Magnetic imaging of shipwrecks.

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Abstract

Proton magnetometer measurements at the surface give an indication of a shipwreck on the seabed. Here I discuss how to interpret such results.

1 Introduction

Ships are mostly made of steel (iron in earlier times). Iron and steel (also nickel and cobalt) are ferromagnetic, which means there is a strong induced magnetism when placed in an external magnetic field. The earth has a magnetic field (this is what makes a magnetic compass work) and so will cause such an induced magnetism. The net effect is to modify the earth's magnetic field in the vicinity of the iron. As an approximation, one can say that the magnetic field of the earth 'prefers' to go through the iron - so the field strength is enhanced where the field enters and leaves the iron and is reduced at the sides. Since the earth's field (in Britain) is downward angled, the ferromagnetic material will tend to increase the field above (and a bit to the south) of the submerged material, while further away the surface field can be reduced.

This modification to the earth's magnetic field is what allows detection at the surface of the presence of iron on the seabed. The induced magnetism of the ferromagnetic material at the seabed produces an anomalous field which decreases as the inverse cube of the distance. However, a large object (a ship) will have an anomalous field that can be detected over a significant area of the surface. This makes for an efficient method to detect such shipwrecks.

Here I discuss whether it is possible and practical to deduce something about the shape of the shipwreck from surface measurements of the magnetic anomaly. This will be especially useful for shipwrecks that are mostly buried below the seabed. To investigate this, I present a discussion of the modelling of a shipwreck and the consequences for the magnetic anomaly. This allows a discussion of how best to interpret readings from a magnetometer taken at the surface.

As an example of this approach, I present my data from several shipwrecks in Liverpool Bay and discuss models that describe the surface anomaly observed. In this case I can also compare with observations, from diving, of the seabed wreck.

The mathematical details of interpreting the surface anomaly are presented in Appendices. Appendix A evaluates the surface magnetic field anomaly from a collection of induced magnetic dipoles representing the shipwreck. Appendix B discusses how such induced dipoles can arise and what their orientation will be. Appendix C gives a discussion of methods to analyse the surface distribution, both in general and with some assumptions. Appendix D reviews creating a data grid from scattered data.

2 Surface magnetic measurements

The earth's magnetic field has a strength and a direction. In the northern hemisphere it is is downward angled. This is called the angle of 'dip' or inclination. It is about 67^0 to the horizontal in Britain in 2010
(see http://www.geomag.bgs.ac.uk/images/fig3.pdf). The horizontal component points to 'magnetic north'. The deviation from true north is called the 'magnetic deviation'. In Britain in 2010 this deviation is small - just a few

degrees. The field strength is around 49000 nT (nanoTesla) in Britain (see http://www.geomag.bgs.ac.uk/images/fig4.pdf)

Magnetometers measure the strength of a magnetic field. Some types of magnetometer measure the component in a given direction. This can be useful for a compass. In a rocking boat, these components will change with the boat's orientation. A more stable quantity is the total intensity. This can be determined by combining the output from three orthogonal directions (the sum of squares is needed) or by using a magnetometer which measures this directly - such as a proton magnetometer. Here I will mainly consider data obtained with a proton magnetometer since this can be accurate (to within 1 nT) and equipment is available designed for use at sea. Data can be obtained every few seconds (typically 2 secs) with GPS position and magnetic intensity logged. It may also be feasible to log depth to the seabed (and possibly also depth of the magnetometer head if this is allowed to sink)

To avoid detecting the magnetism of one's own vessel, a magnetometer head is towed behind. Since position is detected (by GPS) on the vessel itself, a correction needs to be made for the fact that the magnetometer signal is from a position that was the vessel's position a short time ago.

The magnetic anomaly from a seabed shipwreck depends on the depth and the size of the wreck. A coaster in 30 metres may have a signal as small as a few hundred nT whereas a several thousand ton ship can have a signal of several thousands of nT. The signal is significant (say 10 nT or more) over quite a large area and this is why a proton magnetometer is such a successful wreck finder. I will discuss in more detail the nature of the surface anomaly and whether it is possible to deduce more detail about the wreck - such as its orientation underwater.

3 Induced magnetisation

Ferromagnetic materials will have an induced magnetism when placed in a magnetic field. For ferromagnetic materials such as soft iron, this induced magnetism is the main effect. As discussed later, steel can become semi-permanently magnetised (again by the earth's magnetic field during construction or from subsequent voyages). This permanent magnetism can also be represented in the same way as the induced magnetism, by a dipole distribution.

In summary, the ferromagnetic material of the shipwreck will, in the earth's magnetic field, acquire an induced magnetism. This can be represented by a distribution of magnetic dipoles. The dipoles have a strength and a direction. The task is to estimate this dipole distribution in the case of a shipwreck and then evaluate the modification of the earth's magnetic field at the surface (called the surface anomaly) caused by those induced dipoles. See Appendix A for details of determining the surface anomaly given the dipole distribution.

The simplest case, discussed in Appendix B1, is of a spherically symmetric distribution of ferromagnetic material. This is a mathematician's ship! Even so, it can yield some insight since it can be solved exactly. One finds that the induced magnetic dipole is directed along the earth's field and is actually effec-

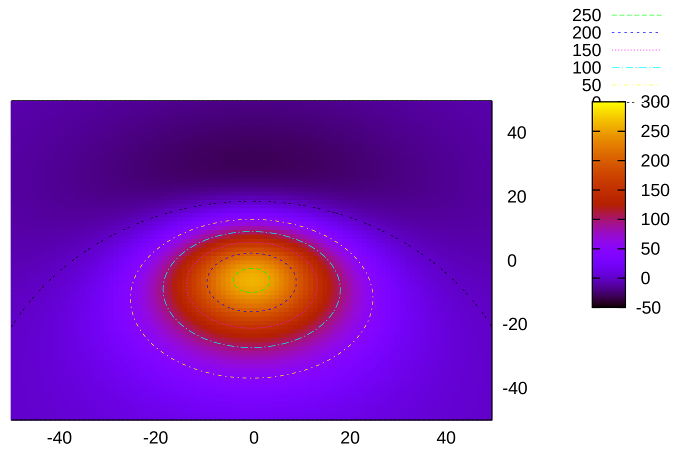

Figure 1: Magnetic field intensity anomaly (arbitrary units) at the surface with distances in metres for a point magnetic source at the origin and at depth 30 metres. Magnetic north is upwards, the angle of dip used is 67^0.

tively located at the centre of the sphere of material. The surface distribution from this induced dipole is easy to calculate (see Appendix A) and is illustrated in fig. 1. This shows that the field intensity is increased in a region above the source and is decreased (especially northwards) away from the source. If the angle of dip were 90^0 (as at the north magnetic pole) the intensity would be increased inside a circle centred on the source and of radius $1.414z$ where z is the depth and the intensity would be decreased outside that radius. In Britain where the angle of dip is quite large (around 67^0) the situation is similar but distorted as shown by fig. 1.

The most characteristic feature, for detection purposes, is that the maximum is slightly south of the source (4 metres in the example) and the most negative value (the minimum) is only 12% of the magnitude of the maximum and is situated further north (32 metres in the example) of the source.

If a shipwreck is modelled as a collection of spherical components which are not close to each other, then the surface distribution will be given by adding together the distributions from each of the components. Although this model would represent a scattered field of cannon balls on the seabed, it is actually a rather unrealistic way to represent an iron shipwreck: rather there will be flat steel plates forming the hull, bulkheads and decks.

I now discuss the induced magnetism in a rectangular steel plate (see Appendix B3). The essential feature is that the magnetic field has to obey certain conditions at the surface between the plate and the water. These imply that the induced dipoles must lie primarily oriented along the plate (i.e. tangential to the surface of the plate). This is a new feature compared to the spherical situation: the induced dipoles will not necessarily be directed along the earth's magnetic field.

For example, a horizontal plate will thus have dipoles with no vertical component and this gives a very different surface distribution - see fig. 2 for an illustration. Here the negative peak (anomaly reducing the intensity of the earth's field at the surface) is more prominent. Other cases are discussed in Appendix B3.

Thus for a shipwreck made of iron/steel plates, it is not likely, in general, that the induced magnetism will lie along the earth's magnetic field direction.

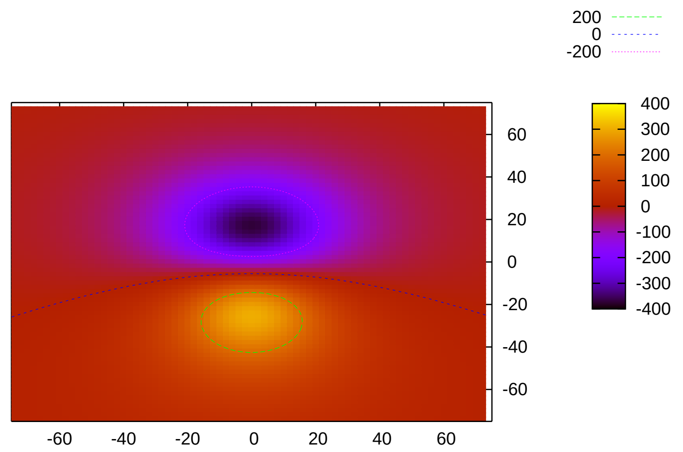

Figure 2: Magnetic field intensity anomaly (in nT) at the surface with distances in metres for a horizontal flat rectangular plate (10m × 40m, longest side lying N-S, thickness t with $t(\mu_r - 1) = 40$ metres) as magnetic source, centred at the origin and at depth 30 metres. Magnetic north is upwards, the angle of dip used is 67^0.

In general, the modification of the magnetic field intensity at the surface can be evaluated given the distribution and orientation of induced magnetic dipoles in the ferromagnetic material of the shipwreck (as described in Appendix A). Approximately this will be related to the distribution of iron/steel underwater. At large distances from the wreck, the net anomaly will have the form of a magnetic dipole centred on the wreck. The only information then available will be the total strength and the orientation of this dipole. The total strength is a useful clue to the tonnage of the wreck. At first sight, the dipole would be oriented along the earth's magnetic field, so giving little information about the wreck. This is not actually the case for a wreck containing steel (or iron) plates, so some extra information on the nature of the wreck can be gleaned. As I will show, if the depth is not much greater than the length of the wreck, the surface anomaly can give additional information about the distribution of material on the seabed.

Given an extended source (big ship, etc) then one solves for the induced magnetism combining the contributions from all the iron/steel in the wreck. This can be evaluated by computer, in principle, if one knows that distribution: see Appendix B4. However, if the wreck has collapsed, turned over while sinking, been partly demolished by explosives, been salvaged, been moved by wave motion,... one will not have any accurate model of the distribution of ferromagnetic material.

In practice one will need to make a simplifying model of the wreck: a flat rectangular plate, a cylinder, a shoe box, a rectangular cross-section tube,....

Some idea of the distribution of material on the seabed can be obtained in general, as described in Appendix C, provided sufficiently precise surface data are available. This is discussed later. If one makes a stronger assumption, that all the ferromagnetic material lies at the same depth and is all magnetised in the same direction, then it is possible to extract the material distribution on the seabed from the surface measurement without constructing a model. This is discussed in Appendix C1. One always can try this approach but if regions on the seabed come out with a negative amount of material, this shows that the assumption was not valid. This approach is related to the method known as 'return to pole' which is described in Appendix C2.

4 Permanent magnetisation effects

What is quite well established is that a ship, a few years after construction, that voyages primarily in the northern hemisphere, will have a semi-permanent magnetisation in the vertical (downward) direction. This semi-permanent magnetism arises from stresses in the steel of the vessel caused by wave motion, engine vibration, gunfire, heat, cargo loading, etc. It can change over a period of a year or so. Ship's compasses are corrected for semi-permanent and induced magnetism and this is a source of much expertise. Also detection of submarines by their magnetic signature is well studied.

For example, submarines are regularly 'depermed' - have their permanent magnetisation removed - by using a strong oscillating magnetic field that decreases in strength. This reduces their magnetic 'presence' so they cannot be as easily detected by magnetometers. During World War II, a method of counteracting the magnetic 'presence' of a ship needed to be developed - to avoid setting off magnetic mines. This involved a permanent electric current encircling the ship close to the waterline which produced a vertical magnetic field which cancelled out the semi-permanent magnetisation (and also the vertical component of the induced magnetisation to some extent). This was known as 'degaussing'. Later on, deperming was also used.

A coda to this is that when magnetic mines were made more sensitive, the degaussing currents were switched to maximise the ship's magnetic signal - causing the mines to explode some distance away from the ship.

The magnetisation from construction (heating/cooling and hammering can both give rise to a permanent magnetisation) is not easy to predict. What emerges from the reports on magnetic effects in real ships (and submarines) is that the vertical semi-permanent magnetic effect is usually the dominant one after a few years at sea. What is quite unclear to me is what happens after a ship sinks. It no longer moves around and is not subject to vibration or wave slamming (if sufficiently deep). So one possibility is that the pre-sinking semi-permanent magnetisation remains. Another possibility is that it decays slowly and what remains is the induced magnetisation caused by the earth's magnetic field direction at the wrecksite which is what we have been discussing above.

High tensile steel would be expected to retain permanent magnetism for longer than mild steel or iron. I can find no reliable quantitative data on this for steel plates used in ship building. One way to explore further is too look at known wrecksites and see how the surface magnetic intensity can be understood.

Another complication is that for an old wreck, the direction of the earth's field will have changed during its period on the seabed. In the Liverpool area, magnetic north was -21^0 from true north in 1877 and is now (2011) much closer (-2^0) to true north.

This discussion of semi-permanent magnetism in ships shows that even more uncertainty is present in trying to predict the magnetic presence of a shipwreck. The semi-permanent magnetism can be represented by a dipole distribution, just like the induced magnetism. If one instead takes the approach that the wreck is a magnetic dipole source of unknown strength, direction and distribution, then it is still possible to analyse this from data on the surface anomaly.

5 Fitting surface data

Given an accurate set of data (position of measurement and value of magnetic intensity shift), it should be possible to deduce something about the distribution of magnetic material on the seabed. The general case is discussed in Appendix C. If the surface data are sufficiently precise and extensive, it will be possible to interpolate them accurately and hence make a 2-dimensional Fourier transform of them. This allows in general, as discussed in Appendix C, to evaluate quantities of interest which may help to deduce the nature of the wreck. One example considered in Appendix C is the surface anomalous magnetic energy distribution. This is less sensitive (than the original data on the magnetic field anomaly) to the orientation of the dipoles induced in the shipwreck. The surface distribution of this quantity thus tracks the underlying distribution of material more closely - allowing the orientation of the shipwreck to be inferred, for example.

As discussed above, if the only magnetic contribution is an induced magnetic dipole distribution (or a semi-permanent magnetism) in a known direction, then one can extract the distribution at the seabed directly if one assumes that all material is at the same depth. See Appendix C1.

To go beyond this case, one needs to model the distribution (and direction) of the magnetic source and then fit the parameters of the model to the data taken at the surface. By fitting, I mean minimising the χ^2 which is the sum of the square of (data – model)/error. This will require models of the shipwreck

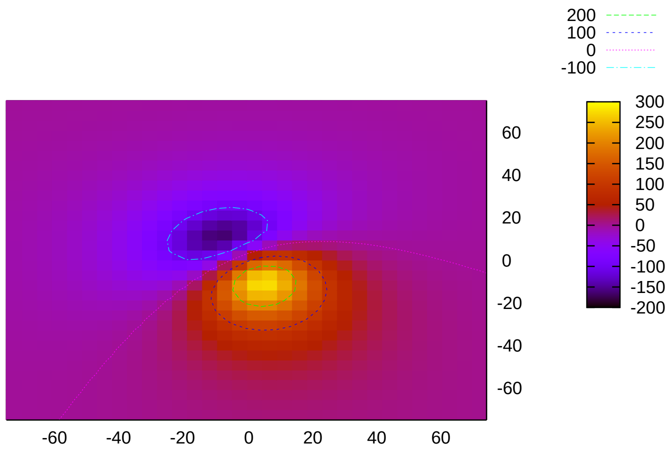

Figure 3: Magnetic field intensity anomaly (in nT) at the surface with distances in metres from a 1 dipole fit to the observed data for the wreckage of a *Tower Base* in 25m in Liverpool Bay.

(unless detailed specifications exist) and I present some examples here that will be used in the comparison section. See refs. [7, 8] for other attempts at this.

The simplest model is of a single magnetic dipole at the seabed. Adjusting the position and orientation (see later discussion of *Tower Base*) gives the surface distribution shown in fig. 3

A model which reproduces many of the features of the data (see section on wreck of *Ystroom*) is shown in fig. 4. This model has an open ended tube with rectangular cross-section (8m wide, 3m vertical) of length 40m and with plates having a combination of thickness t and relative permeability μ_r such that $t(\mu_r - 1) = 10$m. The tube lies 135^0-315^0. The dipoles induced in this model are evaluated following the methods described in Appendix B4, yielding the result in fig. 4. It is also of interest to evaluate the surface energy distribution for this model since this quantity has a simpler interpretation. The result is shown in fig. 5. This shows a distribution with a maximum which appears less long than the 40m of the model, because of saturation effects as discussed in Appendix B3. One can, in principle as discused in Appendix C, also deduce the magnetic energy distribution on a level which is below the surface. In fig. 6 this distribution is shown for a level 5 metres above the seabed for the same model. This illustrates that the shape of the model is now more accurately reproduced.

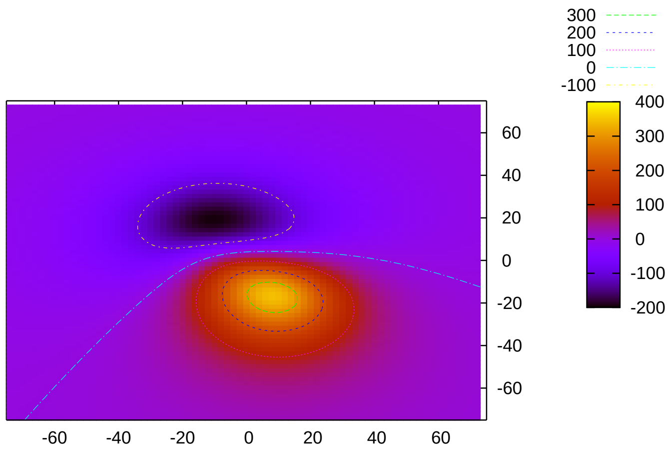

Figure 4: Magnetic field intensity anomaly (in nT) at the surface with distances in metres for a model of the wreck of the *Ystroom* at 26m in Liverpool Bay. Details are in the text.

6 Examples of shipwrecks

Here I illustrate these ideas with some practical cases from Liverpool Bay. More details on the wrecks involved are given in my books [4]. Here the data has been obtained using my boat and towing a proton magnetometer behind. The boat's position is determined by GPS. Typical boat speed through the water is 3.5 to 5 knots and the measurements are taken after a 2 secs polarisation time. Corrections are made for the length of the towing cable and for the current. Depth to the seabed is noted by echo-sounder.

6.1 Ystroom

As a first example, I use data taken by me at the surface over the wreck of the *Ystroom*. This was a Dutch coaster of 400 gross tons that sank from mine damage in 1940. The wrecksite now has no wreckage rising more that a metre or so above the seabed - so the wreck has collapsed and/or become sanded in. For more detail about this wreck, see the book 'Wrecks of Liverpool Bay' [4]. The seabed was at 24 metres depth when I made the magnetic measurements and the wreckage is mostly under the seabed - so I use 26 metres as the average depth of the wreckage. The raw data are shown in fig. 7 after correction for the cable length. They have been gridded using both an inverse power weighting and planar interpolation of a Delaunay triangulation (see Appendix D). Furthermore they have been smoothed by requiring high wave numbers to decrease at the expected rate as discussed in Appendix C. Similar results are obtained from

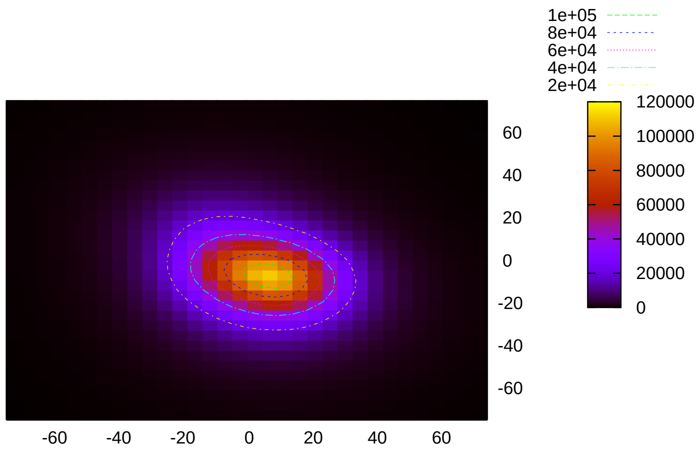

Figure 5: Magnetic energy density anomaly (in nT^2) at the surface with distances in metres for a model of the wreck of the *Ystroom* at 26m in Liverpool Bay. Details are in the text.

these different smoothing methods. Typical results are illustrated in fig. 8.

It is interesting to compare this measured surface anomaly directly with that obtained from the tube model and presented in fig. 4. The overall distribution is rather similar.

Another way to process the data is to extract a quantity which is more simply related to the underlying material distribution. From the gridding discussed above, the two-dimensional Fourier transform was evaluated which allows to calculate the surface energy density - shown in fig. 9. This directly shows the orientation of the wreck, in agreement with the comparison with the model distribution (shown in figs. 4 and 5).

6.2 Tower Base

Another example studied is a *Tower Base* [4] which is the dumped remains of the base of a world war II anti-aircraft fort. This is quite localised (about 35m long and 5m wide). The (smoothed) surface data are illustrated in fig. 10 and a fit using one dipole (so 3 dipole components plus the latitude and longitude as parameters, assuming the depth is 25 metres) has already been shown in fig. 3. This dipole has a horizontal component pointing to 326^0 and a vertical downward component.

6.3 Bouboulina

Bouboulina sank in 1867 after a boiler explosion while at anchor in the Mersey. She was being fitted out as a Greek naval vessel after being built at Liverpool as

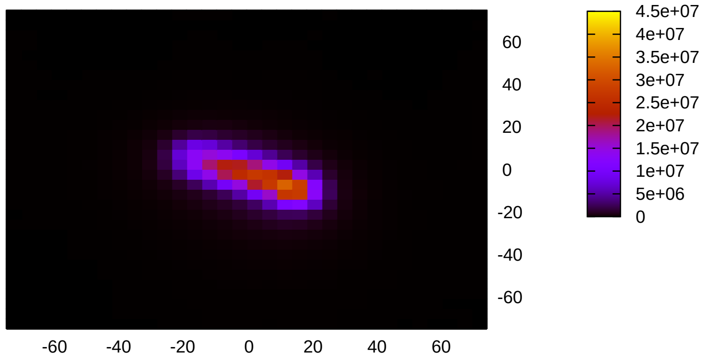

Figure 6: Magnetic energy density anomaly (in nT^2) at at height of 5 metres above the seabed with distances in metres for a model of the wreck of the *Ystroom* at 26m in Liverpool Bay. Details are in the text.

an American Civil War blockade runner, *Colonel Lamb*. For more detail see the book Lelia [5] about Liverpool's part in blockade running in the American Civil War. She was an iron and steel paddle steamer of 699 tons (gross). She split in two after the boiler explosion and the stern section was salvaged. The bow section was reduced in height to cause less obstruction. Though charted, no sign of wreckage above the seabed is now visible (with echo-sounder or side-scan sounder). A substantial magnetic signal is present, however. Since she was at anchor and the current runs $150^0/330^0$ at this location, her remains would be expected to lie down current. At the time (near HW) I investigated this site, the depth to the seabed was 26 metres.

The (smoothed) surface anomaly data are shown in fig. 11. After taking the two-dimensional Fourier transform, the energy density distribution can be reconstructed - see fig. 12. This shows a distribution running NW-SE as expected. The magnitude of the anomaly signal (over 1000 nT) is consistent with a large iron wreck and the orientation fits with expectation. As an example, I continue the energy density distribution to a depth corresponding to 5 metres above the seabed. This sharpens the distribution as shown in fig.13. This suggests that two areas of strong magnetic signal are dominant. A fit using 2 dipoles (at fixed depth so 10 parameters) gives a good description of the raw data - and confirms the orientation as NW-SE with a separation of about 30 metres.

The position I find for the wreckage is close (0.05nm) to that charted. This is certainly the remains of an American Civil War blockade runner.

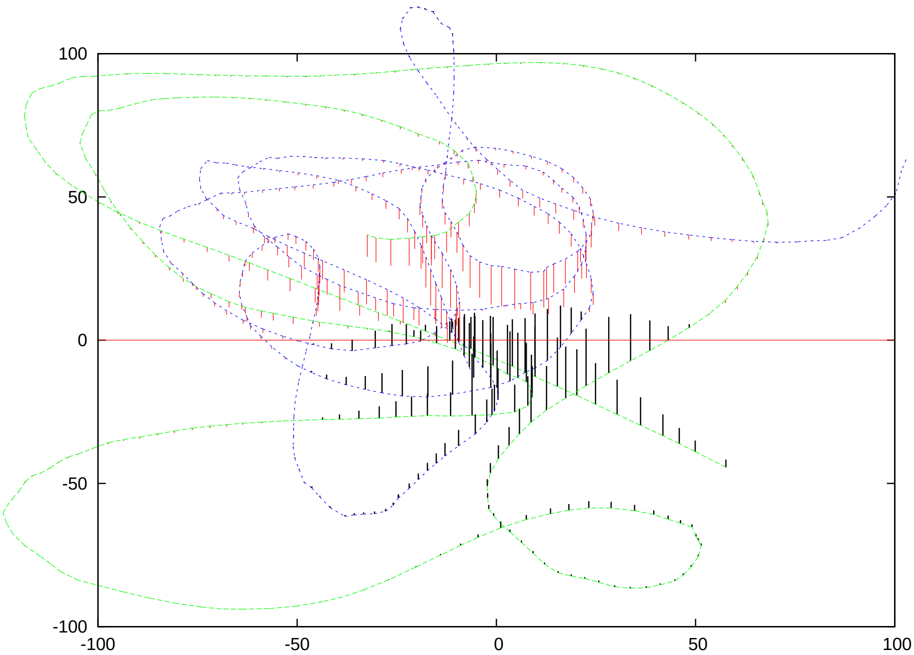

Figure 7: Raw magnetic field intensity anomaly data (in nT) at the surface with distances in metres for the wreck of the *Ystroom* at 26m in Liverpool Bay. The dotted lines show the boat track (2 separate tracks) while the vertical bars (black for positive, red for negative) show the magnetic anomaly. The data have been corrected for the difference between the measured (boat) GPS position and the position of the sensor towed behind.

7 Conclusions

A scattered distribution of roughly spherical ferrous material - such as iron cannon balls - is likely to be magnetised along the earth's magnetic field and so can be modelled with that assumption. Methods then exist which allow to extract the seabed distribution from the surface observations (see Appendix C1 and C2).

Shipwrecks of iron or steel vessels are likely to have substantial iron or steel plates and these will be magnetised dominantly along the surface of the plate. Without a detailed knowledge of the construction of the wreck (as it lies on the seabed), it will be difficult to model this accurately. As a guide, I present some simple models: a rectangular tube, a shoebox, etc. These can help to give an understanding of the expected surface distribution. The rectangular tube is shown to give a surface distribution which is quite similar to that of the wreck of the *Ystroom*. In some cases, it may be possible to fit some of the parameters of such a model to the measured surface data. For example the *Tower Base* can be fitted adequately with one dipole on the seabed while the *Bouboulina* can be well described by 2 dipoles. Note that these dipoles are found not to be aligned with the earth's field.

If no model assumption is justified, it is possible to use data processing (basically a two-dimensional Fourier transform) of the measured surface dis-

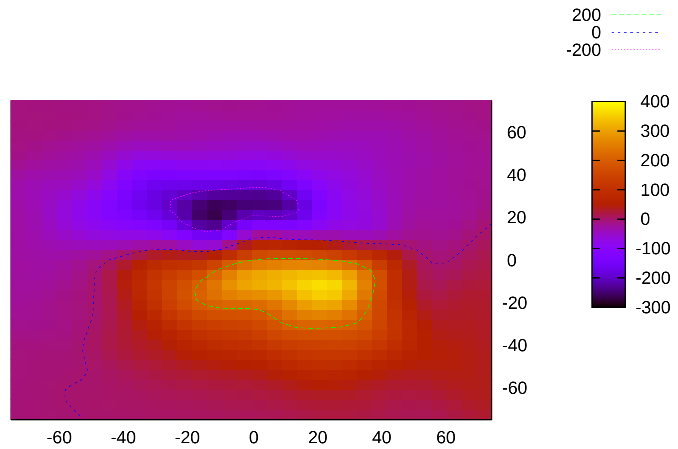

Figure 8: Magnetic field intensity anomaly (in nT) at the surface with distances in metres for the wreck of the *Ystroom* at 26m in Liverpool Bay. Details of smoothing used are in the text (box size of 300×300 metres with step size of 4.7 metres).

tribution to help visualise the seabed distribution. I have presented the case of the surface magnetic energy which can be evaluated and which gives a less biased picture of the seabed magnetic source than the measured anomaly itself (see Appendix C). Examples are given for the wrecks of the coaster *Ystroom* and the paddle steamer *Bouboulina*. These plots give a clear indication of the orientation and length of the wrecks. In principle, it is possible to obtain this information at a lesser height above the wreck (than the surface) which will give even clearer information, although this procedure can be unstable unless the surface data are very precise.

Appendix A: Surface intensity distribution

The discussion of static magnetic problems is well known [1] but still quite subtle: the mathematics assumes a knowledge of vector calculus. There is also a review [2] focussing on geophysical applications with some discussion of programming. A set of operations to analyse magnetic data are also available [3] and the manual (.pdf) can be found on the web.

The magnetic flux field B is measured in Tesla, while the magnetic field intensity H is measured in amp/metre^2. Both of these fields are vectorial: they have a direction as well as an intensity. In free space $B = \mu_0 H$ with $\mu_0 = 4\pi 10^{-7}$. This curious value of μ_0 arises since the units used include the usual electric units (volts, amps) combined with the metre-kilogram-second

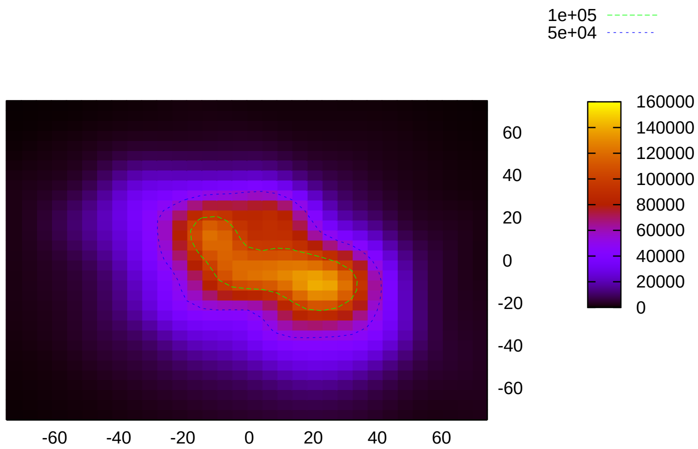

Figure 9: Squared magnetic field intensity anomaly (in nT^2) at the surface with distances in metres for the wreck of the *Ystroom* at 26m in Liverpool Bay. Obtained from a 2-dimensional Fourier transform of the (interpolated) measured data using a 64×64 grid.

system.

In material, the relationship is $B = \mu H$ where the relative permeability is $\mu_r = \mu/\mu_0$. For most materials μ_r is close to 1.0, but ferromagnetic materials have large values (50-10000). The situation in a material can be considered as arising from an induced magnetisation (density of magnetic dipoles) M where $H = B/\mu_0 - M$. Thus the induced magnetism $M = (\mu_r - 1)H$ which is a key relation for this discussion - see Appendix B. For strong fields, the relationship between B and H is complicated for ferromagnetic materials (because of hysteresis effects) but the earth's field is sufficiently weak that a simple linear relationship is appropriate..

From Maxwell's equations, since there are no permanent electric currents, $\nabla \times \mathbf{H} = 0$ and this allows to introduce a magnetic scalar potential ϕ with $\mathbf{H} = -\nabla\phi$. This scalar potential satisfies the Laplace equation $\nabla^2\phi = 0$. Another Maxwell equation gives $\nabla.\mathbf{B} = 0$. These relations imply the boundary conditions: that tangential components of H and the normal component of B are continuous at a boundary.

The field due to a collection of magnetic dipoles can most easily be evaluated using the magnetic scalar potential ϕ.

The magnetic scalar potential at $\mathbf{r}$ due to a (hypothetical) unit magnetic source at $\mathbf{s}$ would be

$$V(\mathbf{r}) = 1/(4\pi d(\mathbf{r}, \mathbf{s}))$$

where $d(\mathbf{r}, \mathbf{s})$ means the distance from point $\mathbf{s}$ to $\mathbf{r}$. For a magnetic dipole of

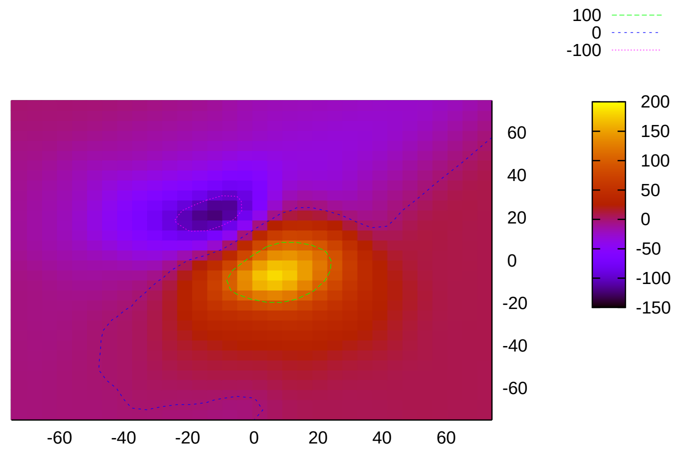

Figure 10: Magnetic field intensity anomaly (in nT) at the surface with distances in metres as observed (and smoothed) for the wreckage of a *Tower Base* in 25m in Liverpool Bay.

strength and direction $\mathbf{p}(\mathbf{s})$ at $\mathbf{s}$, the potential is

$$\phi(\mathbf{r}) = \mathbf{p}(\mathbf{s}).\nabla_s \ 1/(4\pi d(\mathbf{r},\mathbf{s}))$$

The general case can be represented by a sum (over source locations $\mathbf{s}$) of such dipoles:

$$\phi(\mathbf{r}) = \sum_s \mathbf{p}(\mathbf{s}).\nabla_s \ 1/(4\pi d(\mathbf{r},\mathbf{s}))$$

The magnetic field can then be obtained from this potential

$$\mathbf{H}(\mathbf{r}) = -\nabla_r \sum_s \mathbf{p}(\mathbf{s}).\nabla_s \ 1/(4\pi d(\mathbf{r},\mathbf{s}))$$

This magnetic field must be added to the earth's magnetic field $\mathcal{H}$ which is in the direction given by direction cosines $\mathbf{h} = (h_x, h_y, h_z)$. Taking magnetic north as the positive y-axis, and the positive z-direction as upwards, gives $h_x = 0$, $h_y = \cos(\psi)$ and $h_z = -\sin(\psi)$ where ψ is the angle of dip, currently 67^0 in the UK. The magnetic field intensity fluctuation observed at the surface will be dominated by the component $\mathbf{h}.\mathbf{H}(\mathbf{r})$ in practice, namely the intensity anomaly will be $\delta H(x,y) = -\mathbf{h}.\nabla_r \sum_s \mathbf{p}(\mathbf{s}).\nabla_s \ 1/(4\pi d(\mathbf{r},\mathbf{s}))$.

This can be evaluated as

$$\delta H(x,y) = \sum_s (3\mathbf{h}.\mathbf{R} \ \mathbf{p}(\mathbf{s}).\mathbf{R} - \mathbf{h}.\mathbf{p}(\mathbf{s}) \ R^2)/(4\pi R^5) \qquad (1)$$

with $\mathbf{R} = \mathbf{r} - \mathbf{s}$.

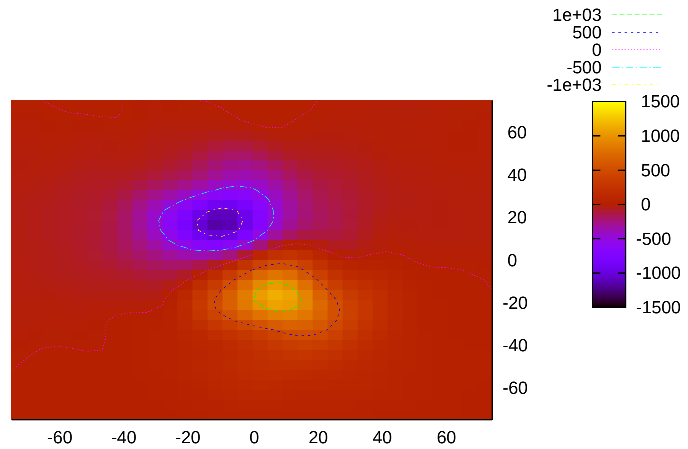

Figure 11: Magnetic field intensity anomaly (in nT) at the surface with distances in metres as observed (and smoothed) for the wreckage of *Bouboulina* (ex-*Colonel Lamb*) in 26m in the Mersey.

For a single magnetic source, one can choose a coordinate system centred on that source. Then $\mathbf{s} = \mathbf{0}$ and $\mathbf{R} = (x, y, z)$ where z will be the depth of water if the source is at the seabed and the intensity at the surface is evaluated. Furthermore, treating the magnetic deviation by defining y as magnetic north with x as perpendicular (so approximately as longitude):

$$\delta H(x,y) = \frac{3(h_y y + h_z z)(p_x x + p_y y + p_z z) - (h_y p_y + h_z p_z)(x^2 + y^2 + z^2)}{4\pi(x^2 + y^2 + z^2)^{5/2}}$$

A greater simplification occurs again if one assumes that the induced magnetic dipole $\mathbf{p}$ is directed also along the earth's magnetic field direction. Then $\mathbf{p}(\mathbf{0}) = \mathbf{h}p(0)$ where $p(0)$ is the dipole strength. So in this case

$$\delta H(x,y) = p(0)\ \frac{3(h_y y + h_z z)^2 - (x^2 + y^2 + z^2)}{4\pi(x^2 + y^2 + z^2)^{5/2}}$$

This expression is illustrated in fig. 1. Note that since $B = \mu_r \mu_0 H$ with $\mu_r \approx 1.0$ and constant in air, one can quote the anomalous magnetic field as the ratio $\delta H/\mathcal{H}$ or as the same number in terms of the magnetic flux ratio $\delta B/\mathcal{B}$. Here $\mathcal{B}$ is the earth's magnetic flux (48800 nT in the UK).

Appendix B: Induced dipole distributions

Appendix B1: Spherical distributions

A case that can be solved exactly: a sphere of ferromagnetic material of radius R with relative permeability μ_r in a external magnetic field H, assuming

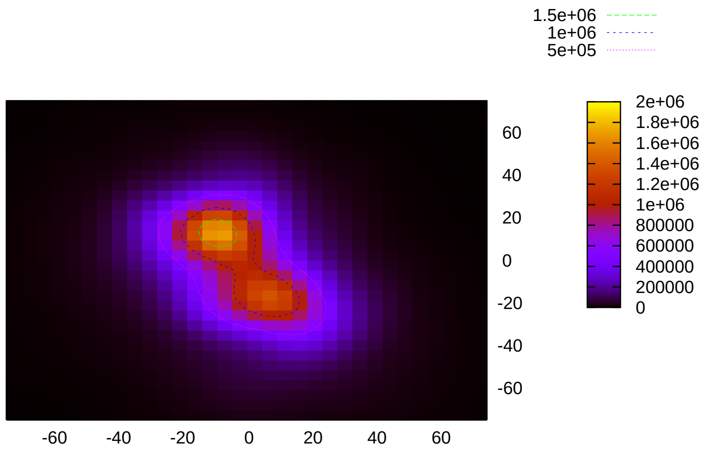

Figure 12: Magnetic energy distribution (in nT^2) at the surface with distances in metres evaluated for the wreckage of *Bouboulina* in 26m in the Mersey.

that $B = \mu H$ both externally and internally with a ratio of permeabilities μ_r. The magnetic scalar potential satisfies Laplace's equation with appropriate boundary conditions on the surface. Choose spherical polar coordinates with the external field along the polar direction, then the magnetic scalar potential inside the sphere is

$$\phi = -hr\cos\theta$$

and outside it is

$$\phi = -Hr\cos\theta + \frac{p}{4\pi r^2}\cos\theta$$

in this case, the induced dipole moment p is located exactly at the centre of the material, this is not a general feature.

Matching the tangential magnetic field ($H_t = -\frac{\partial\phi}{r\partial\theta}$) and the radial magnetic flux ($B = \mu H_r = -\mu\frac{\partial\phi}{\partial r}$) at $r = R$ gives internal magnetic field

$$h = \frac{2}{\mu_r + 1}H$$

and a dipole strength of

$$p = \frac{4\pi R^3}{3}(\mu_r - 1)H\frac{3}{\mu_r + 2} \quad (2)$$

For comparison the induced dipole strength from the external field alone is $p = \frac{4\pi R^3}{3}(\mu_r - 1)H$ which gives the same as the exact result (eq. 2) above for small $\mu_r - 1$, as to be expected.

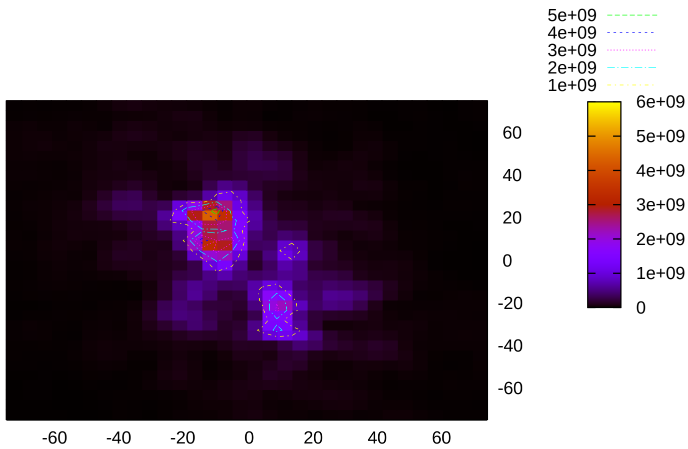

Figure 13: Magnetic energy distribution (in nT^2) at the surface with distances in metres evaluated at a level 5 metres above the seabed for the wreckage of *Bouboulina* in 26m in the Mersey.

Note also that as μ_r becomes big, the external magnetic field becomes normal to the sphere at its surface, since the tangential component of H which is

$$-\frac{\partial \phi}{r \partial \theta} = H \sin\theta - p \frac{\sin\theta}{4\pi R^3}$$

at the surface is zero when $p = H 4\pi R^3$. This induced dipole expression agrees with the exact expression (eq. 2) for large μ_r. Moreover this induced dipole is basically independent of μ_r provided $\mu_r > 10$.

There is a more general way to see why there is a maximum dipole moment induced. A dipole of strength p aligned along the external field H will produce a field of strength $-p/(4\pi r^3)$ transversly to its location and distance r from it. This field is antiparallel to the external field and cannot be so strong as to reverse it, so we have $p < 4\pi r^3 H$ where r is the least distance from the dipole induced in the material to the exterior, transversly: namely R in the example above. This discussion makes it clear that the same reasoning can be applied to a hollow sphere (a spherical shell) with the same maximum induced dipole expression, where R is now the outer radius and r the inner radius. This case can actually also be solved exactly, giving induced dipole

$$p = \frac{(2\mu_r + 1)(\mu_r - 1)}{(2\mu_r + 1)(\mu_r + 2) - (r/R)^3(\mu_r - 1)^2} 4\pi(R^3 - r^3)H \tag{3}$$

For small $\mu_r - 1$, this expression again agrees with the naive expectation of the volume of material times the permeability, namely $p \to 4\pi(\mu_r - 1)(R^3 - r^3)H/3$

This expression (eq. 3) also again has the property that it saturates at very large μ_r at a value $4\pi R^3 H$ as we argued above. However, it can be rewritten in a form more useful for a thin shell, with $(r/R)^3 = 1 - 3t/R$ where t is the thickness of the shell (if thin).

$$p = \frac{(\mu_r - 1)(2\mu_r + 1)}{3\mu_r + 2(\mu_r - 1)^2 t/R} 4\pi R^2 t H \tag{4}$$

The criterion is now that saturation occurs when $t\mu_r >> R$. For plates used in ship-building with thickness $t \approx 0.03$ metres; $\mu_r \approx 50 \rightarrow 1000$; and size of structure $R \approx 10$ metres; then the criterion is not fully satisfied: so one does not necessarily have full saturation.

The induced dipole, because of the spherical symmetry, appears to be located at a point, the centre of the sphere. Thus the surface magnetic anomaly is that given by a point dipole (see Appendix A and fig. 1). One can estimate the magnitude of the surface anomaly in a typical case: taking a sphere of radius 1.7 metres which has volume about 20 m^3 and mass of about 160 tons at depth 30 metres. For typical μ_r values, there is saturation and this gives an induced dipole $p = 4\pi R^3 H$ directed antiparallel to H which produces a magnetic field of approximately $2p/(4\pi z^3)$ at the surface where z is the depth. This gives an anomaly $0.00035H$ which is very small (about 17nT given the UK value of H). Note that a spherical shell of the same outer radius but of thickness 0.05 metre with $\mu_r = 800$ gives 94% of the same surface anomaly with only 9% of the volume, and thus mass, of ferromagnetic material.

What is learnt from these cases, is that the dipole induced by the external field of the earth is modified by the magnetic field produced by that induced dipole itself. This can give rise to a saturation, the induced dipole does not increase further after $\mu_r \approx 10$ in the case of a solid object. For a thin shell, the criterion is that saturation occurs when $t\mu_r >> R$ and this inequality is often not satisfied for ship building plates.

This discussion shows that it is not the volume of ferromagnetic material that determines the induced dipole strength, but rather the spatial extent. So a relatively thin shell can have a similar induced dipole to a solid sphere.

In the case of the sphere and spherical shell, the symmetry of the situation ensures that the induced dipole is aligned with the external field. This is not generally the case, as I illustrate with a discussion of thin plates - see Appendices B3 and B4.

Appendix B2: a cylindrical rod and tube

When a ferromagnetic object has infinite extent in one direction, the problem becomes effectively two-dimensional and thus easier to solve exactly. One application of this is to a long hollow cylinder, which will be a useful model to compare with computer methods to be discussed later.

Consider first an infinite solid rod of radius R with relative permeability μ_r and external magnetic field H perpendicular to the axis. Choose cylindrical coordinates, then the potential inside the rod is (using known solutions of the two-dimensional Laplace equation)

$$\phi = -hr\cos\theta$$

and outside it is

$$\phi = -Hr\cos\theta + \frac{p}{2\pi r}\cos\theta$$

Matching the tangential magnetic field ($H_t = -\frac{\partial\phi}{r\partial\theta}$) and the radial magnetic flux ($B = \mu H_r = -\mu\frac{\partial\phi}{\partial r}$) at $r = R$ gives internal magnetic field

$$h = \frac{2}{\mu_r + 1}H$$

and a dipole strength per unit length, directed antiparallel to the external field H

$$p = 2\pi\frac{\mu_r - 1}{\mu_r + 1}R^2 H \tag{5}$$

For comparison the induced dipole strength from the external field alone is $p = \pi R^2(\mu_r - 1)H$ which gives the same as the exact result above for small $\mu_r - 1$, as to be expected.

Again as μ_r becomes big (> 10), the induced dipole strength saturates and the external magnetic field becomes normal to the cylinder at the surface. As before, a general argument shows why there is a maximum dipole moment induced. A dipole of strength p aligned along the external field H will produce a field of strength $-p/(2\pi r^2)$ transversly to its location and distance r from it. This field is antiparallel to the external field and cannot be such as to reverse it, so $p < 2\pi r^2 H$ where r is the least distance from the dipole induced in the material to the exterior, transversly: namely R in the example above.

This discussion makes it clear that the same reasoning can be applied to a hollow cylinder with the same maximum induced dipole expression, where R is now the outer radius. For a cylindrical shell of 1 mm thick, radius 1 metre with $\mu_r = 999$, one finds this maximum is already nearly attained (83%). So a thin cylindrical shell can have a similar induced dipole moment to a solid cylinder of the same outer radius. This makes it clear that the volume of ferromagnetic material is less important than the spatial extent in regard to the maximum dipole induced.

The exact result is also known in this case (inner radius r, outer R):

$$p = \frac{2\pi(\mu_r^2 - 1)(R^2 - r^2)}{(\mu_r + 1)^2 - (r/R)^2(\mu_r - 1)^2} \tag{6}$$

and again the criterion for saturation is that $t\mu_r >> R$ where the thickness $t = R - r$. In the above numerical example $t\mu_r = 1$ and $R = 1$ (both in metres), so there is partial saturation.

Appendix B3: flat plates

The boundary conditions at a surface between water and ferromagnetic material are that (i) tangential components of the magnetic field H are continuous and (ii) normal components of the magnetic flux $B = \mu H$ are continuous. Ferromagnetic material has a very high permeability μ and this implies that the induced magnetisation is parallel to the surface. For a thin sheet of magnetic material, such as the plate of a ship, the earth's magnetic field will induce magnetic dipoles which have an orientation that lies in the plate.

For example, since the earth's magnetic field lies in direction pointing to magnetic north but downward angled, then a horizontal plate will be magnetised with induced dipoles pointing north, while a vertical plate oriented W-E will have induced dipoles pointing vertically downwards.

The general case is less simple since the induced dipoles will themselves produce a magnetic field which needs to be taken into account consistently. I present a general method for achieving this computationally below in Appendix B4.

I first discuss the relatively simple case of a thin infinite flat plate of width l and thickness t with relative permeability μ_r. If the external magnetic field H is in the plane of the plate and perpendicular to the infinite extent, one can solve the problem analytically though not easily. The key to understanding the situation is that the dipole density, $(\mu_r - 1)H$ per unit volume, induced by H will be oriented along the width direction and be uniform. The field on the plate at distance x from one edge caused by that induced distribution will be given by

$$\frac{1}{2\pi} Ht(\mu_r - 1)(\frac{1}{x} + \frac{1}{l - x})$$

If this field (which is directed against the external field) is too big then the induced dipole distribution will be reduced. The largest effect will be near the edges when $x < t(\mu_r - 1)/(2\pi)$. This can be modelled as an effective reduction in the magnetised length from l to $l - t(\mu_r - 1)/\pi$. For example with $t(\mu_r - 1) = 40$ metres, the length reduction would be approximately 13 metres which is indeed what one gets from the exact solution when $l > 40$ metres. For example, I find, from computer evaluation (with $t(\mu_r - 1) = 40$ metres again, see Appendix B4 for details) that a 100 metre long plate (which would naively have an induced dipole 10 times that of a 10 metre long plate) actually has an induced dipole 40 times bigger (since the self consistency effects reduce the dipole for the 10m plate substantially).

This analysis shows that the largest dipole will be induced

- when the component of the earth's magnetic field tangential to a plate is greatest.
- and when the length of the plate in that direction is longest.

The earth's magnetic field component vertically is 2.36 times as great as that horizontally (north) in UK latitudes. In most shipwrecks the vertical length of plates is relatively short, compared to the horizontal length of plates. The extra length compensates for the lower magnetic field component, this implies that horizontal induced dipoles are likely to be at least as important, if not more so, than vertical ones.

Here I make an estimate of the absolute signal from a shipwreck. Consider a horizontal steel flat plate on the seabed of extent 40 metres by 10 metres and thickness $t = 0.05$ metres (this is thicker than a typical plate, so is supposed to account for several). It has volume 20 m^3 and a mass of about 160 tons. This mass is similar to that of a small coaster. The induced dipole moment is $p = 20(\mu_r - 1)H_N$ where H_N is the component of the earth's magnetic field in the magnetic north direction (so $H\cos(\psi)$). The field produced by such a dipole at the surface (horizontally, pointing south) above the dipole is $p/(4\pi z^3)$ and the anomaly (here a reduction in intensity) will be the component of this along the earth's magnetic field, so $p\cos(\psi)/(4\pi z^3)$. This gives a ratio of the anomaly to the total earth's field intensity of $20(\mu_r - 1)\cos^2(\psi)/(4\pi z^3)$ which gives, setting depth $z = 30$ metres and $\mu_r = 800$, a ratio of 0.007. This is similar to what I expect from experience of actual shipwrecks, namely a signal of around 300nT in a total of 49000nT, giving an observed ratio of 0.006.

A computer evaluation of this case (actually with $(\mu_r - 1)t = 40$ metres, see Appendix B4) gives a peak (actually a trough) anomaly of -372nT if the plate lies N/S and -136nT if the plate lies E/W. The largest signal, here a reduction, is not actually directly above the centre of the wreck but displaced to the north. The smaller signal for a plate lying E/W comes because the self-consistency is more stringent in that case (the end effects are more serious if the length in the appropriate direction is only 10 metres not 40 metres as discussed above). See fig. 2 for an illustration of the surface distribution for the N/S case.

For a vertical plate of the same size and thickness (lying with centre at depth 30 metres, so half under the seabed), the peak anomaly is now positive at 763nT if the plate is N/S and 599nT if it is E/W. These signals are larger since the earth's field has a relatively larger vertical component (if the angle of dip is 67^0).

Even though the earth's magnetic field lies in a (magnetic) north - vertical plane, induced dipoles with a E/W orientation can occur in general. For instance a similar vertical plate lying NE/SW has a net induced dipole in direction $(0.48, 0.48, -0.74)$ so the horizontal component is directed along the direction of the plate - as it must be.

Appendix B4: Self consistent treatment of flat plates

To determine the magnetic anomaly produced by ferromagnetic material, one also has to take into account, consistently, the additional magnetic induction caused by the field produced by the induced dipoles themselves. This is computationally feasible, if one knows the exact location and permeability of all magnetic material. Here I follow a method recommended for use with ships [6].

A thin plate of ferromagnetic material (thin means that the thickness is small compared to other dimensions involved) can be treated relatively easily as a thin shell.

Treating the ferromagnetic shell as a series of elements with surfaces S_i (here taken as flat for simplicity of presentation) labelled $i = 1, \dots n$, one wishes to determine the magnetic dipoles $\mathbf{p_i}$ induced on each element by the external field $\mathbf{H}$. Here the induced dipoles will be treated as located at the centre of each surface element. Since only the tangential component $\mathbf{H_t}$ of the external field contributes, integrating over the surface of the element i gives the induced dipole:

$$\mathbf{p_i} = k \int_{S_i} dS_i \;\; (\mathbf{H_t} - \nabla_\mathbf{t} \phi_i) \tag{7}$$

where $k = (\mu/\mu_0 - 1)t$ represents the thickness of the material t and the relative permeability $\mu_r = \mu/\mu_0$. The second term on the right hand side represents the surface component of the magnetic field created by the induced dipoles themselves. This is the term that I discussed qualitatively in the approximate discussion above in Appendix B3.

Now ϕ_i is the magnetic static potential on the surface element S_i from all the induced dipoles (here treated as pointlike). So

$$\phi_i = \sum_{j=1}^{n} \mathbf{p_j}.\ \nabla G_{ij} \tag{8}$$

Here G_{ij} is the Greens function $(1/(4\pi|\mathbf{r}_i - \mathbf{r}_j|))$ from the source at the centre of element j to a coordinate in element i. Using Gauss' theorem on the surface of element i, one can re-express the second term in eq. 7, since

$$\int_{S_i} dS_i \; \mathbf{n}.\nabla_\mathbf{t} \phi_i = \int_i \mathbf{dl_i}.\mathbf{n} \; \phi_i$$

where $\mathbf{dl_i}$ is a length element of the closed boundary curve encircling S_i and is oriented normal (and outwards) to the curve but tangential to S_i. For a rectangular surface element, the component in the direction ($\mathbf{n}$) of the length of a side will be given by the difference of the integrals along the two sides normal to it. It is convenient to describe the dipole moment $\mathbf{p_i}$ of each element i by its components in the two orthogonal directions of the sides (for a rectangular element).

In this formulation, the unknowns are the $2n$ induced moments p_i (which are tangential to the surface elements so have only two components each). Then one can solve for these induced dipoles p_i by inverting a $2n \times 2n$ matrix and hence determine the anomalous magnetic field $(-\nabla\phi)$ anywhere from them.

I have written computer programs to evaluate these dipoles. As a check one can compare with known cases (such as a cylindrical shell) and one can vary the number of elements used and check for convergence of the result as the number is increased. The codes run almost instantly with hundreds of elements.

The result from one example is shown for the surface magnetic anomaly in fig. 4 and for the surface energy anomaly in fig. 5. This example is a rectangular cross-section tube (8m wide, 3m vertical, 40m long) with open ends and lying horizontally with orientation NW-SE. The thickness t satisfies $t(\mu_r - 1) = 10$m. This was evaluated using a model with many small rectangular elements as described above (96 elements are sufficient).

Appendix C: analysing the surface field

Here I first consider the general case: assuming only that the magnetic dipole sources are located in a limited area near the seabed - so away from the surface. In general, there might also be some geophysical source of magnetism in the rocks under the seabed - this is called a 'magnetic anomaly' on the sea charts. Here I assume that the magnetic dipoles are in a localised region - as would be the case for a shipwreck.

The anomalous magnetic field H at the surface then can be derived from a potential ϕ (where $\mathbf{H} = -\nabla\phi$) which satisfies the Laplace equation ($\nabla.\nabla\phi = 0$). The observed intensity shift δH is given by $\mathbf{h}.\mathbf{H}$ where $\mathbf{h}$ is the (known) direction of the earth's magnetic field.

From the property that $\nabla.\mathbf{H} = 0$, it follows, from the Gauss theorem, that

$$\int dxdy\ H_z = 0$$

Also, since H can be derived from a potential, it follows that

$$\int dx\ H_x = \int dy\ H_y = 0$$

Thus the observed anomaly satisfies

$$\int dxdy\ \delta H = 0$$

This is a useful cross-check on any data taken.

At large distance (r) from the shipwreck, the anomalous magnetic field (given by eq. 1) will decrease as $1/r^3$ or faster. This can also be used to cross-check data.

A very useful way to analyse data is with the two-dimensional Fourier transform:

$$\delta H(u, v, z) = \int dxdy\ e^{-iux-ivy}\delta H(x, y, z)$$

Here u and v will be referred to as wave-numbers. The inverse of this transform is given by:

$$\delta H(x, y, z) = \frac{1}{4\pi^2}\int dudv\ e^{iux+ivy}\delta H(u, v, z)$$

In terms of the potential and its Fourier transform $\phi(u, v, z)$, we have

$$\phi(x, y, z) = \frac{1}{4\pi^2}\int dudv e^{iux+ivy}\phi(u, v, z)$$

and the Laplace equation constraint implies that

$$\frac{\partial^2\phi(u, v, z)}{\partial z^2} = (u^2 + v^2)\phi(u, v, z) \qquad (9)$$

Defining $k^2 = u^2 + v^2$, then $\partial\phi(u,v,z)/\partial z = -k\phi(u,v,z)$ since ϕ must decrease at large values of z. This enables evaluation of the gradient of ϕ and hence the magnetic field associated with it:

$$\mathbf{H}(u,v,z) = (-iu,\ -iv,\ k)\phi(u,v,z)$$

Thus the potential can be extracted from the observed magnetic field anomaly which is measured at the surface (z is the height of the surface above the seabed):

$$\phi(u,v,z) = \frac{\delta H(u,v,z)}{-iuh_x - ivh_y + kh_z}$$

Note that when $u = 0$ and $v = 0$, then $k = 0$ and hence the division is ill-defined. This case corresponds to the sum of δH over all space which must be zero as discussed above. So there is a 0/0 situation: the average value of $\phi(x,y,z)$ over x and y is not determined. This is resolved by the fact that an overall constant change to $\phi(x,y,z)$ is not physically important - only differences of potential values are significant. Moreover, in practice, one may not need to extract the potential itself, but quantities such as magnetic field components derived from differences of it.

In order to illustrate the significance of the two-dimensional Fourier transform, I present the transform of the general case given in eq. 1 in Appendix A. Since the transform of $(x^2 + y^2 + z^2)^{-1/2}$ is $2\pi \exp(-kz)/k$ where $k^2 = u^2 + v^2$, one can evaluate

$$\delta H(u,v,z) = \sum_s \ \mathbf{h}.\mathbf{K}\ \mathbf{p_s}.\mathbf{K}\ e^{-ius_x - ivs_y} e^{-k(z-s_z)}/(2k)$$

where the wave-number vector $\mathbf{K} = (-iu, -iv, k)$ and the sum is over dipoles of strength and direction $\mathbf{p_s}$ located at (s_x, s_y, s_z)

The most noticeable feature of this expression is the factor $\exp(-kz)$ which can be deduced on general grounds from the Laplace equation as shown in eq. 9. This factor controls the rate at which the Fourier transform decreases for large wave-number. Large wave-number is related to finer structure: so this can be seen as the quantitative expression of the fact that the surface distribution will be smoother, the deeper is the wreck. This can be used to smooth data - by requiring that the wave-number distribution falls off in magnitude as $k \exp(-kz)$ at large k.

One can also estimate the range of important wave-numbers since $k \exp(-kz)$ is maximum at $k \approx 1/z$. If a total spatial size $L \times L$ with grid step interval $s \times s$ is used to process the data, the range of sizes of wave-number will be from $2\pi/L$ to π/s. Ideally the minimum non-zero wave number $(2\pi/L)$ should be several times smaller than the typically important wave-number $(1/z)$. This implies $L > 2\pi z$. So for a depth z of 25 metres the box size used should be of linear size greater than 160 metres.

As well as the potential (and derived quantities) at the surface (height z above the wreckage), one can evaluate the potential at a lesser height ($z_0 < z$

and z_0 greater than the vertical coordinate of any part of the wreckage itself) above the wreck since

$$\phi(u, v, z_0) = e^{k(z-z_0)}\phi(u, v, z)$$

Since $k^2 = u^2+v^2$, this will enhance contributions with larger u and v which correspond to more detailed structures in x and y. Thus, at least in principle, one can continue the surface data down to a small height z_0, such that z_0 is above every part of the wreck, which will show up the detailed spatial structure more clearly. This, of course, relies on having sufficient precision in the data taken at the surface. Only in that case will the high wave-number components reproduce the required rate of drop off (as $\exp(-kz)$). As an estimate, if the data are available with a step size of s metres, then the enhancement $\exp k(z - z_0)$ will be $\exp \pi(z - z_0)/s$ for the largest wave-numbers. If $z - z_0 > s$, this large enhancement factor will multiply the very small contribution from such large wave-numbers, but the numerical procedure of obtaining the Fourier transform for such large wave-numbers may introduce errors that de-stabilize the analysis. Hence possible instabilities can arise in continuing to lower depths by an amount greater than the spatial resolution s.

Returning to the discussion of an appropriate combination to evaluate, once the potential has been evaluated, one suitable combination is $\mathbf{H}.\mathbf{H}$, which is proportional to the total anomalous magnetic field energy at the surface. This quantity is less sensitive to the orientation of the magnetic dipoles at the seabed than the observed anomaly itself ($\delta H = \mathbf{h}.\mathbf{H}$). Indeed, from a dipole $\mathbf{p}$ at relative position $\mathbf{R}$,

$$\mathbf{H}.\mathbf{H} = p^2 \frac{3\cos^2\theta + 1}{16\pi^2 R^6}$$

where θ is the angle between the directions of $\mathbf{p}$ and $\mathbf{R}$. Thus the signal is always positive and varies by a factor of, at most, 4 for any θ at fixed distance R. This makes the surface signal easier to interpret, compared to δH which can have sign changes, etc. Furthermore, this quantity will show quite a localised surface distribution: for instance, a point source at depth z will spread out on the surface so that the peak lies above the point source and the signal drops to 50% within a circle of radius $z/2$ (for the case of a vertical dipole).

This can be illustrated with the rectangular tube model of a shipwreck introduced in Appendix B4, yielding the result in fig. 4 for the surface anomaly. For the same model, the surface energy distribution is shown in fig. 5. This is closer to the underlying material distribution but shows a distribution with a maximum which appears less long than the 40m of the model, because of saturation effects as discussed in Appendix B3. One can, in principle as discused above, also deduce the magnetic energy distribution on a level which is below the surface. In fig. 6 this distribution is shown for a level 5 metres above the seabed for the same model. This illustrates that the shape of the model is now more accurately reproduced.

As a practical example, I show the distribution of $\mathbf{H}.\mathbf{H}$ on the surface for the wreck of the *Ystroom* in fig. 9. This shows a distribution lying NW-SE, in agreement with the result of comparing models with the observed surface anomaly distribution.

Another combination, $\sqrt{\mathbf{H}.\mathbf{H}}$, has an even smaller dependence on the orientation of seabed dipoles (a factor of 2 at maximum) and would be a possible quantity to present. For a dipole at the seabed, the combination $\mathbf{H}.\mathbf{H}$ has a more peaked spatial distribution at the surface, and this is why I favour it.

Using the above formalism to evaluate $\mathbf{H}.\mathbf{H}$ at a lesser height (z_0) above the seabed, gives rather noisy and unstable results (unless very accurate data exist or unless the data are processed taking account of the expected decrease with wave number) if evaluated more than a few metres below the surface, as expected from the discussion above.

Appendix C1: deducing the bottom distribution

Here I consider a simple model: all the ferro-magnetic material lies at the same depth below the surface (z) and it is all magnetized along the same direction $\mathbf{a}$. Then $\mathbf{p}(\mathbf{s}) = \mathbf{a}p(s)$ where $p(s)$ is the dipole strength.

This assumption could be appropriate to a wreck consisting of a scattered collection of iron cannon balls, for example. They will then all be magnetised along the earth's magnetic field direction which is the special case that the direction $\mathbf{a}$ is that of the earth's magnetic field itself ($\mathbf{h}$).

From Appendix A the observed deviation in magnetic field intensity will be given by eq. 1

$$\delta H(x,y) = \sum_s (3\mathbf{h}.\mathbf{R}\ \mathbf{p}(\mathbf{s}).\mathbf{R} - \mathbf{h}.\mathbf{p}(\mathbf{s})\ R^2)/(4\pi R^5)$$

where one introduces a magnetic dipole density $\rho(s_x, s_y)$ to describe the seabed distribution and makes use of the fixed direction of the induced dipoles. So

$$\delta H(x,y) = \int ds_x ds_y\ (3\mathbf{h}.\mathbf{R}\ \mathbf{a}.\mathbf{R} - \mathbf{h}.\mathbf{a}\ R^2)\rho(s)/(4\pi R^5)$$

This expression has the form of a convolution

$$\delta H(x,y) = \int ds_x ds_y\ G(x - s_x, y - s_y)\rho(s_x, s_y)$$

where G is a known function since z has been fixed at the depth below the surface and $\mathbf{h}$ and $\mathbf{a}$ are both known. Our aim is to measure $\delta H(x,y)$ and thence deduce $\rho(s_x, s_y)$. This can be achieved by taking the 2-dimensional Fourier transform of the expression, using

$$G(u,v) = \int dxdy\ e^{-iux-ivy} G(x,y) = \mathbf{h}.\mathbf{K}\ \mathbf{a}.\mathbf{K}\ e^{-kz}/(2k)$$

$$\delta H(u,v) = \int dxdy\ e^{-iux-ivy} \delta H(x,y)$$

Then $\rho(u,v) = \delta H(u,v)/G(u,v)$ and so one can reconstruct

$$\rho(s_x, s_y) = \frac{1}{4\pi^2} \int dudv\ e^{ius_x + ivs_y} \rho(u, v)$$

However, as noted in the previous section, the correction $1/G(u, v)$ will be very large for large wave number (k) because of the exponential factor $\exp(kz)$. This will render the approach unstable unless very precise data are available to give an accurate Fourier transform of $\delta H(x, y)$.

Appendix C2: Return to pole derivation

At the magnetic north pole, where the magnetic field is vertical, the magnetic intensity pattern on the surface above a magnetic item can be interpreted more easily. An isolated magnetised region will show an enhancement directly above it and a decrease all around. So, roughly, the area of increased magnetic intensity will correspond to the presence of iron below. This makes interpretation of the surface anomaly easier.

However, Britain is not at the north magnetic pole. With some key assumptions, it is possible to process data so that they are in the form they would take if they were measured at the north pole. This is known as 'Return to pole'.

As discussed in Appendix A, the magnetic field intensity change observed at the surface will be given in terms of the magnetic scalar potential V from a (hypothetical) unit point magnetic source at $\mathbf{s}$ by

$$\delta H(x, y) = -\mathbf{h}.\nabla_r \sum_s \mathbf{p}(\mathbf{s}).\nabla_s\ V(\mathbf{R})$$

with $\mathbf{R} = \mathbf{r} - \mathbf{s}$ where $\mathbf{r} = (x, y, z)$ is the coordinate of observation and $\mathbf{s}$ is the location of the magnetic dipole source (of strength and direction given by $\mathbf{p}(\mathbf{s})$.

The key assumption is that the magnetic dipoles induced are all aligned along the earth's magnetic field direction given by $\mathbf{h}$. Again (as in Appendix C1) this assumption is most appropriate for a scattered collection of wreckage - rather than a ship with iron plates. So $\mathbf{p}(\mathbf{s}) = \mathbf{h}p(s)$ where $p(s)$ is the dipole strength at $\mathbf{s}$ and $\mathbf{h}$ is independent of $\mathbf{s}$. One may include this factor of $p(s)$ within the sum over sources defining a scalar potential Φ (note not the same as ϕ introduced in the introduction to Appendix C): $\Phi = \sum_s p(s) V(\mathbf{R})$

Now consider the Fourier transform (in the two horizontal spatial coordinates x and y of $\mathbf{r}$). Here z is the vertical coordinate of $\mathbf{r}$.

$$\Phi(x, y, z) = \frac{1}{4\pi^2} \int dudv\ e^{iux + ivy} \Phi(u, v, z)$$

Since the potential satisfies Laplace's equation $\nabla.\nabla\Phi = 0$. This implies that

$$(u^2 + v^2)\Phi(u, v) = \frac{\partial^2 \Phi(u, v)}{\partial z^2} = k^2 \Phi(u, v)$$

This allows the gradients to be evaluated, giving the magnetic field intensity anomaly

$$\delta H(x, y, z) = \frac{1}{4\pi^2} \sum_s \int dudv\ ((-iu, -iv, k).\mathbf{h})^2 p(s) e^{iux + ivy} \Phi(u, v, z)$$

In contrast the field at the surface when the earth's magnetic field is vertical (at the pole) is given by $\mathbf{h} = (0, 0, 1)$. Hence

$$\delta H_{pole}(x, y, z) = \frac{1}{4\pi^2} \sum_s \int dudv\ k^2 p(s) e^{iux+ivy} \Phi(u, v, z)$$

Within our assumptions, these two expressions are related by a factor of $k^2/(-ih_x u - ih_y v + h_z k)^2$ where $k^2 = u^2 + v^2$. This factor is independent of the source $\mathbf{s}$ and so one does not need to know the distribution of magnetic material in order to make the correction.

Thus the u, v Fourier transform of the measured intensity $\delta H(x, y, z)$ can be corrected by this factor where $\mathbf{h}$ is known. For simplicity, neglecting the magnetic deviation and choosing y as latitude, $h_x = 0$, $h_y = \cos(\psi)$, $h_z = -\sin(\psi)$ where ψ is the angle of dip.

Appendix D: Interpolating point data

Assume one has logged data (such as the magnetic intensity at the surface) at a set of locations. Data values are $v(x_i, y_i)$ with $i = 1, \ldots n$ representing the n values logged at locations (x_i, y_i). One needs to interpolate these values for several purposes: to make a Fourier transform, to draw a smooth surface on a plot, etc.

One straightforward way to do this is to make a weighted average of the data.

$$v(x, y) = \frac{1}{W} \sum_{i=1}^{n} v(x_i, y_i) w(x - x_i, y - y_i)$$

with $W = \sum_{i=1}^{n} w(x - x_i, y - y_i)$.

A sensible weight to use is one that emphasises close values preferentially:

$$w(x - x_i, y - y_i) = \frac{1}{((x - x_i)^2 + (y - y_i)^2)^p}$$

Choosing $p = 1$ corresponds to using an inverse square distance as weight. Choosing very large p corresponds to using the nearest data point. A reasonable compromise occurs using an inverse power such as $p = 2$.

Note that this approach is appropriate for interpolating data but it will not reproduce the property that a magnetic field intensity should drop off with a known power (the inverse cube) at large distances unless this condition is explicitly imposed.

A more sophisticated way to interpolate data is to introduce a triangular network in x, y and use planar interpolation (in x, y, v) in each such triangle. The usual choice of triangles is that corresponding to the Delaunay triangulation (this has no other vertex points in the circumcircle of any triangle). To set up such a triangulation, one must avoid repeated vertices and sets of three or more vertices lying on a line. It can also be helpful to add a few vertices near the boundary (with v value zero) if there are not measurements in that region.

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