algebraic-geometrical intuition working both synthetically, i.e. without coordinates, and in coordinates in terms of polynomials with coefficients in an algebraically closed field.

We will discuss some basic constructions of algebraic geometry, such as projections, blowing up, Segre embedding and products, families of varieties, linear series, tangent and vector bundles, differential forms and the canonical class. In the end we will also touch upon cohomology groups and how they can help to understand the geometry of algebraic varieties in the context of their topology and differential forms.
**GALOIS THEORY (MATH449)**

**Credits:** 15 / **Semester:** semester 2  
This module introduces the theory of polynomial equations of one variable: Galois Theory. This theory provides criteria when a polynomial equation can be solved in radicals, when a geometric construction can be performed by a ruler and a compass.

**INTRODUCTION TO STRING THEORY (MATH423)**

**Credits:** 15 / **Semester:** semester 2  
Introduction to String Theory.

**INTRODUCTION TO MODERN PARTICLE THEORY (MATH431)**

**Credits:** 15 / **Semester:** semester 2  
Modern particle theory is combining special relativity, quantum mechanics and field theory to describe all the fundamental subatomic particles and their interactions. The module develops the relevant concepts that enter into the Standard Model of particle physics. The key concept in modern physics is that of invariance under local symmetries and the conservation laws that they give rise to. The module covers the basic elements that describe modern particle theory, including: Lorentz and Poincare symmetries, which underlie special relativity; Hamilton and Lagrange formalism of classical mechanics and fields, which underlie the modern formalism; basic elements of relativistic quantum mechanics including the Dirac and Klein–Gordon equations; field quantisation; global and local symmetries; global and local symmetry breaking and the Higgs mechanism; unitary groups and the classification of elementary particles; basic elements of grand unified theories and phenomenological aspects. The students will be introduced to many of the modern ideas in Particle Physics at an accessible level.

Programme details and modules listed are illustrative only and subject to change.

---

**YEAR FOUR**

One of the project modules MATH499 or MATH490 must be taken in year four. Some modules are only delivered in alternate years.

---

**OPTIONAL MODULES**

**LINEAR DIFFERENTIAL OPERATORS IN MATHEMATICAL PHYSICS (MATH421)**

**Credits:** 15 / **Semester:** semester 1
This module is concerned with linear partial differential equations (PDEs) that arise in mathematical physics, and advanced methods for solving them. There is a particular focus on methods that use singular solutions, which satisfy the PDE at all but a finite number of points. We will study three canonical PDEs: Laplace’s equation, the heat equation and the wave equation. In each case we will see how the solution to complicated problems can be built up from solutions to simpler problems, typically in the form of an infinite series or an integral.

**QUANTUM FIELD THEORY (MATH425)**

**Credits: 15 / Semester: semester 1**

Quantum Field Theory provides the mathematical language of modern theoretical particle and condensed matter physics. Historically Quantum Field Theory was developed to be the consistent theory of quantum mechanics and special relativity. The mathematical techniques developed in this course form the theoretical basis for varied fields such as high energy particle physics or superconductivity.

**VARIATIONAL CALCULUS AND ITS APPLICATIONS (MATH430)**

**Credits: 15 / Semester: semester 1**

In the same way that calculus is concerned with the extremisation of functions, variational calculus deals with the extremisation of functionals, or “functions of functions”. Variational calculus underpins much of modern mathematical physics and applied mathematics. This module provides the fundamental background theory on variational calculus, which is accompanied by a range of physical examples. The course is delivered via thirty-six lectures and twelve tutorials. There will be ten problem sheets, which will underpin your learning, enhance your understanding of the topics covered, and prepare you for the final examination. Each problem sheet will contribute 1% toward your final mark for this module. The final written examination contributes 90% toward your final mark for Math430.

**REPRESENTATION THEORY OF FINITE GROUPS (MATH442)**

**Credits: 15 / Semester: semester 2**

REPRESENTATION THEORY OF FINITE GROUPS

**PROBABILITY AND ANALYSIS (MATH465)**

**Credits: 15 / Semester: semester 1**

This module aims to teach students the modern theoretical and applicable methods and tools of the vast field of Probability Theory (Stochastics) that are at the intersection of many mathematical disciplines. It aims to help students obtain solid foundations in the field per se and its applications. There will be emphasis both on stochastic and analytical tools, as the interplay between them is the foundation of any solid knowledge of the subject.

**STOCHASTIC ANALYSIS AND ITS APPLICATIONS (MATH483)**
This module provides the foundations of stochastic analysis. Many of the basic results are considered in detail, in particular the ones that play a crucial role in applications such as mathematical finance. Students taking this module will study conditional expectations, martingales, Brownian motion, Brownian bridge, the reflection principle and scaling, stopping times, Ito's integral and stochastic calculus, stochastic differential equations (linear and nonlinear), martingale representation, Girsanov theorem, and Feynman-Kac formula. Applications include stochastic control, optimal investment, and mathematical finance. All the theoretical results are illustrated with numerical examples from various fields of applications.

**MATH499 – PROJECT FOR M.MATH. (MATH499)**

Credits: 15 / Semester: whole session

This is a one-semester module for Year 4 G101 Mathematics MMath students. Research is performed in an advanced topic in a particular area of Mathematics under the supervision of a member of staff, which is followed by preparation of a report and an oral presentation. It is hoped that this will provide further insights into advanced subjects and additional experience in handling specialist literature.

**MATH490 – PROJECT FOR M.MATH. (MATH490)**

Credits: 30 / Semester: whole session

This is a two-semester module for Year 4 G101 Mathematics MMath students. Research is performed in an advanced interesting topic which should lead to acquiring knowledge useful for potential continuation of mathematical studies through a PhD. Students who took a Year 3 project module have the opportunity to continue research in the same topic.

**ADVANCED TOPICS IN MATHEMATICAL BIOLOGY (MATH426)**

Credits: 15 / Semester: semester 2

Mathematics can be applied to a wide range of biological problems, many of which involve studying how systems change in space and time. In this module, an example selection of mathematical applications will be presented chosen from staff research interests involving developmental biology, epidemic dynamics & biological fluid dynamics.

**WAVES, MATHEMATICAL MODELLING (MATH427)**

Credits: 15 / Semester: semester 2

This module introduces some of the generic ideas that underpin the analysis of waves in physical systems. Both linear and nonlinear models are discussed. Quasi-linear hyperbolic first-order systems of equations are introduced leading to the study of Riemann invariants, simple waves and shock solutions. Some knowledge of Vector Calculus (e.g. MATH225 Vector calculus with applications in fluid mechanics) is useful.
ASYMPTOTIC METHODS FOR DIFFERENTIAL EQUATIONS (MATH433)
Credits: 15 / Semester: semester 2
This module provides an introduction into perturbation theory for partial differential equations. This theory has a wide, and growing, range of applications in the study of electromagnetism, elasticity, heat conduction, the propagation of waves, and the study of cracks in materials.

ELLiptic Curves (MATH444)
Credits: 15 / Semester: semester 2
This module provides an introduction to the problems and methods in the theory of elliptic curves.

RIEMANN Surfaces (MATH445)
Credits: 15 / Semester: semester 1
This module will introduce students to a beautiful theory at the core of modern mathematics. Students will learn how to handle some abstract geometric notions from an elementary point of view that relies on the theory of holomorphic functions. This will provide those who aim to continue their studies in mathematics with an invaluable source of examples, and those who plan to leave the subject with the example of a modern axiomatic mathematical theory.

GEOMETRY OF CONTINUED FRACTIONS (MATH447)
Credits: 15 / Semester: semester 2
This module covers a wide variety of methods of geometric continued fraction theory.

INTRODUCTION TO STRING THEORY (MATH423)
Credits: 15 / Semester: semester 2
Introduction to String Theory.

MANIfOLDS, HOMOLOGY AND MORSE THEORY (MATH410)
Credits: 15 / Semester: semester 1

HIGHER ARITHMETIC (MATH441)
Credits: 15 / Semester: semester 2
This module provides an introduction to topics in Analytic Number Theory, including the worst and average case behaviour of arithmetic functions, properties of the Riemann zeta function, and the distribution of prime numbers.

ALGEBRAIC GEOMETRY (MATH448)
Algebraic geometry is a classical and nowadays vast area of mathematics. It deals with geometric figures given as roots of polynomial equations. Such figures live in projective spaces and are called algebraic varieties. Because of the algebraic nature of the defining equations, varieties can be considered over different ground fields.

Non-singular projective varieties over complex numbers are complex manifolds. As such, they are also smooth manifolds, so that the methods of complex-analytic and differential geometry are applicable to them. If varieties are given by polynomials with coefficients in the field of rational numbers, they encode deep arithmetical phenomena. This is why algebraic geometry marvelously merges different kinds of geometry and number theory into one big field. Algebraic geometry itself splits into several major subareas, such as birational geometry, intersection theory, deformation theory, moduli problems, arithmetic geometry, non-commutative geometry and much more. In the last decades the role of algebraic geometry in theoretical physics is steadily increasing. Within this advanced and demanding one-semester module the students will learn basics of algebraic geometry, being concentrated on the detailed elaboration of some instructive examples illustrating fundamental concepts and phenomena. Our purpose would be to train our algebraic-geometrical intuition working both synthetically, i.e. without coordinates, and in coordinates in terms of polynomials with coefficients in an algebraically closed field.

We will discuss some basic constructions of algebraic geometry, such as projections, blowing up, Segre embedding and products, families of varieties, linear series, tangent and vector bundles, differential forms and the canonical class. In the end we will also touch upon cohomology groups and how they can help to understand the geometry of algebraic varieties in the context of their topology and differential forms.

GALOIS THEORY (MATH449)

This module introduces the theory of polynomial equations of one variable: Galois Theory. This theory provides criteria when a polynomial equation can be solved in radicals, when a geometric construction can be performed by a ruler and a compass.

INTRODUCTION TO MODERN PARTICLE THEORY (MATH431)

This module introduces the theory of polynomial equations of one variable: Galois Theory. This theory provides criteria when a polynomial equation can be solved in radicals, when a geometric construction can be performed by a ruler and a compass.
Modern particle theory is combining special relativity, quantum mechanics and field theory to describe all the fundamental subatomic particles and their interactions. The module develops the relevant concepts that enter into the Standard Model of particle physics. The key concept in modern physics is that of invariance under local symmetries and the conservation laws that they give rise to. The module covers the basic elements that describe modern particle theory, including: Lorentz and Poincare symmetries, which underlie special relativity; Hamilton and Lagrange formalism of classical mechanics and fields, which underlie the modern formalism; basic elements of relativistic quantum mechanics including the Dirac and Klein–Gordon equations; field quantisation; global and local symmetries; global and local symmetry breaking and the Higgs mechanism; unitary groups and the classification of elementary particles; basic elements of grand unified theories and phenomenological aspects. The students will be introduced to many of the modern ideas in Particle Physics at an accessible level.

**FURTHER METHODS OF APPLIED MATHEMATICS (MATH323)**

**Credits: 15 / Semester: semester 1**

Ordinary and partial differential equations (ODEs and PDEs) are crucial to many areas of science, engineering and finance. This module addresses methods for, or related to, their solution. It starts with a section on inhomogeneous linear second-order ODEs which are often required for the solution of higher-level problems. We then generalize basic calculus by considering the optimization of functionals, e.g., integrals involving an unknown function and its derivatives, which leads to a wide variety of ODEs and PDEs. After those systems of two linear first–order PDEs and second–order PDEs are classified and reduced to ODEs where possible. In certain cases, e.g., ‘elliptic’ PDEs like the Laplace equation, such a reduction is impossible. The last third of the module is devoted to two approaches, conformal mappings and Fourier transforms, which can be used to obtain solutions of the Laplace equation and other irreducible PDEs.

**CARTESIAN TENSORS AND MATHEMATICAL MODELS OF SOLIDS AND VISCOUS FLUIDS (MATH324)**

**Credits: 15 / Semester: semester 1**

This module provides an introduction to basic concepts and principles of continuum mechanics. Cartesian tensors are introduced at the beginning of the module, bringing simplicity and versatility to the analysis. The module places emphasis on the importance of conservation laws in integral form, and on the fundamental role integral conservation laws play in the derivation of partial differential equations used to model different physical phenomena in problems of solid and fluid mechanics. Some knowledge of Vector Calculus (e.g. MATH225 Vector calculus with applications in fluid mechanics) is useful.

**QUANTUM MECHANICS (MATH325)**

**Credits: 15 / Semester: semester 1**
The development of Quantum Mechanics, requiring as it did revolutionary changes in our understanding of the nature of reality, was arguably the greatest conceptual achievement of all time. The aim of the module is to lead the student to an understanding of the way that relatively simple mathematics (in modern terms) led Bohr, Einstein, Heisenberg and others to a radical change and improvement in our understanding of the microscopic world.

RELATIVITY (MATH326)

Credits: 15 / Semester: semester 1

Einstein's theories of special and general relativity have introduced a new concept of space and time, which underlies modern particle physics, astrophysics and cosmology. It makes use of, and has stimulated the development of modern differential geometry. This module develops the required mathematics (tensors, differential geometry) together with applications of the theory to particle physics, black holes and cosmology. It is an essential part of a programme in theoretical physics. Together with MATH325 (Quantum mechanics) it covers the basics of modern theoretical physics. Possible follow up modules in theoretical physics are MATH423 (Introduction to string theory), MATH425 (Quantum field theory) and MATH431 (Introduction to modern particle theory). MATH326 is essential for students who consider doing a project on black holes or cosmology. Students following a pure mathematics or applied mathematics pathway might be interested in MATH326 because of its applications of differential geometry, and take it together with MATH349 (Differential geometry).

MATHEMATICAL BIOLOGY (MATH335)

Credits: 15 / Semester: semester 1

In the current age of big data, mathematics is becoming indispensable in order for us to make sense of experimental results and in order to gain a deeper understanding into mechanisms of complex biological systems. Mathematical models can provide insights that cannot be gained through experimental work alone. This module will focus on teaching students how to construct and analyse models for a wide range of biological systems. Mathematical approaches covered will be widely applicable.

NUMBER THEORY (MATH342)

Credits: 15 / Semester: semester 1

Number theory begins with, and is mainly concerned with, the study of the integers. Because of the fundamental role which integers play in mathematics, many of the greatest mathematicians, from antiquity to the modern day, have made contributions to number theory. In this module you will study results due to Euclid, Euler, Gauss, Riemann, and other greats: you will also see many results from the last 10 or 20 years. Several of the topics you will study will be familiar from MATH142 (Numbers, groups, and codes). We will go into them in greater depth, and the module will be self-contained from the point of view of number theory. However, some background in group theory (no more than is in MATH142) will be assumed.

GROUP THEORY (MATH343)
The module provides an introduction to the modern theory of finite non-commutative groups. Group Theory is one of the central areas of Pure Mathematics. Being part of Algebra, it has innumerable applications in Geometry, Number Theory, Combinatorics and Analysis, but also plays a very important role in Theoretical Physics, Mechanics and Chemistry. The module starts with basic definitions and some well-known examples (the symmetric group of permutations and the groups of congruence classes modulo an integer) and builds up to some very interesting and non-trivial constructions, such as the semi-direct product, which makes it possible to construct more complicated groups from simpler ones. In the final part of the course, the Sylow theory and its applications to the classification of groups are considered.

**DIFFERENTIAL GEOMETRY (MATH349)**

Credits: 15 / Semester: semester 1

Differential geometry studies distances and curvatures on manifolds through differentiation and integration. This module introduces the methods of differential geometry on the concrete examples of curves and surfaces in 3-dimensional Euclidean space. The module MATH248 (Geometry of curves) develops methods of differential geometry on examples of plane curves. This material will be discussed in the first weeks of the course, but previous familiarity with these methods is helpful. Students following a pathway in theoretical physics might find this module interesting as it discusses a different aspect of differential geometry, and might take it together with MATH326 (Relativity), MATH410 (Manifolds, homology and Morse theory) and MATH446 (Lie groups and Lie algebras).

**APPLIED STOCHASTIC MODELS (MATH360)**

Credits: 15 / Semester: semester 1

Stochastic processes are ways of quantifying the dynamic relationships of sequences of random events. Stochastic models play an important role in elucidating many areas of the natural and engineering sciences. They can be used to analyse the variability inherent in biological and medical processes, to deal with uncertainties affecting managerial decisions and with the complexities of psychological and social interactions, and to provide new perspectives, methodology, models and intuition to aid in other mathematical and statistical studies. This module is intended as a beginning course in introducing continuous-time stochastic processes for students familiar with elementary probability. The objectives are: (1) to introduce students to the standard concepts and methods of stochastic modelling; (2) to illustrate the rich diversity of applications of stochastic processes in the science; and (3) to provide exercises in the applications of simple stochastic analysis to appropriate problems. The module is complementary to MATH362 (Applied probability), in which discrete-time processes are studied. Those who plan to go on to MSc study in financial mathematics will find this module a very useful preparation for modules such as MATH481 (Interest rate theory), MATH482 (Stochastic modelling in finance), MATH483 (Stochastic analysis and its applications) and MATH484 (Advanced numerical analysis for financial mathematics).
APPLIED PROBABILITY (MATH362)

Credits: 15 / Semester: semester 1

To give examples of empirical phenomena for which stochastic processes provide suitable mathematical models. To provide an introduction to the methods of probabilistic model building for dynamic events occurring over time. To familiarise students with the usual techniques in the area of probability modelling.

LINEAR STATISTICAL MODELS (MATH363)

Credits: 15 / Semester: semester 1

This module follows on directly from MATH263 (Statistical Theory and Methods I), extending the work there on linear regression and analysis of variance, and then going beyond these to generalised linear models. The module emphasises applications of statistical methods, while the companion module MATH361 (Theory of Statistical Inference) focuses on more theoretical aspects. Statistical software is used throughout as familiarity with its use is a valuable skill for those interested in a career in a statistical field. It is helpful, though not essential, to have taken MATH264 (Statistical Theory and Methods II).

GAME THEORY (MATH331)

Credits: 15 / Semester: semester 2

In this module you will explore, from a game-theoretic point of view, models which have been used to understand phenomena in which conflict and cooperation occur and see the relevance of the theory not only to parlour games but also to situations involving human relationships, economic bargaining (between trade union and employer, etc), threats, formation of coalitions, war, etc.

NUMERICAL METHODS FOR ORDINARY AND PARTIAL DIFFERENTIAL EQUATIONS (MATH336)

Credits: 15 / Semester: semester 2

Many real-world systems in mathematics, physics and engineering can be described by differential equations. In rare cases these can be solved exactly by purely analytical methods, but much more often we can only solve the equations numerically, by reducing the problem to an iterative scheme that requires hundreds of steps. We will learn efficient methods for solving ODEs and PDEs on a computer.

NETWORKS IN MATHEMATICAL BIOLOGY (MATH338)

Credits: 15 / Semester: semester 2
Networks are familiar to us from many real-world systems such as the internet, power grids, transportation and biological networks. The underpinning mathematical concept is called a graph and it is no surprise that the same issues arise in each area, whether this is to identify the most important or influential individuals in the network, or to prevent dynamics on the network (e.g. epidemics) or to make the network robust to the dynamics it supports (e.g. power grids and transportation). In this module, we learn about different classes of networks and how to quantify and describe them including their structures and their nodes. Much of our detailed understanding of networks and their features will come from analysis of idealised random networks which nevertheless are often good representations of those seen in the real world. We will consider real-world biological applications of network theory, in particular focusing on epidemics.

**COMBINATORICS (MATH344)**

**Credits:** 15 / **Semester:** semester 2

Combinatorics is a part of mathematics in which mathematicians deal with discrete and countable structures by means of various combinations, such as permutations, ordered and unordered selections, etc. The seemingly simple methods of combinatorics can raise highly non-trivial mathematical questions and lead to deep mathematical results, which are, in turn, closely related to some fundamental phenomena in number theory.

**THE MAGIC OF COMPLEX NUMBERS: COMPLEX DYNAMICS, CHAOS AND THE MANDELBROT SET (MATH345)**

**Credits:** 15 / **Semester:** semester 2

A “dynamical system” is a system that changes over time according to a fixed rule. In complex dynamics, we consider the case where the state of the system is described by a single (complex) variable, and the rule of evolution is given by a holomorphic function. It turns out that this seemingly simple setting leads to very rich, subtle and intricate problems, some of which are still the subject of ongoing mathematical research, both at the University of Liverpool and internationally. This module will provide an introduction to this fascinating subject, and introduce students to some of these problems. In the course of this study, we will encounter many results about complex functions that may seem “magic” when compared with what might be expected from real analysis. A highlight of this kind is the theorem that every polynomial is “chaotic” on its Julia set. We will also see how this “magic” can help us understand phenomena that at first seem to have no connection with complex numbers at all.

**TOPOLOGY (MATH346)**

**Credits:** 15 / **Semester:** semester 2
Topology is the mathematical study of space. It is distinguished from geometry by the fact that there is no consideration of notions of distance, angle or other similar quantities. For this reason topology is sometimes popularly referred to as ‘rubber sheet’ geometry. It was introduced by Poincaré, under the name of analysis situs, in 1895 and became one of the most successful areas of 20th century mathematics. It continues to be an active research area to this day, and its insights and methods underlie many areas of modern mathematics. More recently, new applications of topological ideas outside mathematics have been developed, in particular to provide qualitative analysis of large data sets. This module introduces the basic notions of topological space and continuous map, illustrating them with many examples from different areas of mathematics. It also introduces homotopy theory, the study of paths in a space, which has become one of the most fundamental areas of modern mathematics.

**THEORY OF STATISTICAL INFERENCE (MATH361)**

**Credits: 15 / Semester: semester 2**

This module introduces fundamental topics in mathematical statistics, including the theory of point estimation and hypothesis testing. Several key concepts of statistics are discussed, such as sufficiency, completeness, etc., introduced from the 1920s by major contributors to modern statistics such as Fisher, Neyman, Lehmann and so on. This module is absolutely necessary preparation for postgraduate studies in statistics and closely related subjects.

**MEDICAL STATISTICS (MATH364)**

**Credits: 15 / Semester: semester 2**

**MEASURE THEORY AND PROBABILITY (MATH365)**

**Credits: 15 / Semester: semester 2**

This module is important for students who are interested in the abstract theory of integrating and in the deep theoretical background of the probability theory. It will be extremely useful for those who plan to do MSc and perhaps PhD in Probability, including financial applications. If you plan to take level 4 module(s) on Financial Mathematics next year, MATH365 can be very helpful.

**MATHEMATICAL RISK THEORY (MATH366)**

**Credits: 15 / Semester: semester 2**

To provide an understanding of the mathematical risk theory used in practise in non-life actuarial depts of insurance firms, to provide an introduction to mathematical methods for managing the risk in insurance and finance (calculation of risk measures/quantities), to develop skills of calculating the ruin probability and the total claim amount distribution in some non-life actuarial risk models with applications to insurance industry, to prepare the students adequately and to develop their skills in order to be exempted for the exams of CT6 subject of the Institute of Actuaries (MATH366 covers 50% of CT6 in much more depth).
NETWORKS IN THEORY AND PRACTICE (MATH367)

Credits: 15 / Semester: semester 2

MATH367 aims to develop an appreciation of optimisation methods for real-world problems using fundamental tools from network theory; to study a range of ‘standard problems’ and techniques for solving them. Thus, network flow, shortest path problem, transport problem, assignment problem, and routing problem are some of the problems that are considered in the syllabus. MATH367 is a decision making module, which fits well to those who are interested in receiving knowledge in graph theory, in operational research, in economics, in logistics and in finance. It is a 100% written exam module, and full marks will be awarded for complete answers to all questions.

STOCHASTIC THEORY AND METHODS IN DATA SCIENCE (MATH368)

Credits: 15 / Semester: semester 2

This module raises the awareness of students on how mathematical methods from stochastics can help to deal with problems arising in a variety of areas, ranging from quantifying uncertainty, to problems in physics, to optimisation and decision making, among others. The module summarises probability theory, explain the basics of simulation and sampling and then focuses on learning theory and methods. Specific topics and examples will be presented along with the theory and computer experiments.

STATISTICAL PHYSICS (MATH327)

Credits: 15 / Semester: semester 2

Statistical Physics is a core subject in Physics and a cornerstone for modern technologies. To name just one example, quantum statistics is informing leading edge developments around ultra-cold gases and liquids giving rise to new materials. The module will introduce foundations of Statistical Physics and will develop an understanding of the stochastic roots of thermodynamics and the properties of matter. After successfully completing this module students will understand statistical ensembles and related concepts such as entropy and temperature, will understand the properties of classical and quantum gases, will be know the laws of thermodynamics and will be aware of advanced phenomena such as phase transition. The module will also develop numerical computer programming skills for the description of macroscopic effects such as diffusion by an underlying stochastic process.

PROFESSIONAL PROJECTS AND EMPLOYABILITY IN MATHEMATICS (MATH390)

Credits: 15 / Semester: semester 1

This module gives the opportunity to further develop skills of mathematical problem solving and the application of mathematical results to real-world scenarios through group activities. The module aims to develop skills that are needed when undertaking employment or research, such as working in-depth on a problem over an extended period, writing reports, communicating mathematical results to different audiences and working in collaboration with others. This module will provide employability skills experiences and develop students’ ability to articulate their skills, which will be useful to draw on when applying for jobs.
MATHS SUMMER INDUSTRIAL RESEARCH PROJECT (MATH391)

Credits: 15 / Semester: semester 1, summer

The research internship module is designed to give students the experience of working in a research environment or setting that is quite different from any project work that they undertake in the Department of Mathematics. It should provide an insight into how students may apply skills and experiences later in their career; whether working abroad, in industry or in a scientific setting.

SINGULARITY THEORY OF DIFFERENTIABLE MAPPINGS (MATH455)

Credits: 15 / Semester: semester 1

This module is an introduction to the calculus side of Singularity Theory. Theory of singularities of differentiable maps is a far-reaching generalisation of the study of functions at maxima and minima. It has numerous applications in mathematics, the natural sciences and technology (as in the so-called theory of bifurcations and catastrophes). This module concentrates on the theory and stability of smooth maps, and classification techniques for critical points of smooth functions. Although not pre-requisites, any of MATH244 (Linear algebra and geometry), MATH248 (Geometry of curves), MATH343 (Group theory), MATH349 (Differential geometry) and MATH443 (Curves and singularities) would be helpful. MATH410 (Manifolds, homology and Morse theory) is a follow-up module but may be taken simultaneously.

Programme details and modules listed are illustrative only and subject to change.

HOW YOU'LL LEARN

Your learning activities will consist of lectures, tutorials, practical classes, problem classes, private study and supervised project work.

In year one, lectures are supplemented by a thorough system of group tutorials and computing work is carried out in supervised practical classes. Key study skills, presentation skills and group work start in first-year tutorials and are developed later in the programme. The emphasis in most modules is on the development of problem solving skills, which are regarded very highly by employers.

Project supervision is on a one-to-one basis, apart from group projects in year two.

HOW YOU'RE ASSESSED

Most modules are assessed by a two and a half hour examination in January or May, but many have an element of coursework assessment. This might be through homework, class tests, mini-project work or key skills exercises. You will be taught by internationally recognised experts within a friendly department.

LIVERPOOL HALLMARKS

We have a distinctive approach to education, the Liverpool Curriculum Framework, which focuses on research-connected teaching, active learning, and authentic assessment to
ensure our students graduate as digitally fluent and confident global citizens.
Careers and employability

A mathematically-based degree opens up a wide range of career opportunities, including some of the most lucrative professions.

Typical types of work our graduates have gone onto include as an actuarial trainee analyst in the audit practice, a graduate management trainee risk analyst and as a trainee chartered accountant on a graduate business programme. Employers value mathematicians’ high level of numeracy and problem solving skills.

87.5% OF MATHEMATICAL SCIENCES GRADUATES GO ON TO WORK OR FURTHER STUDY WITHIN 15 MONTHS OF GRADUATION.

Discover Uni, 2018-19.

Recent employers of our graduates are:
- Barclays Bank plc
- Deloitte
- Forrest Recruitment
- Marks and Spencer
- Mercer Human Resource Consulting Ltd
- Venture Marketing Group.

Preparing you for future success

At Liverpool, our goal is to support you to build your intellectual, social, and cultural capital so that you graduate as a socially-conscious global citizen who is prepared for future success. We achieve this by:

- Embedding employability within your curriculum, through the modules you take and the opportunities to gain real-world experience offered by many of our courses.
- Providing you with opportunities to gain experience and develop connections with people and organisations, including student and graduate employers as well as our global alumni.
- Providing you with the latest tools and skills to thrive in a competitive world, including access to Handshake, a platform which allows you to create your personalised job shortlist and apply with ease.
- Supporting you through our peer-to-peer led Careers Studio, where our career coaches provide you with tailored advice and support.
Fees and funding
Your tuition fees, funding your studies, and other costs to consider.

TUITION FEES
Tuition fees cover the cost of your teaching and assessment, operating facilities such as libraries, IT equipment, and access to academic and personal support. Learn more about tuition fees, funding and student finance.

<table>
<thead>
<tr>
<th>UK fees</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Full-time place, per year</td>
<td>£9,250</td>
</tr>
<tr>
<td>Year in industry fee</td>
<td>£1,850</td>
</tr>
<tr>
<td>Year abroad fee</td>
<td>£1,385</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>International fees</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Full-time place, per year</td>
<td>£22,150</td>
</tr>
</tbody>
</table>

Fees stated are for the 2022-23 academic year and may rise for 2023-24.

ADDITIONAL COSTS
Your tuition fee covers almost everything but you may have additional study costs to consider, such as books.

Find out more about the additional study costs that may apply to this course.

SCHOLARSHIPS AND BURSARIES
We offer a range of scholarships and bursaries to help cover tuition fees and help with living expenses while at university.

Scholarships and bursaries you can apply for from the United Kingdom
Select your country or region for more scholarships and bursaries.
# Entry requirements

The qualifications and exam results you’ll need to apply for this course.

<table>
<thead>
<tr>
<th>Your qualification</th>
<th>Requirements</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>About our typical entry requirements</td>
</tr>
<tr>
<td>A levels</td>
<td>AAB</td>
</tr>
<tr>
<td></td>
<td>Applicants with the Extended Project Qualification (EPQ) are eligible for a reduction in grade requirements. For this course, the offer is <strong>ABB</strong> with <strong>A</strong> in the EPQ. You may automatically qualify for reduced entry requirements through our contextual offers scheme.</td>
</tr>
<tr>
<td>GCSE</td>
<td>4/C in English and 4/C in Mathematics</td>
</tr>
<tr>
<td>Subject requirements</td>
<td>Mathematics A level grade A. Applicants must have studied Mathematics at Level 3 within 2 years of the start date of their course. For applicants from England: Where a science has been taken at A level (Chemistry, Biology or Physics), a pass in the Science practical of each subject will be required.</td>
</tr>
<tr>
<td>BTEC Level 3 National Extended Diploma</td>
<td>Applications Considered. Relevant when combined with A level Mathematics grade A</td>
</tr>
<tr>
<td>International Baccalaureate</td>
<td>35 including 6 in Higher Mathematics</td>
</tr>
<tr>
<td>Irish Leaving Certificate</td>
<td>H1, H1, H2, H2, H2, H3 including Mathematics at H1</td>
</tr>
<tr>
<td>Your qualification</td>
<td>Requirements</td>
</tr>
<tr>
<td>-----------------------------</td>
<td>-------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>Scottish Higher/Advanced Higher</td>
<td>Advanced Highers accepted at grades AAB including grade A in Mathematics.</td>
</tr>
<tr>
<td>Welsh Baccalaureate Advanced</td>
<td>Acceptable at grade B or above alongside AA at A level including grade A in Mathematics.</td>
</tr>
<tr>
<td>Access</td>
<td>Considered</td>
</tr>
</tbody>
</table>

**International qualifications**

Select your country or region to view specific entry requirements.

Many countries have a different education system to that of the UK, meaning your qualifications may not meet our entry requirements. Completing your Foundation Certificate, such as that offered by the [University of Liverpool International College](https://www.liverpool.ac.uk/), means you're guaranteed a place on your chosen course.

---

**ALTERNATIVE ENTRY REQUIREMENTS**

- If your qualification isn't listed here, or you're taking a combination of qualifications, [contact us](mailto:contact@liverpool.ac.uk) for advice.
- Applications from mature students are welcome.